Multi-Channel Electrocardiogram Denoising Using a Bayesian Filtering Framework

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Abstract

In some recent works, model-based filtering approaches have been proved as effective methods for extracting ECG signals from single channel noisy recordings. The previously developed methods, use a highly realistic nonlinear ECG model for the construction of Bayesian filters. In this work, a multi-channel extension of the previous approach is developed, by using a three dimensional model of the cardiac dipole vector. The results have considerable improvement compared with the single channel approach. The method is hence believed to be applicable to low SNR multi-channel recordings.

1. Introduction

In some recent works the quasi-periodic behavior of ECG signals has been used to set up a model-based Bayesian filtering framework for ECG denoising [1, 2, 3]. The thereby presented filtering procedures are based on a nonlinear dynamic model originally presented in [4], for the generation of synthetic ECG signals. The dynamic model was later modified and extended to multi-channel ECG recordings of adults and fetal-maternal mixtures [5]. This extension was based on the idea that different ECG recordings can be assumed as noisy projections of the electrical potentials of the heart onto the recording electrode axes. In fact, each of the multi-channel ECGs can be assumed as observations of the hidden states of the cardiac dynamic system. To implement this idea, the dynamics of the cardiac electrical activity was modeled by a rather general 3-dimensional (3D) nonlinear dynamic model, based on the single dipole model of the heart. In this work, the idea of the 3D ECG model is combined with the Bayesian filtering framework presented in [2, 3], to develop a multi-channel ECG denoising framework. It will be shown how the state and observation equations required for Bayesian filters such as the Extended Kalman Filter (EKF) and the Extended Kalman Smoother (EKS) can be extracted from the 3D ECG model, and a given set of noisy recordings. The proposed method is expected to lead to superior results compared with the previous single channel denoising, as we are using additional observation channels. Hence the method is believed to have interesting applications in low SNR conditions such as the extraction of fetal ECG from maternal abdominal recordings, or the removal of ECG artifacts from multi-channel MRI signals.

2. Dipole theory of the heart

The electrical activity of the heart has been modeled with various methods ranging from *single dipole models* (SDM), to *activation maps* [6]. Among these methods, the simplest and yet the most popular is the SDM which is a far field approximation of the cardiac potentials and the ECG and Vectorcardiogram (VCG) are based on it. It is believed that the SDM explains 80%–90% of the representation power of the body surface potentials [7, 8]. According to the SDM, the cardiac electrical activity may be represented by a time-varying rotating vector $\mathbf{d}(t)$ located at the center of the heart. By assuming the body volume conductor as a passive resistive media which only attenuates the source field, any ECG signal is a linear projection of $\mathbf{d}(t)$, onto the direction of the recording electrode axes.

3. A 3D synthetic ECG model

In this section we briefly review a recently developed 3D synthetic ECG generator which is based on the single dipole model of the heart. The model is later used to develop the denoising filters.

3.1. Synthetic cardiac dipole generation

In [5], a synthetic multi-channel ECG generator has been presented. This model is inspired from the original work in [4] and later modified in [2, 3], which suggest the use of Gaussian mixtures to achieve realistic synthetic ECG signals. Considering the Cartesian representation of the dipole vector $\mathbf{d}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$, the model presented in [5] suggests that the x, y, and z coordinates of $\mathbf{d}(t)$ may be modeled with the following dynamic model:

$$
\begin{aligned}
\theta_{k+1} &= (\theta_k + \omega\delta) mod(2\pi) \\
x_{k+1} &= \sum_i \frac{-\delta\alpha_i^x\omega}{(b_i^x)^2}\Delta\theta_i^x exp[\frac{-(\Delta\theta_i^x)^2}{2(b_i^x)^2}] + x_k + \eta^x \\
y_{k+1} &= \sum_i \frac{-\delta\alpha_i^y\omega}{(b_i^y)^2}\Delta\theta_i^y exp[\frac{-(\Delta\theta_i^y)^2}{2(b_i^y)^2}] + y_k + \eta^y \\
z_{k+1} &= \sum_i \frac{-\delta\alpha_i^z\omega}{(b_i^z)^2}\Delta\theta_i^z exp[\frac{-(\Delta\theta_i^z)^2}{2(b_i^z)^2}] + z_k + \eta^z
\end{aligned}
\tag{1}
$$

where δ is the sampling period, $\Delta\theta_i^x = (\theta - \theta_i^x) mod(2\pi)$, $\Delta\theta_i^y = (\theta - \theta_i^y) mod(2\pi)$, $\Delta\theta_i^z = (\theta - \theta_i^z) mod(2\pi)$, and $\omega = 2\pi f$, where f is the beat-to-beat heart rate, and η^x, η^y, and η^z are random additive noises which model the inaccuracies of the dynamic model. Accordingly, the first equation in (1) generates a circular trajectory rotating with the frequency of the heart rate. Each of the three coordinates of the dipole vector $\mathbf{d}(t)$, is modeled by a summation of Gaussian functions with the amplitudes of α_i^x, α_i^y, and α_i^z; widths of b_i^x, b_i^y, and b_i^z; and located at the rotational angles of θ_i^x, θ_i^y, and θ_i^z. The summations in (1) are taken over the number of Gaussian functions used for modeling the shape of the desired dipole, which is not necessarily the same in all the x, y, and z directions. This model of the rotating dipole vector is rather general, since any continuous function, can be modeled with a sufficient number of Gaussian functions.

3.2. Synthetic ECG generation

The dynamic model in (1) is a representation of the dipole vector of the heart (or equivalently the orthogonal VCG recordings). In order to relate this model to realistic multi-channel ECG signals recorded from the body surface, we can use the following simplified linear model:

$$\mathbf{ECG}(t) = H \cdot \mathbf{s}(t) + \mathbf{v}(t), \tag{2}$$

where $\mathbf{ECG}(t)_{N\times1}$ is a vector of the ECG channels recorded from N leads, $\mathbf{s}(t)_{3\times1} = [x(t), y(t), z(t)]^T$ contains the three components of the dipole vector $\mathbf{d}(t)$, and $H_{N\times3}$ is generally a time-variant matrix corresponding to the body volume conductor model and the scalings and rotations of the dipole vector[1]. $\mathbf{v}(t)_{N\times1}$ is the noise vector of the N ECG channels at the time instance of t.

The detection of the R-peaks is a rather typical procedure in ECG analysis. As noted in [3], the R-peaks can be used for the synchronization of the θ_k parameter of the dynamic model with a desired ECG signal. We can in fact, generate a coarse estimate of θ_k, by using the R-peaks as a reference [2, 3].

[1]We have slightly simplified the ECG model presented in [5], for the implementation of the Bayesian filter.

4. Bayesian filter design based on the 3D ECG model

The theoretical backgrounds of Bayesian filters such as the Kalman filter and its extensions, are well studied in the context of estimation theory [9, 10]. In this section it is shown how the synthetic ECG models in (1) and (2) can be used within the Bayesian filtering framework.

4.1. The filter equations

In order to use the Bayesian filter notations, the dynamic model of the cardiac dipole vector in (1) may be represented in a more compact form:

$$
\begin{cases}
\theta_{k+1} = F_0(\theta_k, \omega) \\
x_{k+1} = F_1(\theta_k, x_k, \omega, \alpha_i^x, \theta_i^x, b_i^x, \eta^x) \\
y_{k+1} = F_2(\theta_k, y_k, \omega, \alpha_i^y, \theta_i^y, b_i^y, \eta^y) \\
z_{k+1} = F_3(\theta_k, z_k, \omega, \alpha_i^z, \theta_i^z, b_i^z, \eta^z)
\end{cases}
\tag{3}
$$

where F_0, F_1, F_2, and F_3 correspond to the nonlinear equations presented in (1). For using vector notations, we define the state and observation vectors as follows:

$$
\begin{aligned}
\mathbf{X}_k &\doteq [\theta_k, x_k, y_k, z_k]^T \\
\mathbf{Y}_k &\doteq [\phi_k, \mathbf{ECG}_k^T]^T
\end{aligned}
, \tag{4}
$$

where $\mathbf{ECG}_k$ is the vector of ECG observations at time k, and ϕ_k is a coarse estimate of θ_k, generated from the R-peaks [3]. Finally, the dynamic model noise and observation noise vectors are respectively defined as follows:

$$
\begin{aligned}
\mathbf{W}_k &\doteq [\omega, \alpha_i^x, \alpha_i^y, \alpha_i^z, \theta_i^x, \theta_i^y, \theta_i^z, b_i^x, b_i^y, b_i^z, \eta^x, \eta^y, \eta^z]^T \\
\mathbf{V}_k &\doteq [u_k, \mathbf{v}_k^T]^T,
\end{aligned}
\tag{5}
$$

where u_k is the phase noise due to the uncertainty in the R-peak detection and $\mathbf{v}_k$ is a vector of the ECG observation noises of each channel. All the variables in the definition of $\mathbf{W}_k$ can generally vary with time and the i indexes range over all the number of Gaussians kernels used for modeling the desired waveform. With these definitions, the vector representation of the state dynamics defined in (3), and the observation equations of the Bayesian filter are as follows:

$$
\begin{aligned}
\mathbf{X}_{k+1} &= \mathbf{F}(\mathbf{X}_k, \mathbf{W}_k) \\
\mathbf{Y}_k &= \left[\begin{array}{c|c} 1 & \mathbf{0}^T \\ \hline \mathbf{0} & H \end{array}\right] \cdot \mathbf{X}_k + \mathbf{V}_k
\end{aligned}
\tag{6}
$$

where $\mathbf{F}$ is a vector representation of (3), and H is the $N\times3$ transformation matrix defined in (2). Due to the nonlinearity of the $F_i(\cdot)$'s, linearized versions of them are required for the EKF. The linearizations are the 3D extension of the ones presented in [2, 3], which we shall skip for brevity.

4.2. Parameter selection

An important issue in the design of Bayesian filters is the appropriate selection of the statistical properties of the state variables and noise parameters. The Kalman filter and its extensions are based on the assumption of Gaussian random variables, which means that only up to the second order statistics of the parameters – namely the mean values and covariance matrices – are required. The procedure of the selection of these parameters are explained in details in [2, 3], for the one-dimensional model. A similar approach is used for the hereby presented 3D model, by assuming the state variables and noise parameters of the x, y, and z coordinates to be independent. We can summarize the procedure as follows:

1. Detect the R-peaks of the ECG recordings. These peaks are used to generate the coarse phase measurements ϕ_k.
2. Using the R-peaks, find the average and standard deviation of the ECG waveform through synchronous averaging.
3. By using a nonlinear least square estimation, the mean ECG waveform is used for the extraction of the mean values of the Gaussian kernel parameters defined in (1) [11].
4. The standard deviation of the average ECG waveform is used to find the entries of the covariance matrices of the dynamic model noise $Q_k = E\{\mathbf{W}_k\mathbf{W}_k^T\}$, and measurement noise $R_k = E\{\mathbf{V}_k\mathbf{V}_k^T\}$. We assume all the noise parameters to be Gaussian and uncorrelated, which means that Q_k and R_k are simplified to diagonal matrices.

Note that for online applications or denoising long set of ECG recordings, we can update the dynamic model parameters and the covariance matrices in time, by calculating them from local averages of the ECG recordings.

In the presented 3D model, the definition and calculation of H requires further explanations. Since the SDM is a far field model of the cardiac electrical activity, and the fact that there is no direct means of measuring the true cardiac potentials, there is an intrinsic ambiguity in the definition of H. This problem may be overcome, if we consider that although the matrix H is based on the SDM, we do not need to have the true dipole vector for its calculation. In fact, by using the SDM we are implicitly approximating the cardiac potentials within a 3D space, meaning that any set of multi-dimensional ECG recordings may be mapped to a set of three linearly independent recordings (as with the Dower transformation and its inverse [12]). With these explanations we can consider the $N \times 3$ matrix H as a means of transforming any set of three linearly independent recordings $\mathbf{VCG}(t)_{3\times1}$ to N-channel ECG recordings $\mathbf{ECG}(t)_{N\times1}$. This transformation may be found by using a MMSE estimation as follows:

$$\hat{H} \doteq \underset{H\in R^{N\times3}}{argmin} \|\mathbf{ECG}(t) - H \cdot \mathbf{VCG}(t)\|, \quad (7)$$

which leads to:

$$\hat{H} = E\{\mathbf{ECG}(t)\cdot\mathbf{VCG}(t)^T\}\cdot E\{\mathbf{VCG}(t)\cdot\mathbf{VCG}(t)^T\}^{-1}, \quad (8)$$

where $E\{\cdot\}$ represents time averaging of the whole data or just the most recent ECG cycles, for long term recordings. With this interpretation of H, the parameters of the Gaussian terms in (1) can be directly calculated from the Frank lead VCG recordings.

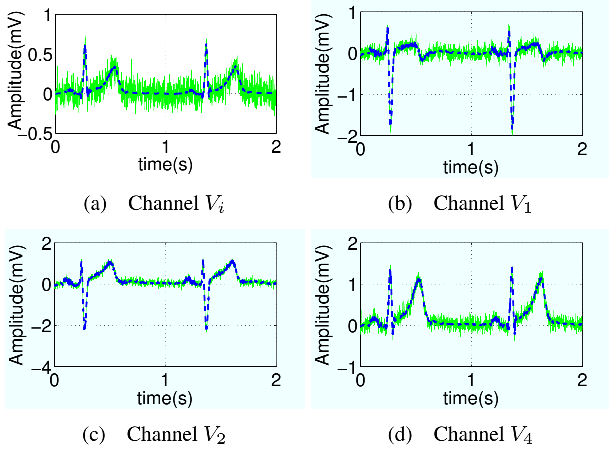
Note that although the presented model is based on the SDM, which is a coarse approximative model, the advantage of the method is that the model uncertainties are considered in the entries of the covariance matrices Q_k and R_k [1, 3].

5. Results

The MIT-BIH PTB Diagnostic ECG Database was used for the evaluation of the proposed method. This database contains the standard 12-lead ECG recordings and the 3 Frank leads. A typical noise-free 30 second segment was visually selected from this database for the evaluation of the method. According to the explanations of previous sections, the optimal parameters of (1) were found from the 3 Frank lead electrodes, by using 5 Gaussian kernels to model each of the Frank lead signals. The H matrix was calculated from the whole 30 seconds of the data using (8). The calculation of the other parameters of the model and Q_k and R_k were done according to the methods previously presented in [3]. To study the performance of the filters, white noise was artificially added to the multichannel ECG recordings and the state variable of (3) corresponding to the VCG channels were estimated, by using the EKF and EKS. Later the denoised state-variables were transformed back to the ECG signals by using the H transformation. Typical results of this procedure which have been achieved by adding white noise with the standard deviation of 0.2mV to each of the ECG channels may be seen in Fig. 1. The pre-filter (SNR_0) and post-filter (SNR_{EKF} and SNR_{EKS}) SNRs can be seen in Table 1 for all the 15 ECG channels, which show considerable improvement compared with the single channel approach (SNR_{prv}).

6. Discussion and conclusions

In this paper, a multi-dimensional extension of a recently developed ECG denoising framework was presented. The proposed method was based on a 3D dynamic model of the cardiac dipole vector. It was discussed that the multi-channel body surface ECG recordings may be considered as projections of the cardiac dipole (or equivalently the standard VCG recordings), onto the recording electrode axes. So within Bayesian filtering frameworks



(a) Channel V_i (b) Channel V_1

(c) Channel V_2 (d) Channel V_4

Figure 1. The noisy (solid line) and EKS denoised (dashed line) signals from some of the ECG channels.

Table 1. The pre-filter (SNR_0) and post-filter (SNR_{EKF}, SNR_{EKS}) SNRs of different channels in dB using the EKF and EKS, vs. the previous single clannel EKS approach (SNR_{prv}). The noisy signals used for the calculation of SNR_0 have been obtained by adding white noise with the standard deviation of 0.2mV to each of the ECG leads.

	Lead	SNR_0	SNR_{prv}	SNR_{EKF}	SNR_{EKS}
Standard 12-leads	V_i	-5.1	7.8	15.8	16.9
	V_{ii}	2.5	9.1	14.7	16.2
	V_{iii}	-1.5	7.9	12.1	13.7
	A_{vr}	-0.6	10.3	15.8	17.2
	A_{vl}	-10.9	8.5	8.4	10.0
	A_{vf}	0.6	10.9	13.7	15.2
	V_1	2.7	12.5	14.4	15.8
	V_2	7.0	15.0	15.5	17.3
	V_3	5.4	13.8	15.3	17.1
	V_4	4.3	13.1	14.9	16.2
	V_5	3.6	12.8	15.3	16.3
	V_6	-0.8	10.1	15.0	16.1
Frank leads	V_x	9.9	16.8	16.7	18.0
	V_y	-2.6	5.5	5.8	7.1
	V_z	8.4	15.9	14.9	16.5

such as the EKF and EKS each ECG recording can be considered as an additional observation of the rotating dipole vector, which can be used in the filtering procedure to improve the estimation performance. The method is hence believed to have interesting applications in low SNR conditions such as the extraction of fetal ECG from maternal abdominal recordings, or the removal of ECG artifacts from multi-channel MRI signals, where conventional filtering schemes do not have satisfactory performance.

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