ECG Denoising with Adaptive Bionic Wavelet Transform

Omid Sayadi [1], Mohammad Bagher Shamsollahi [2]

Abstract **- In this paper a new ECG denoising scheme is proposed using a novel adaptive wavelet transform, named bionic wavelet transform (BWT), which had been first developed based on a model of the active auditory system. There has been some outstanding features with the BWT such as nonlinearity, high sensitivity and frequency selectivity, concentrated energy distribution and its ability to reconstruct signal via inverse transform but the most distinguishing characteristic of BWT is that its resolution in the time–frequency domain can be adaptively adjusted not only by the signal frequency but also by the signal instantaneous amplitude and its first-order differential. Besides by optimizing the BWT parameters parallel to modifying a new threshold value, one can handle ECG denoising with results comparing to those of wavelet transform (WT). Preliminary tests of BWT application to ECG denoising were constructed on the signals of MIT-BIH database which showed high performance of noise reduction.**

Index Terms **– Bionic wavelet transform, Denoising, ECG signal.**

I. INTRODUCTION

The electrocardiogram (ECG) is a time-varying signal reflecting the ionic current flow which causes the cardiac fibers to contract and subsequently relax. The surface ECG is obtained by recording the potential difference between two electrodes placed on the surface of the skin. A single normal cycle of the ECG represents the successive atrial depolarization/repolarization and ventricular depolarization /repolarization which occur with every heartbeat [1]. These can be approximately associated with the peaks and troughs of the ECG waveform labeled P, Q, R, S, and T as shown in Fig. 1(a). Since ECG is mostly contaminated with noise, see Fig. 1(b), extracting useful clinical information from the real noisy signal requires reliable signal denoising techniques.

Extending 1D-adjustable-resolution to 2D-adjustable-resolution for better tradeoff between time and frequency resolutions has been a topic in WT for a long time. Many efforts focus on applying the wavelet transform (WT) to the signal. But research has proven that the choice of mother wavelet function dramatically affects the appearance and quality of the resultant time–frequency representation. In this case attempts have been made introducing adaptivity into WT. Applications of AWT in bio-signal processing, however, are limited by the facts that no entropy criterion is known to be appropriate for biomedical applications and that the computation cost of AWT is usually very high [2].

[1] O. Sayadi , M.Sc. BME., is with Sharif University of Technology, Tehran, Iran. (e-mail: osayadi@ee.sharif.edu).
[2] M.B. Shamsollahi, PhD., is with the Biomedical Engineering Department of Sharif University of Technology, Tehran, Iran. (e-mail: mbshams@sharif.ir).

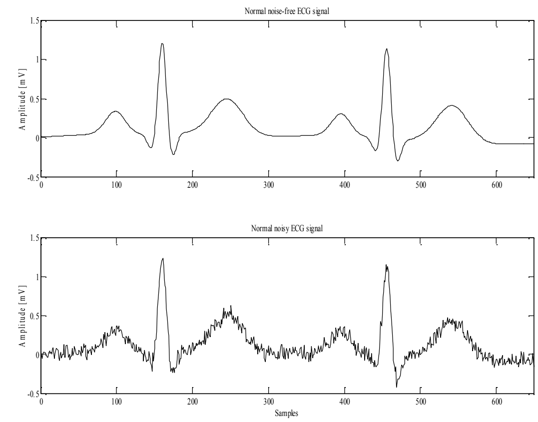


Fig. 1. Human's ECG Signal: (a) Normal noise-free (b) Normal noisy ECG.

Among these methods, BWT, introduced by Yao [2], is mainly developed and being optimized by the human bio-system and has showed promising results in speech processing. In this paper we attempted to apply BWT with new modifications to be properly adjusted for ECG processing.

The paper is organized as follows. Section II summarizes the physiological mechanisms underlying the cardiac cycle and reviews the morphological features, which is reflected in the ECG signal. Section III provides backgrounds on human's auditory model and relates it to the invention of BWT. In section IV BWT is optimized for ECG signal analysis. Section V briefly talks about the denoising technique. Finally, summary and conclusions are provided in section VI.

II. ECG MORPHOLOGY

Each beat of the heart can be observed as a series of deflections away from the baseline on the ECG. These deflections reflect the time evolution of electrical activity in the heart which initiates muscle contraction. A single normal cycle of the ECG, corresponding to one heartbeat may be divided into the following sections:

P is a small low-voltage deflection away from the baseline caused by the depolarization of the atria prior to atrial contraction as the activation (depolarization) wave-front propagates from the SA node through the atria. QRS-complex is the largest-amplitude portion of the ECG, caused by currents generated when the ventricles depolarize prior to their contraction. Although atrial repolarization occurs before ventricular depolarization, the latter waveform (i.e. the QRS-complex) is of much greater amplitude and atrial repolarization is therefore not seen on the ECG. Finally the T-wave is the result of ventricular repolarization, whereby the cardiac muscle is prepared for the next cycle of the ECG [1].

III. Definition of Bionic Wavelet Transform According to Human's Auditory Model

Previously Yao had adopted a nonlinear auditory model [6], in which each point of basilar membrane is modeled by the following equation:

$$\ddot{d}(x,t)+R_{eq}(x,d)\dot{d}(x,t)/L(x)+\omega_0^2(x)d(x,t)=P \quad (1)$$

where

$$R_{eq}=R(x)-G_1(x)\frac{d_{1/2}}{d_{1/2}+|d(x,t)|}R(x) \quad (2)$$

Clearly from (1) a point of basilar membrane is modeled by a BPF that has a nonlinear damping. Furthermore It has been shown that existing a nonlinear compliance is also needed. This way the resulting quality factor of filter-bank is given by:

$$Q_{eq}=R_{eq}^{-1}\sqrt{L(x)/C_{eq}(x)} \quad (3)$$

To introduce this active mechanism into WT, it is sufficient to replace the constant Q_0 of WT by a variable Q_T [3]. In fact passing from WT to BWT is done using a T-function adopted from the ear model. This is the function which brings adaptiveness into the new transform. Equation (4) shows the relation between quality factors for the ear model:

$$Q_{eq}=\left(1-G_2\frac{d_{1/2}}{d_{1/2}+|d|}\right)^{-1}(1+G_2\,|\partial d/\partial t|)^{-1}Q \quad (4)$$

In order to replace the constant quality factor with a variable adaptive quality factor, one can make changes in the mother function of the wavelet transform. Here, the admissible condition for *h(t)* implies that the mother wavelet function has some oscillations. This oscillation can be represented as:

$$h(t)=\frac{1}{\sqrt{a}}\tilde{h}(t)\exp(j2\pi f_0 t) \quad (5)$$

where f_0 is the center frequency of $h(t)$ and $\tilde{h}(t)$ is its envelope function. So the BWT mother function can be defined as follows:

$$h_T(t)=\frac{1}{T\sqrt{a}}\tilde{h}\left(\frac{t}{T}\right)\exp(j2\pi f_0 t) \quad (6)$$

Now with the new mother function, the definition of BWT is:

$$BWT_x(\tau,a)=\int x(t)\,h_T^*\left(\frac{t-\tau}{a}\right)dt \quad (7)$$

$$=\frac{1}{T\sqrt{a}}\int x(t)\,\tilde{h}^*\left(\frac{t-\tau}{aT}\right)\exp\left(-j2\pi f_0\left(\frac{t-\tau}{a}\right)\right)dt$$

One can find that the relation between quality factors of WT and BWT by simply looking at the Fourier transform of their mother functions which is simply stated as $Q_T=TQ_0$. By comparing this relation with equation (4) the T-function is given by:

$$T(\tau+\Delta\tau)=\left(1-\tilde{G}_1\frac{BWT_s}{BWT_s+|BWT_x(\tau,a)|}\right)^{-1}$$

$$\times\left(1+\tilde{G}_2\,|\partial BWT_x(\tau,a)/\partial t|\right)^{-1} \quad (8)$$

In the above equation, $\tilde{G}_1$, $\tilde{G}_2$ and BWT_s are constants. $BWT_x(\tau,a;h)$ is the BWT coefficient at time τ and scale a and $\Delta\tau$ is the calculation step. If the signal and its first-order differential are continuous, BWT can reconstruct the original signal without distortion [2].

IV. Bionic Wavelet Transform Optimization for ECG Analysis

From the definition of BWT there is a major difference in resolution of time-frequency span of analyzing windows. In fact in the WT, for a fixed mother function, all the windows in a certain scale along the t-axis are fixed and the window size of the WT (see the dotted windows) varies with the change of analyzing frequency. However, both the time and frequency resolutions can be different in the BWT even in a certain scale. The adjustment of the BWT resolution in the same scale is controlled by T-function, which is related to the signal instantaneous amplitude and its first-order differential [2].

Fig. 2 shows the time frequency representation for the ECG shown in Fig.1 with both WT and BWT. Notice the smoothing in the BWT representation which is the direct result of windows changes over certain scales.

It only remains to set the BWT parameters efficiently so that it can decompose the signal into finite number of scales and after by, determine the most energetic ones and choose a global or local threshold. In order to optimize the BWT parameters we have used a semi-optimal method considering both analytic and morphological aspects of the analyzed signal. As we are considering ECG signal, we should be aware of its variability.

May be the most important feature for an ECG signal is the frequency range in which its main components occur. Although there are some other components like VLPs, we have focused our interest on P, Q, R, S and T. The resulting frequency range is up to 100 Hz.

Let f_0 be the initial center frequency of the mother wavelet. In [2] it has a value equal to 15165.4 Hz and as the scale goes higher and higher, the center frequency will decrease in the following way:

$$f_m=\frac{f_0}{q^m} \qquad ,q>1 \quad (9)$$

For ECG we do not need such high f_0, so we optimized it simply by running the program for different values of f_0 and then minimizing the gradient of error variance by comparing the results-numerically and morphologically-with each other. It has been found that if the center frequency lies in the range of 360 to 500 Hz there would be no much distortion on the analyzed ECG. Here we have chosen f_0=400 which yields satisfactory results.

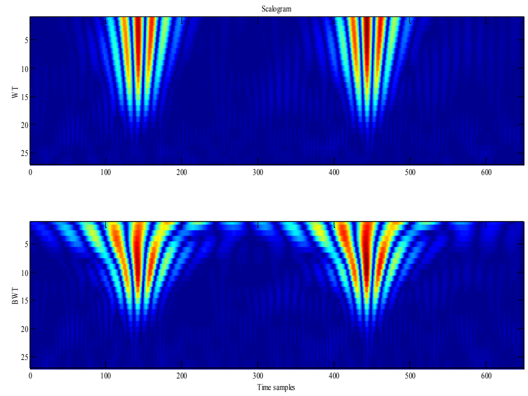


Fig. 2. Time-frequency representation of the ECG signal in Fig. 1: (a) WT (b) BWT.

In our method q is not a global constant but for each signal and for each time-frequency window it has a constant value. Other parameters are exactly the same as what was stated in [7] which used BWT for speech enhancement and denoising. These are $\widetilde{G}_1 = 0.87, \widetilde{G}_2 = 45, BWT_s = 0.8$.

Finally, the calculation step is determined due to the sampling frequency. If we let f_s be the sampling frequency then the step will be $\Delta\tau = 1/f_s$.

V. Denoising Technique

After BWT optimization, the denoising technique illustrated in Fig. 3 is used to reduce the amount of noise contamination in the ECG signal. In implementation, BWT coefficients can be easily calculated based on corresponding WT coefficients by:

$$BWT_x(\tau, a; T) = K \times WT_x(\tau, a) \tag{10}$$

where K is a factor depending on T [2]. Especially, for the real Morlet function $h(t) = e^{-(t/T_0)^2}$, which is used as the mother function in our experiments, where , K is equal to [4] :

$$\frac{\int_{-\infty}^{+\infty} e^{-t^2} dt}{\sqrt{(T/T_0)^2 + 1}} \approx \frac{1.7725}{\sqrt{(T/T_0)^2 + 1}} \tag{11}$$

Here we have used Donoho's proposed approach for denoising including two major categories, hard thresholding and soft thresholding. The choosing of the threshold value can be determined in many ways. Donoho derived the following formula based on white Gaussian noise assumption:

$$thr = \sigma\sqrt{2\log_2^N} \tag{12}$$

where thr is the threshold value, N is the length of the noisy signal, and $\sigma = AMFS/0.6745$, with $AMFS$ denoting the absolute median estimated on the first scale of the bionic wavelet coefficients. We have used the following threshold which is a new modification of (12). Let $T_{fs}(i)$ be the value of T-function in the i-th step of computing. The threshold is formulated as:

$$thr = \frac{\sigma}{\sum_i \alpha_i T_{fs}(i)}\sqrt{2\log_2^N} \tag{13}$$

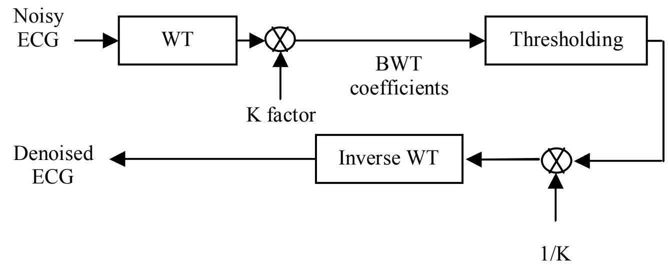


Fig.3. The block diagram of the bionic wavelet transform denoising technique.

In fact a weighted average of T-function values in the first scale of BWT is added to (12) to get better results. After thresholding, the coefficients of BWT are divided by K factor and by taking the inverse WT transform, the denoised version of signal will be reconstructed.

VI. Discussion and Conclusion

To show that BWT is appropriate for ECG denoising we have used two types of ECG signals, both simulated and real ones. We used the MIT-BIH database as the reference for our real signals. The signals were decomposed using BWT up to 30 scales. We have used Morlet wavelet as our wavelet function and its support length is chosen as [-4,4], and 2.5π is chosen as its oscillatory frequency [7].

As mentioned before, for denoising two kinds of thresholding methods, hard and soft, were applied. Besides, for easier comparison we have applied the WT-based denoising with Daubechies wavelets to each signal. Fig. 4 shows some typical results. One can see that in many cases, hard thresholded signal is more similar to the original ECG, which is due to the intrinsic smoothness in BWT. But there are cases like Fig. (4) in which soft thresholded signal has better quality. So it is thoroughly case dependent.

Simulation results provide supportive evidence to claim that BWT has some advantages over the traditional WT. First, BWT has higher sensitivity so it is more probable for WT to have little single-noise-samples remained (refer to Fig. 4(a)). Second, BWT has a smoothing property with respect to its resolution variation over the time-frequency plane, and this is exactly what we are seeking in many denoising techniques, especially true for real ECGs (see Fig. 4(b)). And finally with the appropriate choices for the level of decomposition, i.e. number of scales, and the center frequency, removing other interferences such as powerline or 60 Hz interference is a direct task. To clarify this, we have chosen a real ECG signal (signal 228 of MIT-BIH-Fig. 4(c)) and we have denoised it with both WT and BWT. It is obvious that in WT-based denoised signal (see the second signal) the 60 Hz interference still exists, but in the denoised signal with BWT (see the last signal) baseline wander is completely removed. Of course the penalty is a little loss of smoothness.

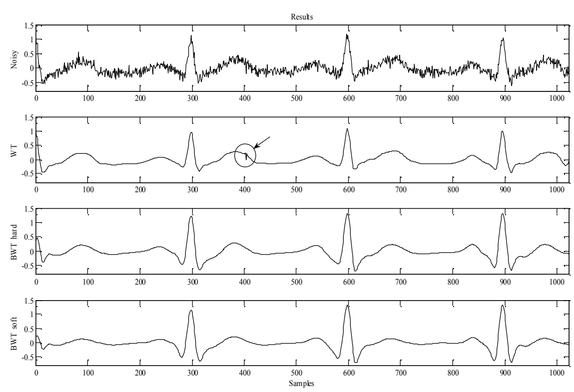
For evaluating the performance of the proposed BWT we have used the SNR improvement measure by the means of the expression:

$$imp[dB] = SNR_{output} - SNR_{input} = 10\log\left(\frac{\sum_i |x_d(i) - x(i)|^2}{\sum_i |x_n(i) - x(i)|^2}\right)$$

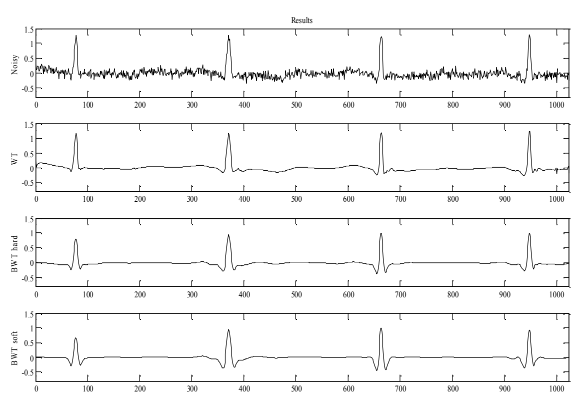
where x denotes the clean ECG, x_d is the denoised signal and x_n represents the noisy ECG signal. Fig 5 compares the improvement values between WT and BWT. For evaluation, both hard and soft thresholding are considered. One can see that in lower input SNRs, soft WT has a better performance. But as the input SNR is increased the BWT improvement increases until in SNRs higher than 8dB, soft BWT passes the WT performance. Besides, for lower SNRs, hard BWT has better performance than hard WT.

In summary, the BWT is adapted for the ECG signal, and a new threshold selection rule is proposed which leads to results comparable to those of WT. The advantages of BWT-based ECG denoising are summarized below:

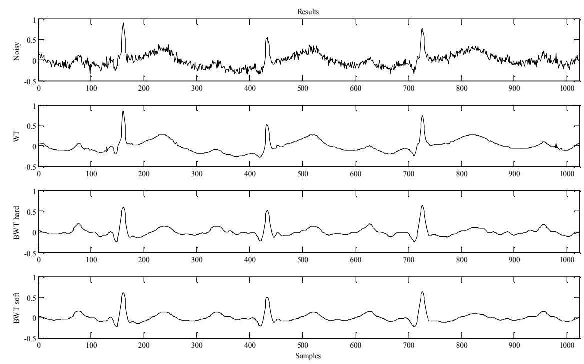
1) BWT denoised signal is a smoothed version.
2) Single artifacts do no longer exist.
3) Interference removal is achieved by properly adjusting the center frequency of mother function and the number of decomposition levels.
4) For higher input SNR more improvement is obtained.



(a)



(b)



(c)

Fig. 4. ECG denoising examples with WT and BWT : (a) Simulated ECG (b) real ECG (c) real ECG with baseline wander. In each figure, the first one is the noisy signal, the second one is the denoised signal with WT – hard thresholding and the last two ones are the denoised signal with BWT-hard and soft thresholding.

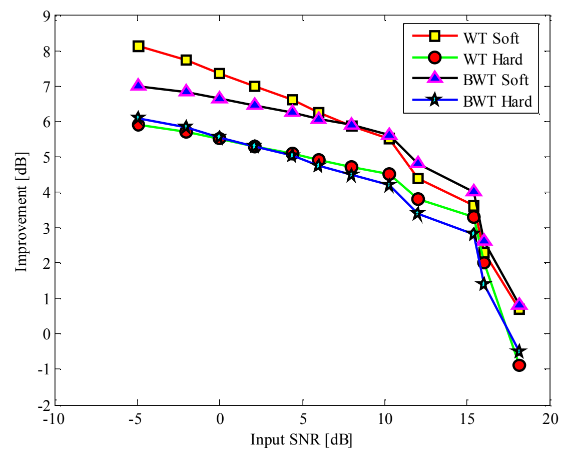


Fig. 5. Improvement vs. input SNR for WT and BWT denoising.

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