Resistivity Parameters Estimation Based on 2D Real Head Model Using Improved Differential Evolution Algorithm

Ying Li, Guizhi Xu, Lei Guo, Lei Wang, Shuo Yang, Liyun Rao, Renjie He, and Weili Yan

***Abstract*—The improved Differential Evolution (DE) algorithm is proposed in this paper to s olve the resistivity parameters estimation problem based on 2D real head model. Our simulations demonstrate that the improved DE algorithm is robust in obtaining high quality reconstruction, and the convergence is much faster than the usual DE algorithm. Furthermore, the selection of the amplification parameters is much easier.**

I. Introduction

THE impedance information of a subject is important in many researches of biomedical engineering. Electrical impedance tomography (EIT) [1], [2] is a useful technique for reconstructing the impedance distribution in the subject from the voltages measured on the surface given the injected current. As a typical inverse problem, EIT imaging is intrinsic non-linear and ill-posed. Usually the solutions can be described in terms of some unknown parameters, and the task is to find the optimal parameters estimation that will cause a minimum error of the cost function, which is always as some sorts of residuals between the measured data and theoretic data.

Differential Evolution (DE) algorithm [3]-[5] is a relative newer evolutionary approach for minimizing possibly nonlinear and non-differentiable functions in continuous space. Since it is first introduced by Price and Storn in 1995, amount of studies demonstrated that it converges faster and with more certainty than many other global optimization methods. Furthermore, it is robust, simple in use, and requires only a few control parameters.

In this paper, we applied DE algorithm and its improvement to resistivity parameters estimation of 2D head section based on real head model. The simulations demonstrated that DE algorithm has the ability for obtaining high quality reconstructions, and the improved DE algorithm has the better performances than the usual DE algorithm.

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II. Resistivity Parameters Estimation Problem

The mathematical model of EIT is given by Maxwell's equations. Since the sufficiently low frequency of the electrical current being applied, the displacement current can be ignored, and only the resistivity (or conductivity) should be considered.

Generally, resistivity parameters estimation using EIT is presented as follows

$$\min f(\boldsymbol{\rho}) = \min\left[(\boldsymbol{u}(\boldsymbol{\rho}) - \boldsymbol{u}_0)^{\mathrm{T}} (\boldsymbol{u}(\boldsymbol{\rho}) - \boldsymbol{u}_0)/2\right] \tag{1}$$

where $\boldsymbol{\rho}$ is the unknown resistivity distribution vector in the object, $\boldsymbol{u}_0$ is the vector of measured voltages on the boundary, and $\boldsymbol{u}(\boldsymbol{\rho})$ is the vector of computed peripheral voltages with respect to $\boldsymbol{\rho}$, which can be obtained using finite element method (FEM).

In the view of mathematics, EIT inverse problem is to finding the parameters in an *m*-dimensional space, where *m* is the freedom of the resistivity distribution. However, since the impedance distribution for lots of biomedical subjects is piecewise constant, such as in the head, we take the piecewise constant model with *D* parts, assuming each part is of constant conductivity. That simplifies the inverse problem into optimization with much fewer parameters, and makes it possible to use the evolutionary optimization methods [5], [6].

III. Improved Differential Evolution Algorithm

As with all evolutionary optimization algorithms, DE operates on a population of candidate solutions, rather than a single solution only. DE's self-referential population reproduction scheme is different from traditional evolutionary algorithms, and results in an adaptive scheme with very good convergence properties.

A. Usual DE Program

In particular, DE maintains a population of constant size that consists of *NP* real-valued vectors, $x_{i,G}$, $i = 1,2,\cdots,NP$, where *i* indicates the index of population and *G* is the generation the population belongs to.

The program of usual DE algorithm is as follows:

1) Initialization: The initial vector population is generated randomly in the entire parameter space. Usually, the unknown parameters are subject to lower and upper boundary constraints, $x_j^{(L)}$ and $x_j^{(U)}$, respectively, that means

$$x_j^{(L)} \le x_j \le x_j^{(U)},\ j = 1,2,\cdots,D \tag{2}$$

Therefore we set the initial population with random values that meet the given boundary constraints:

$$x_{ji,0} = x_j^{(L)} + rand_j[0,1]\cdot(x_j^{(U)} - x_j^{(L)}) \tag{3}$$

where $i = 1,2,\cdots,NP$, $j = 1,2,\cdots,D$, $rand_j[0,1]$ denotes a uniformly distributed random value within range [0,1] that is chosen anew for each *j*.

2) Mutation: DE generates new parameter vectors by adding the weighted difference between two population vectors to a third one.

For each target vector $x_{i,G}$ ($i = 1,2,\cdots,NP$), a mutant vector of basic DE is generated according to:

$$v_{i,G+1} = x_{a,G} + F\cdot(x_{b,G} - x_{c,G}) \tag{4}$$

where random indexes $a,b,c \in \{1,2,\cdots,NP\}$, are integers, mutually different, and are also chosen to be different from the running index *i*. Amplification parameter $F \in (0,2]$ is a real factor, which controls the amplification of the differential variation $(x_{b,G} - x_{c,G})$.

There are some other variants of mutation operation [3]-[5], and in this paper we use the following form:

$$v_{i,G+1} = x_{best,G} + F\cdot(x_{a,G} - x_{b,G}) \tag{5}$$

where $x_{best,G}$ is the best member of the current population.

3) Crossover: The mutated vector's parameters are then mixed with the parameters of another predetermined vector, the target vector, to yield the so-call trial vector. The trial vector is formed as $w_{i,G+1} = (w_{1i,G+1}, w_{2i,G+1},\cdots, w_{Di,G+1})$, where

$$w_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } (randb(j) \le CR) \text{ or } j = rnbr(i) \\ x_{ji,G} & \text{if } (randb(j) > CR) \text{ and } j \ne rnbr(i) \end{cases} \tag{6}$$

where $j = 1,2,\cdots,D$, $i = 1,2,\cdots,NP$, *randb*(*j*) is the *j*-th evaluation of a uniform random number generator with outcome [0,1], $rnbr(i) \in 1,2,\cdots,D$ is a randomly chosen index, which ensures that $w_{i,G+1}$ gets at least one parameter from $v_{i,G+1}$. Parameter $CR \in [0,1]$ is the crossover constant, which has to be determined by the user.

4) Selection: If the trial vector $w_{i,G+1}$ yields a lower cost function value than the target vector $x_{i,G}$, the trial vector replaces the target vector in the following generation; otherwise the old vector $x_{i,G}$ is retained. Each population vector has to serve as the target vector once so that *NP* competitions take place in one generation.

5) The iteration will end when the objective function of the best vector reaches a predetermined value or the evolution reaches the preset generations.

6) Selection of the control parameters: In general, the population size *NP* is choosing from 5*D* to 10*D*, and remains constant during the search process. Parameters *F* and *CR* are usually fixed during the search process. They affect the convergence velocity and robustness of the search process. Suitable values for *NP*, *F* and *CR* can be found by trial-and-error.

B. *Improved DE Algorithm*

In the usual DE algorithm, the amplification parameter *F* always be a constant during the evolution process and must be determined before the evolution. Lots of experiments must be taken to find the suitable value of *F*. In order to deduce user's interferer, we proposed a relative simple improved method to decide the value of amplification parameter *F*.

In the improved DE algorithm, the amplification parameter *F* is selected as follows

$$F_G = rand_G(0,2] \tag{7}$$

where $G = 1,2,\cdots,G_{\max}$, $G_{\max}$ is the maximum generation for convergence, $rand_G(0,2]$ denotes a uniformly distributed random value within range (0,2] that is chosen anew for each generation *G*.

Therefore, it's not necessary to decide the value of the amplification parameter before evolution.

IV. Results of Simulations

In practice, the anatomical prior information can be used in the resistivity parameters estimation problem. In the reconstruction procedure, the boundaries between regions of different resistivities are first determined using information obtained from other high-resolution imaging techniques such as MRI, CT etc; then the resistivity values are determined.

Since the head is usually divided into scalp, skull, cerebrospinal fluid (CSF) and brain, we consider the piecewise constant model with 4 parts, so we have an inverse problem with 4 parameters.

The 2D real head model is used in this paper. The domain is meshed into triangular elements based on the anatomical information of the head from MRI scanner, as shown in Fig. 1, and the numbers of nodes, elements and electrodes are 482, 892 and 16, respectively. For simulation, the resistivity values of scalp, skull, CSF and brain are (2.27, 55, 0.56, 4) $\Omega\cdot\mathrm{m}$.

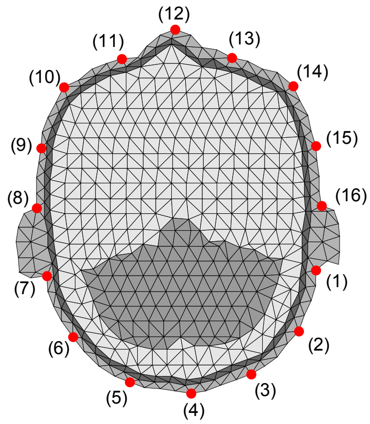

Fig. 1. 2D real head model from MRI scanner

To evaluate the quality of simulation results, the total error (TE) is defined as

$$\text{TE}=\sqrt{\frac{\sum_{i=1}^{m}(\rho_i^{\text{estimated}}-\rho_i^{\text{true}})^2}{\sum_{i=1}^{m}(\rho_i^{\text{true}})^2}}\times 100\% \qquad (8)$$

where the superscript "estimated" and "true" denote the reconstruction values and true values of resistivity, respectively, and m is the freedom of resistivity distribution.

In our study, the bipolar injected current patterns are adopted and the potential differences are computed among all electrodes with respect to one reference. The lower and upper boundary constraints of resistivity values are set to be 0.8 and 1.2 times of the corresponding true resistivity values, respectively. We choose NP=10D and CR=0.9 after some tests. The search process stops when the value of objective function is less than 10^{-6}, or the maximum number of generations reaches 200.

In the usual DE algorithm, we choose F=0.9 after several tests; and in the improved DE algorithm, F is random selected within range $(0,2]$ for each generation.

The comparison results are shown in Table I, where 'Ng' is the abbreviation of number of generation for converging. From Table I, we found that both of the usual DE algorithm and the improved DE algorithm have the ability for high quality reconstruction of resistivity parameters, but the evolution velocity of usual DE algorithm is much slower than the improved DE algorithm.

The evolution processes of the simulated experiments are shown in Fig. 2, where the x-coordinate denotes the iteration number, and the y-coordinate denotes the objective function values in log scale of the best individual in each generation. Fig. 3 shows the corresponding total error values in log scale versus the iteration number. In both Fig.2 and Fig. 3, the dash-dot lines denote the results of the usual DE algorithm, and the solid lines denote the results of the improved DE algorithm. From the figures we can see that in the usual DE, the evolution always pauses in some steps, whereas the random selection of the amplification parameters makes the evolution much quickly.

TABLE I
THE COMPARISON RESULTS OF IMPROVED DE AND USUAL DE

	Estimated Value ($\Omega\cdot$m)				Ng	TE ($\times 10^{-5}$)
	Scalp	Skull	CSF	Brain		
Usual DE	2.2700	55.0001	0.5599	3.9993	87	2.2512
Improved DE	2.2699	54.9998	0.5600	4.0003	40	0.6948

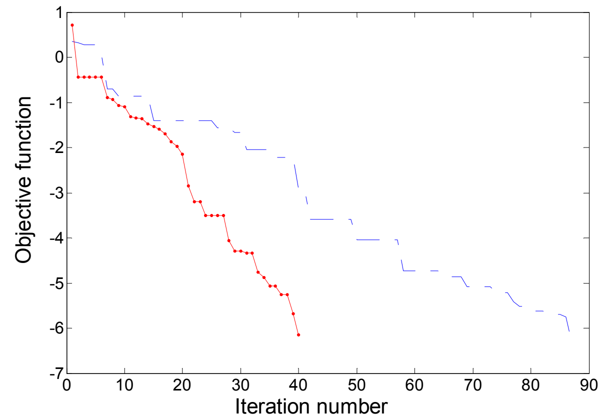

Fig. 2 Comparison of objective function using usual DE and improved DE

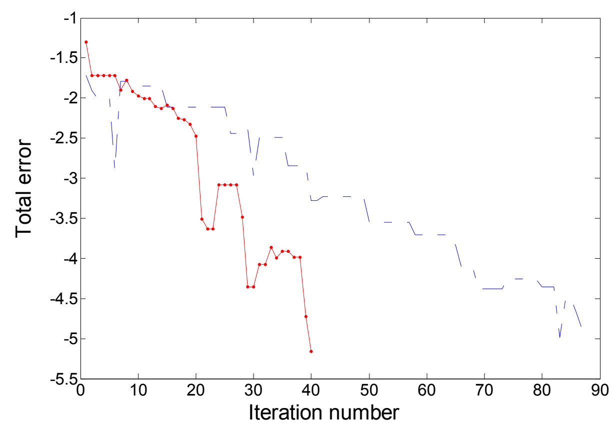

Fig. 3 Comparison of total error using usual DE and improved DE

V. CONCLUSION AND DISCUSSION

In this paper, we proposed an improved DE algorithm and applied it to resistivity parameters estimation problem based on EIT technique. The 2D real head model was used. The simulation results demonstrated that the improved DE algorithm has the superior performance for high quality reconstruction of resistivity parameters. Comparing with the usual DE algorithm, it convergent much faster, and the selection of amplification parameter is much easier.

Furthermore, the improved DE algorithm is hopeful in applying to other optimization problems.

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