Localization of an Equivalent Central Cardiac Electric Dipole for Electrocardiography Applications

C. G. Xanthis, D. G. Drogoudis, P. M. Bonovas, D. Mikroulis and G. A. Kyriacou

Abstract— **An efficient and robust method for the solution of the non-linear and ill-posed inverse problem of electrocardiography is presented. The hearts activity is modeled by a central cardiac electric dipole, which within the present work is allowed only to rotate about a fixed origin. For this purpose a three-dimensional volume conductor model of the human body is constructed based on a classical anatomic atlas. This is excited by an assumed (initial guess) dipole located at the center of the heart. In turn a Least squares optimization scheme is employed, aiming at the matching of the potential distribution calculated on the torso surface to the corresponding distribution measured with the aid of multiple electrodes. The efficiency of the method stems from the employment of arbitrary shaped hexahedral elements within the finite element method for the minimization of the required computational resources while the model realistically reflects the body internal structure. Finally, the algorithm is successfully tested using measured data available online**

I. Introduction

The objective of electrocardiography in general is the qualitative and quantitative representation of the heart's electrical activity exploiting the information provided by the potentials recorded at the body surface. The inverse electrocardiography (ECG) in particular, refers to a pre-determined modelling of the cardiac activity by a variety of equivalent electric sources as a single or double moving/rotating electric dipole, multiple fixed location dipoles, the epicardial potential distribution and the activation of isochrones on the heart surface, e.g. [1].

The aim of the inverse problem of electrocardiography is to restore the heart activity from a given set of body surface potentials. This problem is ill-posed and its solution is generally unstable (especially when multiple source models are employed), i.e. an arbitrary small change of the body surface potentials can cause an arbitrary large change of the equivalent source solution. The measures to stabilize the solution are called regularization and employ a priori information about the functionality of the heart. The solution of the inverse problem provides the researcher with detailed information about the electrophysiological heart activity of a patient and reduces the need for catheter measurements.

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In the dipole approach the activity of the heart is represented by one or two moving-rotating current dipoles. The basic underlying principle is to select the amplitudes and coordinates of these dipoles within an appropriate model of the torso such that calculated torso-surface potential distribution closely matches the measured body-surface-potential distribution. This source model was used by Gulrajani et al, [2, 3], in some earlier investigations and Guard et al, [4]. Also Armoundas et al. [5], and H Bruder et al, [6] used the single moving dipole to simulate the electrical activity of the heart. There are many other research groups, [7,8] in the inverse electrocardiography field which aim at the definition of epicardial potential distribution either by using realistic geometry anisotropic heart models or trying to exploit a priory information. The research status up to 1998 is given in the review paper [1].

A common characteristic of these inverse problems is their ill-posed nature, where this difficulty becomes worst when the number of unknown parameters is increased. Namely, the problem is best conditioned when a single cardiac equivalent electric dipole is considered, which in turn involves a compromise in modeling accuracy. The solution of this inverse problem is generally based on a "volume conductor model" representing the whole body, which enables the solution of the forward problem. Preferably this model should retain high spatial resolution around the heart. The assumed equivalent electric source for heart activity is modeled and the generalized Laplace or Poisson equation (forward problem) is solved to obtain a "calculated data set" for the body surface potentials. At this point a multiple lead (e.g. 24 electrodes or more) electrocardiographer is required to facilitate the "measured data set" on the actual human subject to be diagnosed. In turn an inverse problem solution algorithm like Newton's, Newton-Raphson e.t.c can be employed for the minimization of a cost function, usually in the least squares means. Moreover, like most inverse problems this is a non-linear one. So, the initially assumed equivalent electric dipole parameters (e.g. 3 dipole moment components and 3 dipole coordinates) are iteratively updated until the differences between the measured and calculated data sets become comparable to an accepted error tolerance. The latter may be defined by the measurements errors and their noise as well as the computer modeling inaccuracies.

In the present study, an algorithm for the solution of the inverse ECG problem is proposed. For this purpose, a simple as far as possible, but anatomically realistic torso model has been constructed (which enables the solution of the forward problem) and a single dipole with fixed location (at the center of the heart) has been considered as the basic volume source model. Finally, the inverse problem is based on the Least-Squares (LS) Optimization and the nonlinearity is handled by the implementation of the Levenberg – Marquardt method, [18,19].

It should be pointed out that while the present effort is focused on the localization of a single rotating and not moving electric dipole, the method is general and can be extended t more general source distributions. Beginning this research effort from the single-fixed location dipole constitute the most convenient approach for the establishment of this inverse problem solution. Moreover, the main strength of our approach stems from the appropriately developed body volume conductor model. This reflects the complicated internal structure at the expense of a lot of manual preprocessing efforts for the definition of the appropriate general hexahedral elements. This yields a model of about 10000 elements instead of 400000 to one million cubic elements models based on MRI imaging, which are usually employed by other research groups. Our approach enables the effective solution of the inverse problem which requires multiple solution of the forward problem. To realize how important this is, just recall that the inverse problem non-linearity is handled through iterative solution of a version linearized around a continuously updated solution. One may argue that this approach involves a compromise in the forward problem accuracy, well this is generally true but the effort is directed toward limiting the inaccuracies within the measurements errors. Moreover, a significant effort is devoted toward making the forward problem adaptive to the specific Human subject on which measurements are performed. Also, the currently established model matches the internal structure of a standing or sitting person. Differently modified models for the subjects lying on their left and their right side as asked by medical examinations, will be developed next.

Finally, a series of successfully estimated rotating dipoles tracking the hearts temporal behavior will be presented. High spatial density measured body surface potentials available online, MacLeod et al [20] are exploited for this purpose.

II. Formulation of Forward Problem

A. Torso Model – FEM solution

A three dimensional (3-D) volume conductor modelling is required, which must also be a realistic one, namely accurately reflecting the internal geometrical body structure, in order to get the desirable agreement wit the measurements. From previous works the accurate modelling of human torso needs 400.000 cubic elements or more [9,10]. In order to use a model in inverse problems it is imperative to minimize the elements number. Since, the complexity is mostly due to the curved boundaries between the tissues, the number of elements can be drastically reduced when arbitrary shaped elements are used.

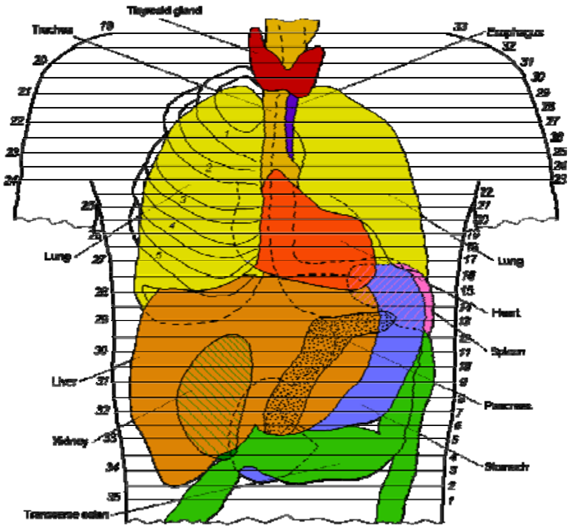


Fig. 1. Vertical view of the torso model, on the left the atlas [11] numbering of cross-sections is given and ont he right is the present numbering.

The present model is based on the Eycleshymer and Shoemaker anatomic atlas [11]. It is constructed into 32 layers (33 cross sections) consisting of totally 9.727 general hexahedral elements and 8.800 nodes [12, 13]. The model starts from the 35th cross section of [11] passing through the 4th waist vertebra and ends upon the 19th cross section passing through the 3rd vertebra of the neck as shown in the vertical cross-section of Fig.1 We renumbered these cross sections from 1 to 33, starting from the bottom just for convenience. The actual distance between successive cross-sections of [11] is 1inch=2.54cm. In order to increase the finite Element method accuracy along the vertical direction (to make it similar to the horizontal one) an additional cross section is inserted between each two successive ones. The tissues boundaries on these new cross sections are approximated through linear interpolation of the original ones above and below the new one. In turn this increases their total number to 33, reducing together the vertical distance to ½ inch (1.27 cm). One of the horizontal cross-sections passing through the heart is shown in Fig.2, this is denoted as section 26 (or as 19th) in Fig.1. For the application of the FEM, to the human torso 3-D model, each cross-section of [11] is subdivided into quadrilaterals following the geometrical boundaries between different tissues. The nodes of the corresponding quadrilaterals of successive cross-sections are properly joined to form the general hexahedral element. Special attention was devoted to the avoidance of badly distorted 3-D elements which causes FEM errors or convergence difficulties during the solution of the resulting system of equations. Such element distortion may occur when the top quadrilateral surface of the element is excessively shifted and/or rotated with respect to its bottom surface (in this case very narrow or very wide angles are formed between the edges of the element). The physiological tissue conductivity values assigned to each general hexahedral element were adopted from Geddes and Baker, [14, 15].

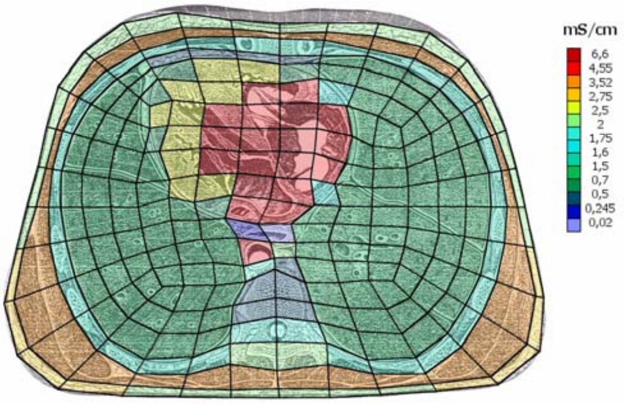


Fig. 2. Cross section 19 passes through the heart.

A trilinear interpolation function for the potential V within each element and a transformation mapping the arbitrary shaped element to the corresponding isoparametric one (2x2x2 cube) is considered. The differentiations and integrations involved in the element matrix calculation are carried out in the transformed domain according to Akin [16]. Furthermore, the finite element analysis techniques are always based on an integral formulation. Therefore the accuracy of the integration of the element matrices is quiet important. Since, the finite element method requires a large amount of integration, it is imperative that one obtain the greatest possible accuracy with the minimum cost. The most accurate numerical method in ordinary use is the Gauss quadrature formula. Since the time required, and thus the cost, it is a function of n^{SPACE} [16] it is desirable to keep n as small as possible where space=3 for 3-D elements. During all investigations n was set equal to 3.

B. Dipole modeling

The model of the equivalent electric dipole for cardiac electric activity consists of six parameters: 3 for dipole coordinates (the dipole origin) and 3 for the dipole moment components Equivalently, a dipole moment $\bar{P} = I \cdot l\hat{p}$, namely with amplitude oriented along an arbitrary direction, is considered as the source exciting the volume conductor model (Fig.3a). Thus, for modeling purposes, dipole in a 3-D space can be represented by a point current source and point sink providing I and –I A, separated by an infinitesimal distance ℓ, as shown in Fig.3b. The Poisson equation for a 3-D space excited by dipole source, which follows from the continuity equation [17] is:

$$\begin{aligned}\nabla\cdot\{\sigma(\boldsymbol{r})\nabla V(\boldsymbol{r})\} &= g(\boldsymbol{r}) = \\ &= \lim_{\ell\to 0} I\left\{\delta(\boldsymbol{r}-\boldsymbol{r}^+)-\delta(\boldsymbol{r}-\boldsymbol{r}^-)\right\}\end{aligned} \tag{1}$$

Where $\delta(\boldsymbol{r}-\boldsymbol{r}^{\pm})$ denotes the delta function centered at $\boldsymbol{r}^{\pm}$ which defines the points of current source and sink (Fig.3b). The above differential equation is also subject to a homogeneous Neumann boundary condition over the body surface (body-air interface) as:

$$\frac{\partial V(\boldsymbol{r})}{\partial \hat{n}} = 0 \tag{2}$$

where $\hat{n}$ is the outward unit vector normal to the body surface when FEM is used for the solution of the generalized Poisson equation the variational principle is usually employed. According to that, the solution of equation (1) subject to the boundary conditions (2) reduces to a functional minimization, e.g. [12],[16]. In turn the inhomogeneous body conductivity is approximated with piece-wise constant distribution, where the conductivity over each FEM element is assumed homogeneous but having different value in each element. In view of this, the integration over the whole solution domain involved in the minimization is reduced to a sum of integrals over each element. This approach is known as “master matrix assembly” and more details on a very effective scheme can be found in our previous work [12]. Focusing on the dipole modeling, for which the method proposed in [17] is employed, let us repeat the linear system of resulting from the minimization – integration over specific element containing the dipole, which results:

$$[\mathrm{K}]^e \cdot [\mathrm{V}]^e = [\mathrm{G}] \tag{3}$$

where $[\mathrm{K}]^e$ is an 8x8 matrix that comes from the left hand side of (1), and $[G]$ is a vector that comes from the integration of right hand side of (1) after multiplying with the element's interpolation functions Hi. According to [17] the excitation with a dipole source is equivalent to the application of currents Ii on the 8 nodes of this element which are actually given by the 6-vector as:

$$\begin{aligned}\mathrm{I}_i &= [G_i] = \\ &= -\lim_{\ell\to 0} I \int_V \left\{\delta(\boldsymbol{r}-\boldsymbol{r}^+)-\delta(\boldsymbol{r}-\boldsymbol{r}^-)\right\}\boldsymbol{H}_i(\boldsymbol{r})\,dv = \\ &= -I\ell(\nabla\boldsymbol{H}_i)\cdot\hat{p}\end{aligned} \tag{4}$$

where Hi denotes the element's interpolation functions which refers to node–i (Fig.3c). In turn, these Ii values will act as the excitation in the final system of equations to be obtained from the master matrix assembly [12, 16]. Interested readers may conduct [12, 13] for further details and particularity for the system solution for which a Gauss-Seidel scheme with an over-relaxation factor is employed.

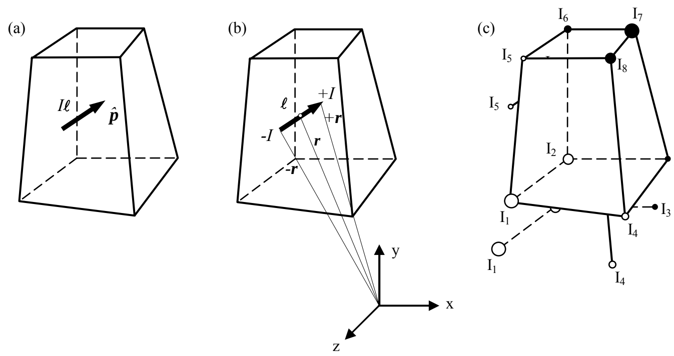


Fig. 3. a) The original 3-D dipole source inside a general hexahedral element. The dipole has amplitude Il and direction p. b) The representation of the dipole as a current source and sink. c) The 8 nodal current values according the dipole.

III. Formulation of Inverse Solution

The solution of the inverse problem is usually based on iterative solutions of the forward problem. Namely, for a given geometry, a given conductivity distribution and a specified internal source, calculate the body surface potentials. In contrary for the inverse problem the body surface potentials are measured, the conductivity and geometry are assumed known (physiological values) and the internal source is sought.

In order to confront the ill-posed nature of the inverse ECG problem, which is accompanied by a non-uniqueness of the solution, a few constraints have to be imposed to minimize the number of solutions. One of these constraints can include source model constraints. Namely, the problem is best conditioned when a single cardiac equivalent electric dipole with fixed location is considered. For this sake, the number of parameters of the cardiac dipole has been restricted from the possible six (3 dipole moment components and 3 coordinates of dipole origin) to three arbitrary moment components with fixed origin. While its location has been placed at the center of the heart [5], the magnitude and orientation of this dipole are variables used to minimize the error of fitting the recorded data in the least square sense. Namely, the inverse problem is based on the Least-Squares (LS) Optimization and the nonlinearity is confronted by the implementation of the Levenberg – Marquardt method.

The required measured data set can be obtained from measurements on electrodes positioned equidistantly over the body surface (measured data - V). The algorithm starts with a guess for an equivalent dipole in the center of the heart and the body surface potentials are obtained (calculated data – U), solving the forward problem. Thereafter, the Levenberg – Marquardt optimization scheme is employed for the final equivalent dipole estimation. So, the initially assumed equivalent electric dipole parameters (e.g. 3 dipole moment components and fixed location) are iteratively updated until the differences between the measured and calculated data sets become comparable to an accepted error tolerance. The least squares method minimizes the summed square of residuals. The residual for the i_{th} data point r_i is defined as the difference between the measured potential value V_i and the calculated potential value U_i, and is identified as the error associated with the data:

$$r_i = V_i - U_i \tag{5}$$

The summed square of residuals is given by

$$F = \sum_{i=1}^{n} r^2 = \sum_{i=1}^{n} (V_i - U_i)^2 \tag{6}$$

where n is the number of data points (electrodes on the body surface) included in the fit and S is the sum of squares (SSQ) error estimate.

The environment of the inverse problem algorithm is programmed in Matlab, while the computationally demanding procedures are implemented as Fortran subroutines. So presently, for the Levenberg-Marquardt scheme we use the functions provided by the Matlab optimization toolbox for the nonlinear least-squares minimization.

IV. Numerical Results

The proposed algorithm was applied for the localization of the equivalent central cardiac dipole for different measured data sets and different electrodes locations and satisfactory results were obtained.

First, the so called "computer test" is carried out. Namely, instead of real measurements the forward problem for the target dipole is solved to obtain a "measured data set», labeled as "measurements". The initially assumed equivalent electric dipole parameters (e.g. 3 dipole moment components) are iteratively updated until the differences between the measured and calculated data sets become comparable to an acceptable error tolerance. A lot of successful localizations, based on these computer-generated data, are carried out and the algorithm is found to converge with accuracy better than 10^{-3} mA.cm after a small number of iterations, as shown in Table1. During this step, a variable number of electrodes were used on the torso model, Fig4.

The actual or in-vivo test constitutes the second step toward the establishment of the method. For this purpose, the algorithm was tested on real multichannel measured data set available on-line [20]. During this procedure, the body surface potentials were measured from 128 electrodes positioned equidistantly over the body surface, for four different body poises (aligned-sitting, aligned-left, aligned-right and aligned-lying).

The dipole tip orbit during the whole cardiac temporal period for the sitting poise is shown in Fig.5a). In Fig.5b) a magnified version is shown to better distinguish the smaller cycles of the dipole. Also Fig.6 presents the respective orbits for the other 3 poises (aligned – left, right, lying). In these figures we can observe the three characteristic points of the cardiac temporal period Q, R, S. We can notice that for each case (Fig5a, Fig.6a,b,c) there is a variation in the shape of dipoles tips orbits as well as a change to of its inclination with respect to the horizontal plane. Note also that for better presentation the scaling is different in its case as well as a change of its inclination with respect to the horizontal plane. Note also that for better presentation the scaling is different in each case.

In order to verify the validity of our algorithm and our model, the calculated potential values during the whole cardiac temporal period are compared with the respective values of the measured potentials. This is shown in Fig.7 for the electrode number 93, which is the lowest front side electrode shown in fig4. As we can see the obtained voltages have exactly the same form with small variations in their values.

Finally, in Fig.8 the moment components (Px, Py, Pz) of the dipole for the "aligned-lying model" are presented.

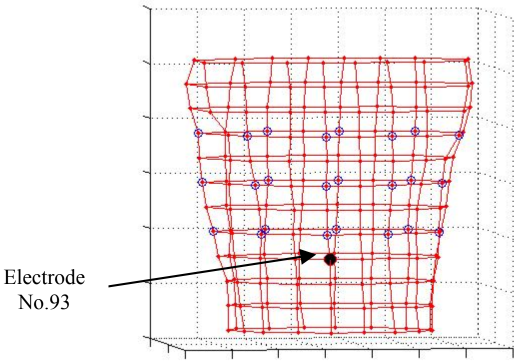
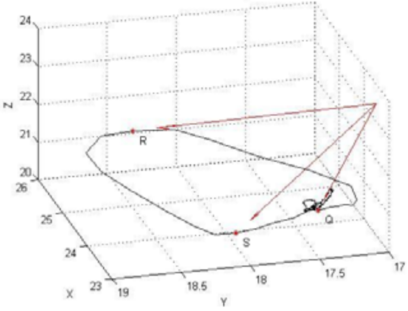
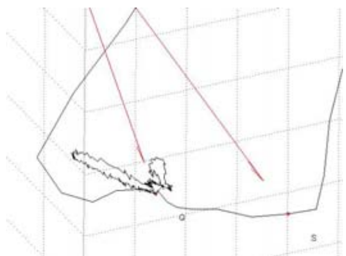


Fig. 4. Position of 24 electrodes over the body surface.



a)

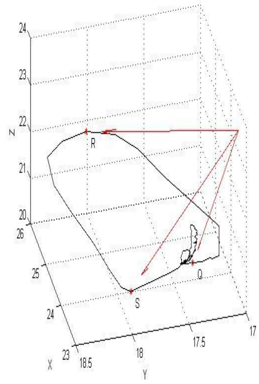


b)

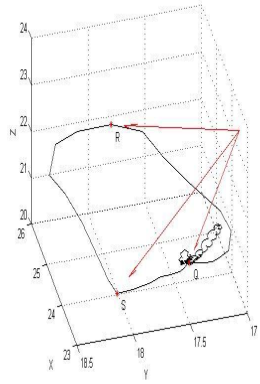
Fig. 5 a) Dipole rotation during the whole cardiac temporal period for the sitting poise case and b) magnified view of the rotation

V. Conclusion

A method solving the inverse problem of electrocardiography based on a single – central cardiac electric dipole allowed only to rotate is successfully implemented. The uses of a single dipole effectively comfort the generally expected non-uniqueness of the problem. The future extensions include improvements in both the inverse and the forward problem. The next step in inverse problem is to allow the single dipole movement additionally to its rotation. The forward problem advancements include its "adaptivity' to the measured subject dimensions and separate models reflecting the subject poises (sitting or various lying alignments).



a)



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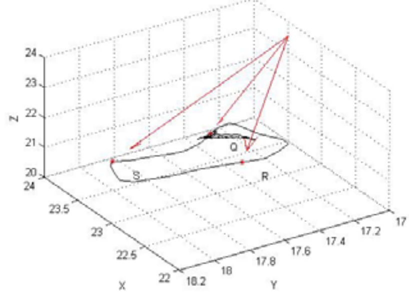


c)

Fig.6. Dipole rotation during the whole cardiac temporal period for a) the left poise case, b) the right poise case and c) the lying poise case.

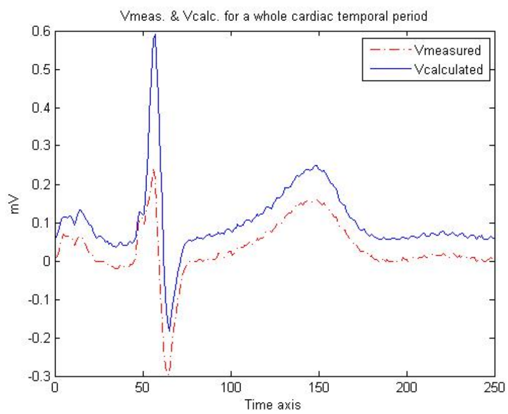
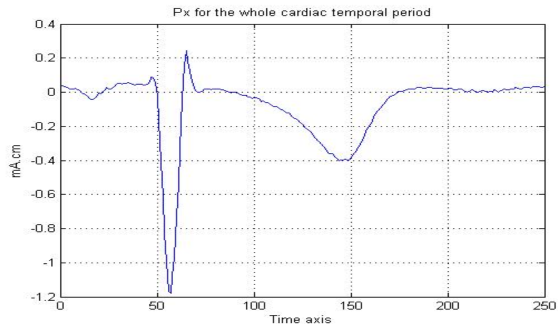


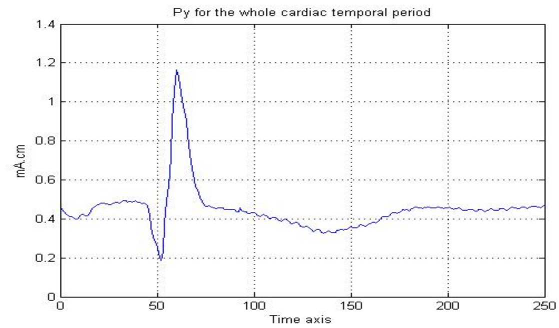
Fig.7 Measured and calculated voltages at the electrode 93 for the whole cardiac temporal period and the sitting poise case.

TABLE 1
Target and reconstructed dipole moments for 24 and 128 electrodes

Number of Electrodes	Target Dipole			Initial guess			Reconstructed Dipole			Number of Iterations
	Px	Py	Pz							
24	0.02456	0.376	0.0042	1	1	1	0.0252843	0.375564	0.00385255	3
128							0.0254103	0.375701	0.00424618	3



a)



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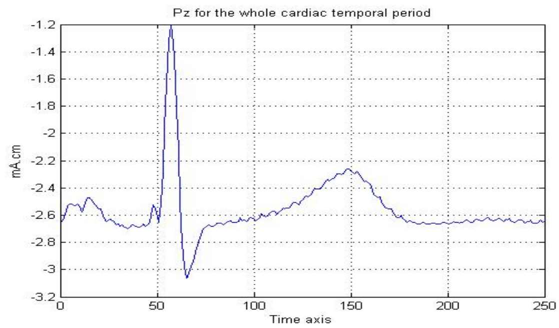

c)

Fig.8 Dipole moment components for the "aligned-lying" case, a) P_x, b) P_y, c) P_z.

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