Modeling and Estimation of Time-Varying Heart Rate Variability during Stress Test by Parametric and Non Parametric Analysis

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Abstract

A methodological framework for simulating real-like HRV during stress test with controlled spectral proprieties has been developed with the purpose to assess SPWVD and time-variant AR analysis. For each method results have been evaluated computing the estimation error for LF and HF components during all the test (mean error of the order of 10% *in all case) and a direct comparison based on the correlation between estimate and original ACF yields very high values (*ρ_{WV}=0, 99 *and* ρ_{AR}=0, 94*). In real data analysis both methods highlight an inversion of the relative spectral balance around the apex of exercise which passes from LF to HF prevalence.*

1. Introduction

Analysis of Heart Rate Variability (HRV) is a non invasive technique that provides an evaluation of the autonomic modulation of cardiovascular activity [1]. The spectrum of HRV signal is characterized by two main spectral components: the low frequency component (LF), range [0,04-0,15 Hz], is considered an index of the sympathetic modulation, and the high frequency (HF), range [0,15-0,4 Hz], is linked to the parasympathetic activity. Recently the analysis of HRV during stress test conditions has attracted much attention and it has been studied as potencial marker of ischemia [2]. To assess HRV during non-stationary conditions and to monitor autonomic control in extreme conditions time-frequency methods must be employed. Among others, time-variant (TV) parametric spectral analysis and Smoothed Pseudo Wigner Ville Distribution (SPWVD) have been widely used. Even if quantitative comparisons of these methods have been already proposed [3], no data is available during stress test. The purpose of this study is to provide a comparison of TV parametric spectral analysis and SPWVD in the analysis of HRV signal during stress stest. At this regard, a methodological framework for simulating HRV signals with known and controlled spectral proprieties has been developed. Using the framework, real-like HRV signals mimicking the characteristics observed during stress test [4] have been generated and used to evaluate the performance of the methods in tracking dynamic changes in LF and HF components. In the time-variant approach, the HRV signal, $x(n)$, is seen as the output of an autoregressive (AR) model:

$$x(n) = -\sum_{l=1}^{q} a_l(n)x(n-l) + b_0(n)\xi(n) \qquad (1)$$

where $a_l(n)$ and $b_0(n)$ are the model coefficients, q is the model order and $\xi(n)$ is a white noise. In (1) n is the time index. The TV spectrum becomes

$$S_{\text{AR}}(f,n) = |H_n(e^{j2\pi f})|^2\sigma_\xi^2 = \left(\frac{b_0(n)\sigma_\xi}{\prod_{k=1}^{q}|e^{j2\pi f} - p_k(n)|}\right)^2 \qquad (2)$$

where the $p_k(n)$ are the time variant model poles and $H_{\text{n}}(z)$ is the AR model transfer function. Time variant analysis is achieved identifying the coefficient $a_l(n)$ and the gain $b_0(n)$ by means of the Recursive Least Squares (RLS) method. Using the RLS algorithm results may be affected by changing the order of the model and the forgetting factor, which sets the memory horizon of the algorithm [3]. Temporal evolution of the spectral components (i.e. the dominant frequencies $f_{\text{LF}}(n)$ and $f_{\text{HF}}(n)$ and powers $P_{\text{LF}}(n)$ and $P_{\text{HF}}(n)$) and ACF $s_{\text{AR}}(n,k)$ are computed by a decomposition based on pole residuals [5].

1.1. Smoothed pseudo Wigner Ville analysis

The Smoothed Pseudo Wigner-Ville Distribution (SP-WVD) is defined as [6]:

$$S_{\text{WV}}(n,m) = 2\sum_{k=-N+1}^{N-1} s_{\text{WV}}(n,k)e^{-j\frac{2\pi k}{N}m} \qquad (3)$$

where $s_{\text{WV}}(n,k)$ is the windowed filtered instantaneous autocorrelation function (ACF) of the analytic signal

$a_x(n)$, computed as:

$$s_{\text{WV}}(n,k) = |h(k)|^2 \sum_{p=-M+1}^{M-1} [g(p) r_{\text{wv}}(n+p,k)] \qquad (4)$$

and $r_{\text{wv}}(n,k) = a_x(n-k)a_x^*(n+k)$. In this equation $g(p)$ and $h(k)$ are the time and frequency smoothing kernels used to reduce interferences, and n and m are discrete time and frequency indexes. The time window length $2M+1$ is adapted to the local spectral properties, increasing or decreasing according to the instantaneous frequency rate of variation [7]. When time and frequency smoothing kernels are, respectively, a rectangular and an exponential smoothing window, $s_{\text{wv}}(n,k)$ may be described, for every n, as a sum of complex damped sinusoids [6]. Their amplitude and frequency can be estimate using the Kumaresan Tufts (KT) decomposition [6]. This decomposition provides a way to track the temporal evolution of main signal components, namely the dominant frequencies $f_{\text{LF}}(n)$ and $f_{\text{HF}}(n)$ and the amplitudes $A_{\text{LF}}(n)$, $A_{\text{HF}}(n)$ of LF and HF components.

1.2. Simulation

In this work, the simulated signal $x(n)$ is obtained using (1). To derive a signal with known and controlled characteristics we proceed as explained in Fig. 1. An ideal time-frequency spectrum $S_{\text{AR}}(f,n)$ is defined by fixing, for each n, the dominant frequencies $f_{\text{LF}}(n)$ and $f_{\text{HF}}(n)$, and the amplitudes $S_{\text{AR}}(f_{\text{LF}}(n),n)$ and $S_{\text{AR}}(f_{\text{HF}}(n),n)$ of the LF and HF components. The evolution of frequencies and amplitudes are derived on the base of some physiological knowledge of ANS response during stress [2],[4]. In particular, frequencies and amplitudes are assumed to vary as in Fig. 2. The $f_{\text{HF}}(n)$ increases linearly during exercise. The HF amplitude initially decreases due to vagal withdrawal with exercise, while some time before the apex of effort it increases as consequence of the augmented ventilation which causes a mechanical stretch of the sinus node synchronous with respiration. The system comes back to the original conditions after a short recovery period.

It is evident form (2), that a desired spectral pattern may be obtained by positioning the model poles in the complex plane (Fig. 3). The spectral amplitude is inversely proportional to the squared distance $|e^{j2\pi f} - p_k(n)|^2$ between the k-pole and the unite circle points. A 6th order AR model is used. A pair of complex conjugate poles for the LF component and two pairs for the HF are set. The HF pole pairs have the same phase but different modules. To locate $p_k(n)$ we solve the following equation system:

$$\angle[p_1(n)] = -\angle[p_2(n)] = f_{\text{LF}}(n) \qquad (5)$$
$$\angle[p_3(n)] = \angle[p_5(n)] = -\angle[(p_4)] = -\angle[p_6(n)] = f_{\text{HF}}(n)$$

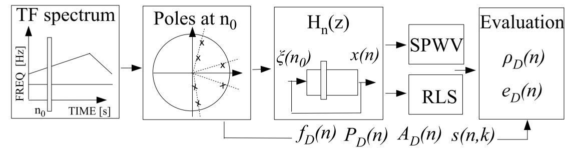

Figure 1. Simulation, analysis and evaluation process

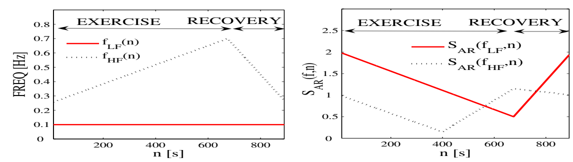

Figure 2. Time evolution of the ideal spectral component

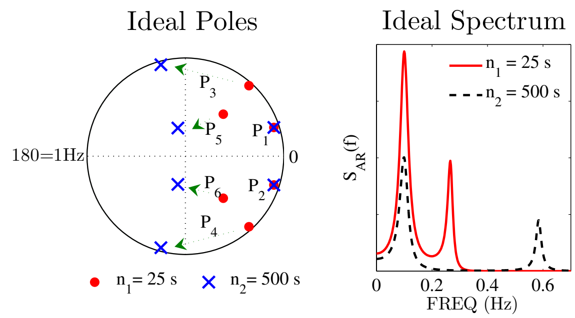

Figure 3. $S_{\text{AR}}(n,k)$ for 2 different n and its polar configuration

$$|p_1(n)| = |p_2(n)| = |p_1(0)|$$
$$\left[\prod_{k=1}^{6} \frac{b_0(n)}{|e^{j2\pi f_{\text{D}}(n)} - p_k(n)|}\right]^2 = S_{\text{AR}}(f_{\text{D}}(n),n)$$

where $\text{D} \in [\text{LF,HF}]$. From the model poles, coefficients $a(n)$ and gain $b_0(n)$ are obtained, (1) is used to filter a zero-mean unit-variance white noise $\xi(n)$ to obtain the simulated signal $x(n)$.

1.3. Evaluation and comparison

A random process can be described only probabilistically. This implies that, to evaluate the methods, mean results should be used analyzing a group of trials $x_i(n)$, where $i = [1,...,L]$. The parametric method can be directly evaluated using as reference the powers ($P_{\text{D}}(n)$) and the frequencies ($f_{\text{D}}(n)$) associated with the pole configurations of the model and computed through the residual method. The estimates $\widehat{P}_{\text{D}}(n)$ and $\widehat{f}_{\text{D}}(n)$ are obtained identifying the mean coefficients $\widehat{a}_l(n)$ and $\widehat{b}_0(n)$ across the L trials $x_i(n)$. The error is then computed as $e_{\text{D}}(n) = \frac{\widehat{P}_{\text{D}}(n) - P_{\text{D}}(n)}{P_{\text{D}}(n)}$.
To evaluate the SPWVD performance the ideal ACF $r(n,k)$ is computed from the poles of the model [5] and filtered with the same *g(p)* and *h(k)* used in (4), obtaining $s(k,n)$. Its KT decomposition provides the dominant frequencies $f_{\text{D}}(n)$ and the amplitudes $A_{\text{D}}(n)$ used as references. Amplitudes $A^2_{\text{D}}(n)$ are evaluated to make them comparable with $P_{\text{D}}(n)$. The error is then computed as:

$e_{\text{D}}(n) = \frac{\widehat{A^2_{\text{D}}(n) - A^2_{\text{D}}(n)}}{A^2_{\text{D}}(n)}$ where $\widehat{A}_{\text{D}}(n)$ is the mean amplitude that characterizes a group of L trials. It is computed decomposing $\widehat{s}_{\text{WV}}(n,k) = \frac{1}{L}\sum_{i=1}^{L} s_{\text{WV},i}(n,k)$.
Since the parametric method yields power estimates while the non parametric yields amplitude estimates the comparison of their performance is not straightforward. To overtake this problem the ACF $s(n,k)$ is proposed as reference for the comparison of both methods. The mean poles identified during parametric analysis are used to compute $\widehat{s}_{\text{AR}}(n,k)$. Then we evaluate the correlation between $\widehat{s}_{\text{WV}}(n,k)$ and $\widehat{s}_{\text{AR}}(n,k)$ and the reference $s(n,k)$ as:

$$\rho_{\text{A}}(n) = \frac{\sum_{k=1}^{M}\left[\widehat{s}_{\text{A}}(n,k)s(n,k)\right]}{\sqrt{\sum_{k=1}^{M}\widehat{s}^2_{\text{A}}(n,k)}\sqrt{\sum_{k=1}^{M}s^2(n,k)}} \qquad (6)$$

where $A \in [AR, SPWV]$.

2. Results

2.1. Simulation results

We have analyzed a group set of $L = 100$ signals and evaluated and compared the results of both methods as explained in section (1.3).
The $x_i(n)$ studied with the AR method have been sampled at 2 Hz. A 4th order model and a forgetting factor implying a memory of 10 s have been used. The estimates $\widehat{f}_{\text{D}}(n)$ and $\widehat{P}_{\text{D}}(n)$ are reported in Fig. 4(a). In the recovery period it yields a slightly poorer estimation due to the increased HF rate of variation, which is roughly 3 times higher than during exercise. For both components the estimation error $\overline{e}_{\text{D}}$, obtained by averaging $e_{\text{D}}(n)$, has been computed during exercise and recovery as reported in Table 1.

Table 1. Estimation error for the AR analysis [%]

	STRESS	RECOVERY	TOTAL
$\overline{e}_{LF}$	$-0,72 \pm 11,45$	$-20,18 \pm 12,8$	$-5,57 \pm 14,5$
$\overline{e}_{HF}$	$-6,92 \pm 15,77$	$-33 \pm 11,55$	$-13,42 \pm 18,65$

Figure 4(b) shows SPWVD results. The estimates $\widehat{f}_{\text{D}}(n)$ and $\widehat{A}_{\text{D}}(n)$ have been obtained from the decomposition of $\widehat{s}_{\text{WV}}(n,k)$. In this case, $x_i(n)$ have been sampled at 4 Hz and a time window $2M+1$ of 10 s has been chosen and adapted as proposed in [7]. This method is able to follow the evolution of the signal even when its components are changing quickly.
The error $\overline{e}_{\text{D}}$, evaluated during exercise and recovery for both components, is shown in Table 2.

In order to compare the methods, the correlation $\rho_{\text{A}}(n)$, defined in (6), is computed between the results obtained with both methods and the model. In Fig. 5 we can see, as a proof of the method pertinence, that, for both methods, the correlation is always very close to one.

Table 2. Estimation error for the SPWVD analysis [%]

	STRESS	RECOVERY	TOTAL
$\overline{e}_{LF}$	$5,49 \pm 6,92$	$3,89 \pm 7,12$	$5,09 \pm 7,01$
$\overline{e}_{HF}$	$-14,13 \pm 12,76$	$3,05 \pm 11,29$	$-9,85 \pm 14,23$

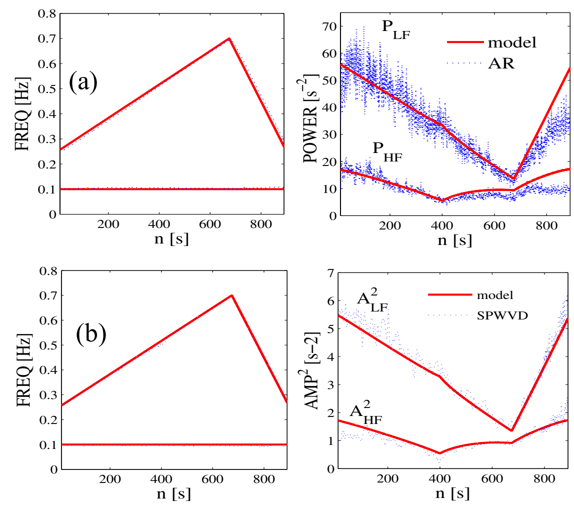

Figure 4. (a) AR and (b) SPWV analysis

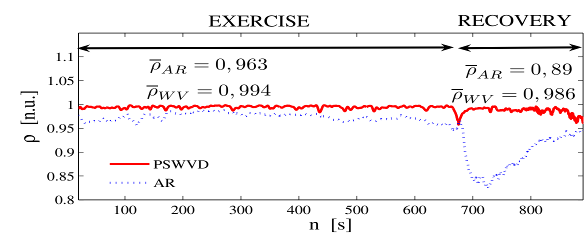

Figure 5. Correlation $\rho_{\text{A}}(n)$ computed for A=[AR,WV]

2.2. Real data analysis

We have applied SPWVD and AR algorithms to real data registered in the Hospital Lozano Blesa of Zaragoza during stress test. Instantaneous heart rate has been obtained using a method based on the integral pulse frequency modulation [8] and its very low frequencies have been filtered out in order to obtain the HRV.
In Fig. 6(a) results of AR analysis are shown for a subject. As done often in literature [1], the powers of the spectral components have been reported in normalized units (n.u.), which represent the relative value of each power component with respect to the total power. This reduces the variability of the estimation allowing a straightforward interpretation of the graphic. A 8th order model has been used and those components whose power was very small have been eliminated, considering them as a spurious contribu-

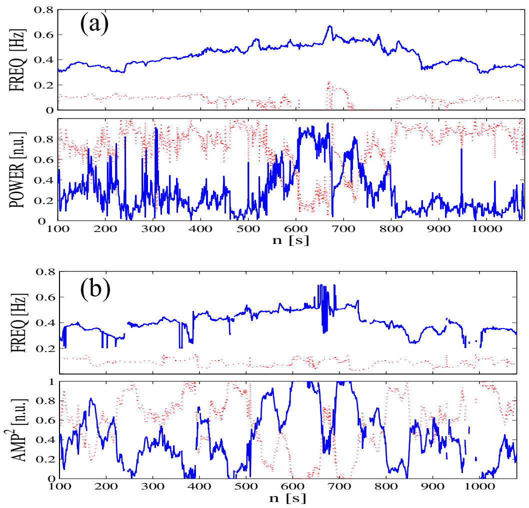

Figure 6. Real HRV AR (a) and SPWVD (b) analysis

tion without physiological relevance.
In Fig. 6(b) results obtained with the SPWVD are displayed. Amplitudes are first squared (see section 1.3) and then normalized.

3. Discussion

Simulation results evidence that poorest estimation performances are observed for AR analysis during recovery period, since the frequency rate increases. This variation can be considered as the main cause of error in estimation. It would be possible to make the RLS algorithm more reactive reducing or making adaptive the forgetting factor, but this would be achieved at the expense of a grater variability. SPWVD results, on the other hand, seems not to be greatly affected neither by the rate of variation of the frequencies nor by its abrupt change. In the simulation presented in this paper the SPWVD globally yields slightly better results: its mean value of correlation is 0,99 while it is 0,94 for the AR method. Considering the temporal average of the estimation error, SPWVD yields much better results during recovery ($3,9\%$ for LF and 3% for HF against, respectively, -20% and -33% for the AR), but not during exercise ($5,5\%$ for LF and $-14,1\%$ for HF against, respectively, $-0,7\%$ and $-6,9\%$ for the AR). In all case the standard deviation of the error is lower for SPWVD.
On real data analysis the methods give very similar information: LF maintains constant frequency of about 0,1 Hz, while HF varies in a way that can be considered coherent with the respiratory frequency. A LF predominance is observed except around the apex of exercise, where the HF power increases and becomes the predominant one. The same behavior have been noticed in other registrations recorded during stress test but never at rest, so that it should be directly connected with the dynamic condition of the test. This confirm the occurrence of the non neuronal mechanism which has already been considered in our simulation model and remarked in literature [4].

4. Conclusion

A stochastic model has been developed to simulate a real-like HRV signals with a great variety of spectral properties, like those observed during stress test, and, more generally, any kind of two-component random process. The performances of parametric and non parametric TV analysis methods have been evaluated and compared using the ACF. The independence on the rate of variation of the frequencies and a minor dependence on parameters can be highlighted as an advantage of the SPWVD with respect to the AR analysis, which seems to be affected by a greater inertia. In real data analysis both methods yields similar results.

Acknowledgements

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