Stability Analysis of Epileptic EEG Signals

Gatien Hocepied, Abdellah Kacha, Francis Grenez, Antoine Nonclercq

***Abstract*— Macroscopic neurophysiologic models, supposed to produce EEG signals, suggest that the epileptic seizures are due to a limit of stability of the system. A technique based on multivariate autoregressive model (MVAR) and stability analysis by means of the eigenvalues of the estimated model is presented in this paper. This method has been used to analyse stability of EEG signals of epileptic patients, especially during epileptic seizure. We computed the maximum of the module of the eigenvalues on short term intervals by using a sliding window in the EEG signals. Results show that these values remain close to 1 and nearly constant during the epileptic seizure and occasionally thereafter, especially in the case of secondary generalized seizures. A new feature (IndexS) has been introduced in the purpose of emphasizing this phenomenon. The number of seizures identified correctly by this index was 11 among a total of 17 seizures.**

I. Introduction

Electroencephalography is a useful tool for physicians. It can be used for diagnostic purposes in many brain's pathologies as epilepsy, sleep troubles, etc.

Epilepsy is the most current pathology in this field and concerns about 1% of the population. One of the characteristics of epilepsy is the presence of repetitive seizures. Clinically, these seizures can take different forms: from muscular moves to critical convulsions called "Grand Mal" or tonic-clonic seizure. In EEG signals, an epileptic seizure is characterized by high-voltage and rhythmic EEG waveforms as illustrated in Fig. 1. High-voltage signals show a phenomenon of high synchronous activities of the brain or a part of it. Epilepsy can be focal (or partial) if the seizure is reduced to a part of the brain, called the focus. If the entire brain is affected, the epilepsy is said to be generalized.

In the literature, different kinds of models have been proposed to simulate EEG signals. Models presented in [1]-[6] use neurophysiologic mechanisms and are called "mean-field models" because of their macroscopic approach. These models are able to represent epileptic-like EEG signals as well as normal EEG signals. They strongly suggest that epileptic activity is related with the instability or quasi-instability of the system leading to rhythmic activities. Investigating this feature of epileptic process in EEG signals could permit to better understand this process and to provide a feature for characterizing the epileptic seizure in computer-based monitoring of EEG.

The detection of epileptic seizure is an important purpose for specialist's diagnostic, because it could save the specialist time by pointing out the seizure periods. In the literature, various features have been investigated with some success [7]. Multiple features can be used in a same algorithm by means of a classifier, like support vector machine, to increase the performance of the detection.

Furthermore, another interesting challenge in the field of EEG signal processing is the ability of predicting a seizure. It could enable in the future to avoid seizures of the patients by means of different techniques. Several linear and nonlinear features have been investigated in [8]-[11]. But at the present time, it's difficult to say if an epileptic seizure is reliably predictable [12], [13].

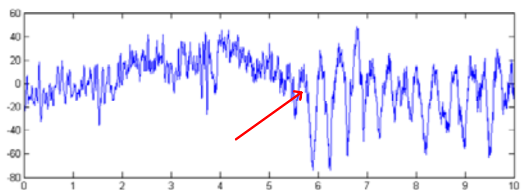

Fig. 1. Segment of EEG signals with an epileptic seizure.

In this paper, we propose to investigate the stability of the EEG signal of patients with epileptic seizures. To that purpose, a multivariate autoregressive (MVAR) model has been used. The model order is reduced by using a state representation and the eigenvalues are computed from the resulting system to investigate the stability of the signal. The maximum of the module of the eigenvalues is computed for each analysis frame along the entire signal, before, during and after epileptic seizure for several epileptic patients.

Similar methods have been used elsewhere in a different framework [14], [15]. In [14] a nonlinear MVAR model has

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been estimated and the maximum of the modules of the eigenvalues of biomedical signals has been computed in order to analyse the stability of breathing patterns of a patient affected by sleep apnea. In the field of oceanographic purposes [15], the eigen-decomposition of sea level by means of linear MVAR model has been studied.

II. THEORY

A. Multivariate autoregressive modelling

In multivariate autoregressive modelling, the current sample is estimated by a weighted sum of past samples of this channel as well as of the other channels. The MVAR representation of a vector process X_n is derived from the scalar version by replacing scalars by matrices. Let $X_n \in \mathbb{C}^{m\times 1} = [x_1(n)x_2(n)...x_m(n)]^T$ be a stationary discrete-time zero-mean m-dimensional stochastic process. The vector process X_n can be modelled as the output of a multichannel system driven by a *m*-dimensional zero-mean white Gaussian noise $E_n \in \mathbb{C}^{m\times 1}$ with a covariance matrix $C \in \mathbb{C}^{m\times m}$.

$$X_n = \sum_{i=1}^{p} A_i X_{n-i} + E_n \qquad (1)$$

Expression (1) is the MVAR model of the process, *p* is the model order and A_i, $i = 1,\dots,p \in \mathbb{C}^{m\times m}$ are the model parameters to be estimated.

Adding state variables, a p-order MVAR model can be written as a 1-order model [16]. It consists in replacing X_{n-i} with $i > 1$ by new states variables. This leads to:

$$\hat{X}_n = J\hat{X}_{n-1} + \hat{E}_n, \qquad (2)$$

$\hat{X}_n = [X^T{}_n X^T{}_{n-1} ... X^T{}_{n-p+1}]^T \in \mathbb{C}^{mp\times 1}$ is the augmented state vector, $\hat{E}_n = [E^T{}_n \ \ 0 \ \ ... \ \ 0]^T \in \mathbb{C}^{mp\times 1}$ is the augmented noise vector and *J* is the system matrix expressed as:

$$J = \begin{bmatrix} A_1 & A_2 & \dots & A_p \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \in \mathbb{C}^{mp\times mp} \qquad (3)$$

Equation (2) is called the state representation of the system.

B. Eigenvalues and stability

The system matrix *J* can also be written as:

$$J = PLP^{-1}, \qquad (4)$$

where the columns of $P \in \mathbb{C}^{mp\times mp}$ are the eigenvectors P_k. *L* is a diagonal matrix that contains the *mp* eigenvalues:

$$L = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \lambda_{mp} \end{bmatrix} \in \mathbb{C}^{mp\times mp} \qquad (5)$$

The state vector can be expressed as a linear combination of the eigenvectors:

$$\hat{X}_n = \sum_{k=1}^{mp} \tilde{X}_n(k) P_k = P\tilde{X}_n, \qquad (6)$$

with coefficient vector $\tilde{X}_n = \left[\tilde{X}_n(1)...\tilde{X}_n(mp)\right]^T \in \mathbb{C}^{\text{mpx1}}$

The noise vector can be expressed in a similar way as

$$\hat{E}_n = \sum_{k=1}^{mp} \tilde{E}_n(k) P_k = P\tilde{E}_n, \qquad (7)$$

with coefficient vector $\tilde{E}_n = \left[\tilde{E}_n(1)...\tilde{E}_n(mp)\right]^T \in \mathbb{C}^{\text{mpx1}}$

By replacing (6) and (7) in (2), we get:

$$\tilde{X}_n = L\tilde{X}_{n-1} + \tilde{E}_n. \qquad (8)$$

The covariance matrix C' of the noise vector $\tilde{E}_n$ can be expressed as:

$$C' = P^{-1}CP \qquad (9)$$

Therefore, we have a new system with *mp* independent equations that defines the *mp* modes of the model:

$$\tilde{X}_n(k) = \lambda_k \tilde{X}_{n-1}(k) + \tilde{E}_n(k) \quad \text{for } k = 1\dots mp. \qquad (10)$$

These equations are now coupled only via the covariance matrix C' of the noise coefficients.

An eigenvalue λ is a complex number that defines a specific mode of the system:

$$\lambda = re^{i\omega}, \qquad (11)$$

with *r* denoting the damping ratio and ω the frequency. An eigenvalue with imaginary part defines an oscillatory damped mode and a real eigenvalue defines a non-oscillatory damped mode. When max ($|\lambda|$) < 1, the system is said to be asymptotically stable [17]. Thus, we expect that the eigenvalues of the matrix *J* could give a good feature of the stability of the system. For more details on the eigen-decomposition, see [13].

III. Methodology

A. EEG database

The study was carried out on 11 patients suffering from partial seizure (Temporal Lobe Epilepsy) followed, in several cases, by secondary generalizations. The 11 patients present a total of 17 seizures.

EEG signals of epileptic patients have been recorded on 25 electrodes at the University Hospital in Brussels (Belgium) at the sampling frequency *fs* = 250 Hz. The epileptic seizures have been labelled by a neurologist using both EEG signals and video monitoring. The electrode montage is referential: the voltage is measured between a reference electrode and each one of the 25 electrodes. A 10/10 system is used. The positions of each electrode are shown in Fig. 2.

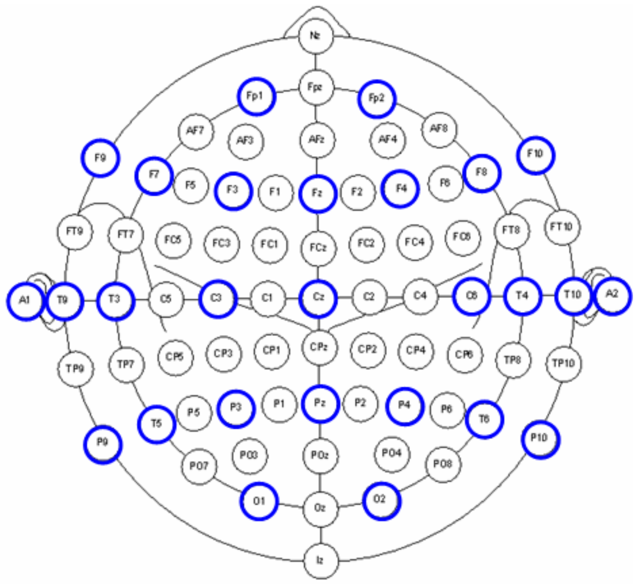

Fig. 2. System 10/10. Used Electrodes have been marked with bold circles.

B. Estimation of the multivariate autoregressive coefficients

In order to estimate the multivariate autoregressive coefficients, we used the multivariate generalization of the Burg method for monovariate autoregressive model estimation, called Vieria-Morf method. Schlögl [18] showed that this method is the best among other methods for 6-channels EEG. We used this algorithm from the open source TSA Matlab toolbox.

C. Selection of the model order

The selection of the model order is not a trivial problem. The order is chosen according to a trade-off between fitness of the model and computational cost. Several criterions for AR model order selection are available in the literature.

The order estimated from one of these criterions should give an insight on the dimension of the inherent system. We could expect that a more complex signal (for instance noise-like signal), will require a higher order. In this paper, the Schwartz Bayesian criterion (SBC) is used for the selection of the order p of the model. Indeed, for MVAR model, the SBC is superior to the Akaike information criterion (AIC) and the final prediction error criterion (FPE) as discussed in [19].

In order to select a model order, we computed Schwartz Bayesian criterion on many time windows of 10 s on long-term (several hours) EEG signals of different patients. We fixed $p = 25$, which is the most frequently obtained value for the criterion (Fig. 3).

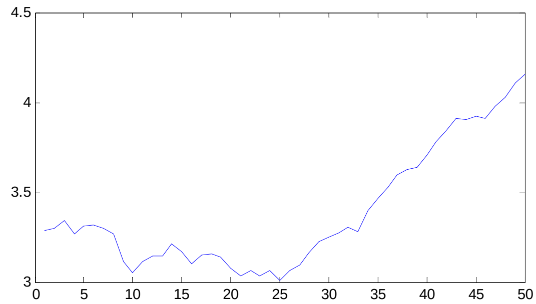

Fig. 3. Selection of the model order by means of SBC. The SBC is plotted as a function of the order for a time window of 10 s (2500 samples for each of 25 channels).

D. Implementation of the method

Each channel has been filtered using a high-pass filter with a 0.15 Hz cut-off frequency in order to remove low-frequency artefacts [20], together with a 70 Hz low-pass filter .This frequency range is commonly used for epilepsy. A 50 Hz notch filter has been used to eliminate the power line interference from of the EEG signals.

We used sliding analysis windows of 10 s along the signal with a time shift of 2 s between successive windows. We computed for each window the MVAR model by means of Vieria-Morf estimator on the 25 channels and then the eigenvalues of the system matrix *J*.

The maximum of the module of the eigenvalues has been calculated for each window by excluding non-oscillatory modes, i.e. real eigenvalues, as explained previously.

IV. Results and Discussion

The maximum of the module of the eigenvalues computed according to the method described in section II are shown as a function of time for 2 patients with 2 seizures each in Fig. 4a, 5a and 6a. The 2 seizures of patient A are partials and secondary generalized. Patient B presents 2 complex partial seizures taking place in a short period of time. The starts of the seizures are marked by an arrow in Fig. 4, 5 and 6.

We filtered the values with a 5-tap moving average filter in order to smooth the traces of the maximum of the module of the eigenvalues. The resulting values are displayed in Fig. 4b, 5b and 6b.

We can see a different behaviour during the seizure period (ictal period) compared to the non-seizure period (interictal period). In Fig. 4b, 5b and 6b, the values remain close to 1 and form a plateau during and occasionally after the seizure. To emphasize this behaviour, we computed the following index:

$$IndexS(i) = \log\left[\frac{1}{n}\sum_{k=-\frac{n-1}{2}}^{\frac{n-1}{2}}\left(\left|1-\left|\lambda_{\max}(i-k)\right|\right|\right)\right]^{-1}, \qquad (12)$$

where $\lambda_{\max}(k)$ is the maximum of the modules of the eigenvalues for time window *k*. This index is computed on *n* windows around the current window of the smoothed trace and permits to illustrate this transition phenomenon. In order to keep appropriate time resolution, the number of windows involved in the computation of the index has been *n*=5. The results are shown in Fig. 4c, 5c, and 6c.

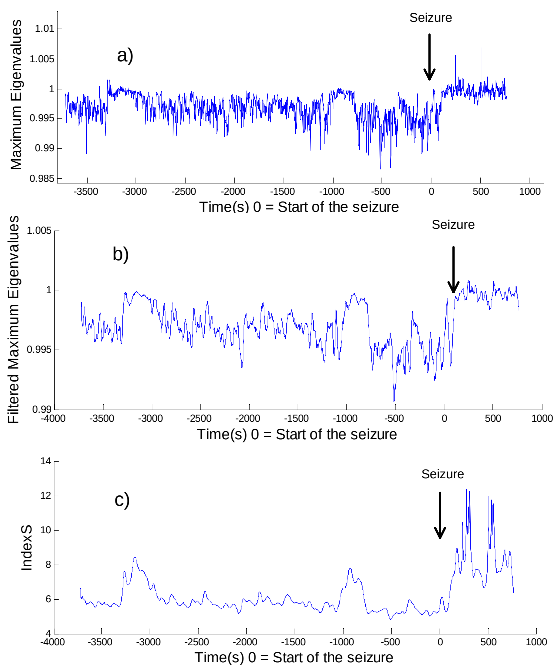

Fig. 4. First seizure of patient A: a) maximum of the module of eigenvalues b) filtered values by the moving average filter c) IndexS.

In Fig. 4c, 5c and 6c, we can see that the index permits to emphasize the phenomenon previously observed and reaches a maximum during the epileptic seizure and occasionally thereafter, especially in the case of secondary generalized seizures as can be seen for patient A in Fig. 4 and 5. The number of seizures correctly identified by this index was 11 among the total of 17 seizures of the database.

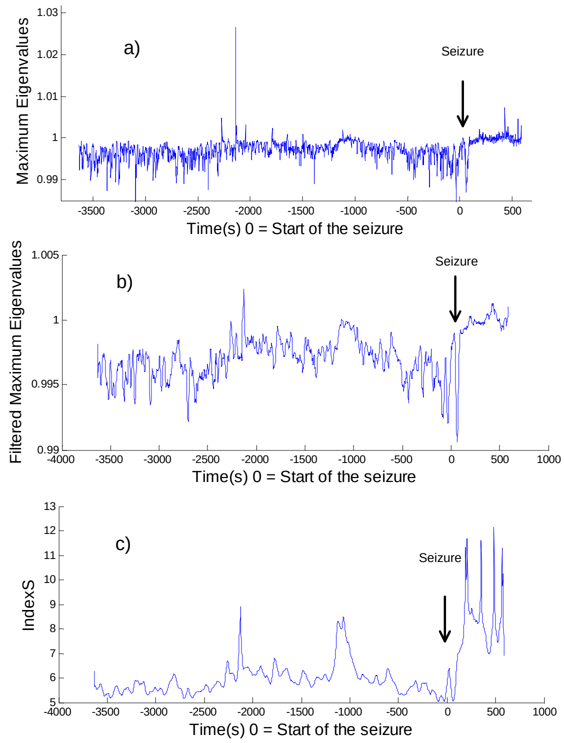

Fig. 5. Second seizure of patient A: a) maximum of the module of eigenvalues b) filtered values by the moving average filter c) IndexS.

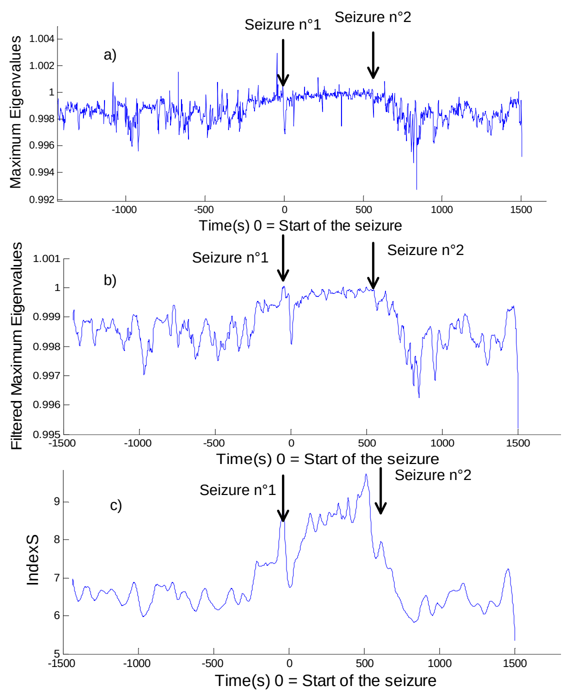

Fig. 6. The 2 seizures of patient B: a) maximum of the module of eigenvalues b) filtered values by the moving average filter c) IndexS.

V. Conclusion

This paper has investigated a technique based on MVAR model for stability analysis by means of the eigenvalues of the underlying model. As far as we know, this method has never been used in epileptic EEG analysis. Even if, at the present time, the epileptic process is poorly understood, this technique is based on neurophysiologic considerations endorsed by macroscopic models of generation of the EEG.

Results show a different behaviour of the maximum of the module of the eigenvalues of the MVAR model after the start of the seizure and often after the seizure itself (Fig. 4 and 5). This last observation is surprising and needs further investigations. Results obtained in this paper corroborate the assumptions about stability of epileptic system.

In the future, this technique needs to be statistically tested in the purpose of establishing a detection feature. In order to compare the proposed approach to other features of seizure detection for scalp EEG recordings, the different methods needs to be tested on the same data set and the false alarm rate has to be fixed (by means of ROC-curve) to determine the sensibility as in [7].

Prelimining results suggest that prediction purposes seem to be excluded with this method. It may be the model is too simple to show a phenomenon before the seizure. Actually, several authors [11], [21]-[24] suggest that phenomena before seizure could be due to nonlinear mechanisms. Using more complex models such as nonlinear autoregressive model or neurophysiologic based models with nonlinear mechanisms could improve the method presented in this study.

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