Effect of Heart Motion on the Solutions of Forward and Inverse Electrocardiographic Problem - a Simulation Study

Y Jiang, D Farina, O Doessel

Institute of Biomedical Engineering, Karlsruhe Institute of Technology, Karlsruhe, Germany

Abstract

Solving the forward problem of electrocardiography provides a better understanding of electrical activities in the heart. The inverse problem of electrocardiography enables a direct view of cardiac sources without catheter interventions. Today the forward and inverse computation is most often performed in a static model, which doesn't take into account the heart motion and may result in considerable errors in both forward and inverse solutions. In this work a dynamic heart model is developed. With this model the effect of the heart motion on the forward and inverse solutions is investigated.

1. Introduction

Forward and inverse problems of electrocardiography are mostly solved in a static model, which is usually built from MRI or CT data of the heart in diastolic state. However, neglecting the heart motion introduces a certain amount of modeling error. This can lead to instability in the forward and inverse solutions. A dynamic heart model is developed in the current project and used to study the contribution of the heart motion to the simulated ECG and the influence of neglecting cardiac dynamics on the inverse reconstructions. Tikhonov 0-order regularization and GMRES method are applied to solve the inverse problem for epicardial potentials in both dynamic and static models.

2. Methods

2.1. Anatomical model

In the present study the data-set of the Visible Human Project of National Library of Medicine [1] is utilized. The highly detailed data are segmented and classified to more than 40 tissue classes and converted to a voxel-based model with a resolution of $2mm \times 2mm \times 2mm$. The excitation conduction system, i.e., bundle branches, fascicles, and Purkinje fibers as well as the fiber orientation are introduced in the ventricles. From the voxel-based model a tetrahedron mesh is generated and applied in the forward and inverse computations (see Fig. 1) [2].

The heart dynamics in the computer model is implemented through mesh deformation. The nodes that belong to the heart are shifted appropriately to reproduce the contraction and twist of the heart. Using this method 16 states during the whole heart cycle, i.e., from diastolic state to systolic state and then back to diastolic state, are created. Three states of the heart motion are shown in Fig. 1.

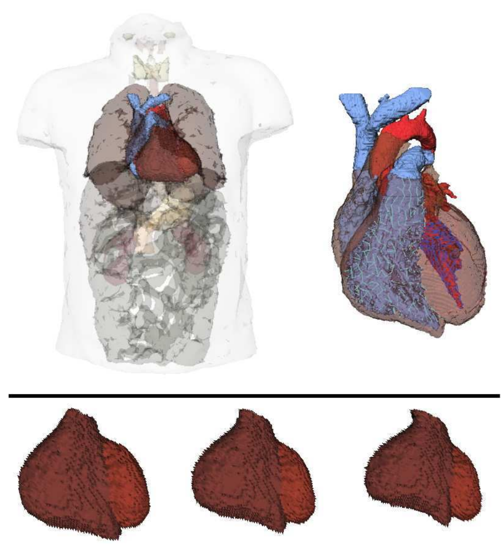

Figure 1. Visible human thorax model (upper left), the heart model (upper right) and the deformation of ventricles from diastolic state to systolic state (bottom).

2.2. Cellular automaton

A cellular automaton is used to simulate the excitation propagation in the heart [3]. A voxel in the myocardium can be activated by the neighboring active voxels. After activation the state transition of this element, i.e., change of transmembrane voltage, depends on the action potential curves, which are extracted from the ten Tusscher's cardiac cell model [4]. The transmural heterogeneity of myocardium is also considered. The simulation is performed in the voxel-based heart model with the spatial resolution of $1mm \times 1mm \times 1mm$ with the time step of $4ms$.

2.3. Forward problem

The distributions of the transmembrane voltage during the cardiac cycle produced by the cellular automaton are interpolated to the correspondent nodes in the tetrahedron mesh. The potential distributions on the body surface are obtained by solving the bidomain equation

$$\nabla \cdot ((\sigma_i + \sigma_e)\nabla \Phi_e) = -\nabla \cdot (\sigma_i \nabla V_m) \qquad (1)$$

with the finite element method, where σ_i and σ_e are the intracellular and extracellular conductivity tensors; Φ_e and V_m are the extracellular potential and transmembrane voltage.

2.4. Inverse problem

In this investigation the epicardial potential is selected as source model. In this case the inverse problem is formulated as

$$\mathbf{A}\mathbf{x} = \mathbf{y}, \qquad (2)$$

where $\mathbf{x}$ is the source vector (the potential distribution on the epicardium), $\mathbf{y}$ is the measurement vector (ECG) and $\mathbf{A}$ is called lead-field matrix, which describes the relation between the source and the measurement signal.

Due to the ill-posed nature of the inverse problem of electrocardiography the following regularization techniques are employed in order to improve the stability of the inverse solution.

2.4.1. Tikhonov regularization

Tikhonov regularization is a classical method to solve the inverse problem of electrocardiography [5]. It uses the general information about the solution for constraints besides the $\ell 2$-norm residual, in that the following minimization problem is considered:

$$x_\lambda = \arg\min_x (\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_\mathbf{2}^\mathbf{2} + \lambda^\mathbf{2} \|\mathbf{L}\mathbf{x}\|_\mathbf{2}^\mathbf{2}), \qquad (3)$$

where the regularization operator $\mathbf{L}$ is the identity matrix $\mathbf{I}$ in the case of 0-order, and λ is termed regularization parameter, controlling the weight attributed to the constraint condition $\|\mathbf{L}\mathbf{x}\|_\mathbf{2}^\mathbf{2}$. The optimal λ is found according to the criterion of "L-curve" [6].

2.4.2. Generalized minimal residual method

GMRES method is an iterative regularization method. In the iteration process the solution is sought, that satisfies

$$\mathbf{x_j} = \arg \min_{\mathbf{x} \in \mathbf{K_j}(\mathbf{A}', \mathbf{b})} \|\mathbf{b} - \mathbf{A}' \cdot \mathbf{x}\| \qquad (4)$$

where $\mathbf{K_j}$ indicates the j-th Krylov subspace and with $\mathbf{A}' = \mathbf{A}^\mathbf{T} \cdot \mathbf{A}$ and $\mathbf{b} = \mathbf{A}^\mathbf{T} \cdot \mathbf{y}$ [7, 8].

3. Results

3.1. Forward simulation

The forward problem is solved on the models including the dynamic heart and the static heart, respectively. In the dynamic case the heart stays in the diastolic state till the end of QRS-complex. Then the ventricles start to deflate and twist and reach the systolic state at the middle of the T-wave. After that the heart goes back to the diastolic state slowly. In the static case the heart is in the diastolic state during the whole cardiac cycle. Because in both cases the heart is in the diastolic state during QRS-complex, the present investigation focuses mainly on the T-wave.

From the BSPMs in Fig. 2 a considerable difference can be seen. The negative area is mostly on the back in the dynamic case in comparison with the static case, due to the twist of heart and the rotation of heart vector. The difference between the ECG signals obtained with the dynamic heart model and the static heart model is shown in Fig. 3.

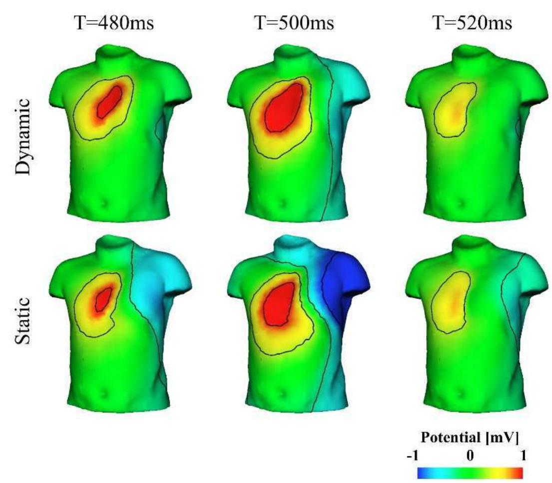

Figure 2. The BSPMs simulated with the dynamic heart model (upper) and with the static heart model (bottom) at 3 time instants during the T-wave.

3.2. Inverse solution

In order to solve the inverse problem incorporating the heart motion, the lead-field matrix is calculated for each

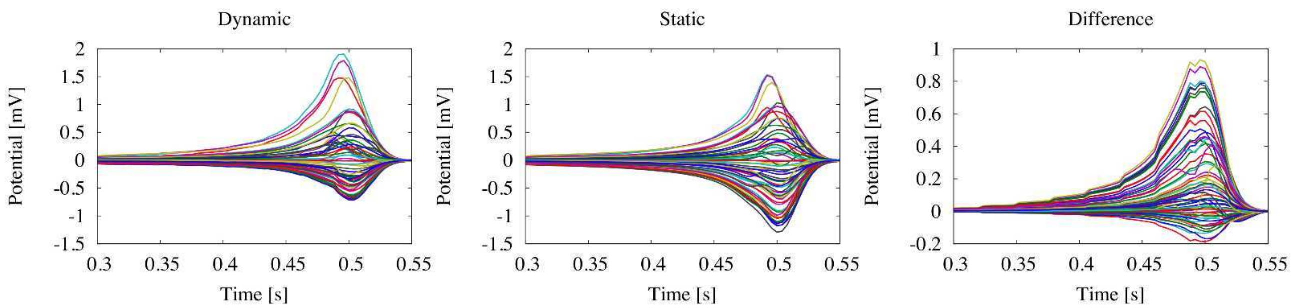

Figure 3. The 64 channel ECGs derived from the forward solutions made with the dynamic heart model (left) and with the static heart model (center) and the difference between them (right) during the ST-segment and the T-wave.

one of the 16 states during the whole cardiac cycle. The inverse problem for a given time instant is then solved with the corresponding lead-field matrix of the current state. For the static case the lead-field matrix of the diastolic state is deployed all the time. A simulated ECG in sinus rhythm made with the dynamic heart model, which can be considered as realistic ECG in the current simulation study, is taken as the input of inverse problem. The epicardial potentials are reconstructed in the dynamic and static models using the Tikhonov 0-order and GMRES. The reconstructions and the simulated epicardial potentials (reference) are presented in Fig. 4. The correlation between the inverse solutions and the reference, which reflect the quality of the reconstruction, is plotted in Fig. 5.

4. Discussion and conclusions

The contribution of the heart motion to ECG is significant, as can be seen in Fig. 3. It amounts up to 50% in amplitude at the middle of the T-wave. It states that the heart motion cannot be neglected in the forward problem of electrocardiography for the accurate simulation of repolarization process.

From the point of view of inverse electrocardiographic problem it is of advantage to incorporate the heart motion into the inverse calculation. In Fig. 4 the reconstructions using Tikhonov 0-order with dynamic heart model are considerably better than those obtained with static heart model and an improvement of 0.1 in correlation coefficient during the T-wave is shown in Fig. 5. But there is no significant difference between the reconstructions with and without the consideration of the heart motion when the GMRES method is used, as seen in Fig. 4 and Fig. 5. It also proves that the GMRES method is more stable against modeling errors.

In further work a more accurate dynamic model will be created from a 4D MRI data-set. Then the forward and inverse computation will be performed in this model. Other regularization methods and source models, e.g., MAP-based regularization and activation time reconstruction, will be tested on the dynamic model. It is also of interest to study the effect of respiration on the forward and inverse solutions.

Acknowledgements

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Address for correspondence:

Yuan Jiang
Institute of Biomedical Engineering,
Karlsruhe Institute of Technology
Kaiserstrasse 12
76131 Karlsruhe, Germany
Yuan.Jiang@kit.edu

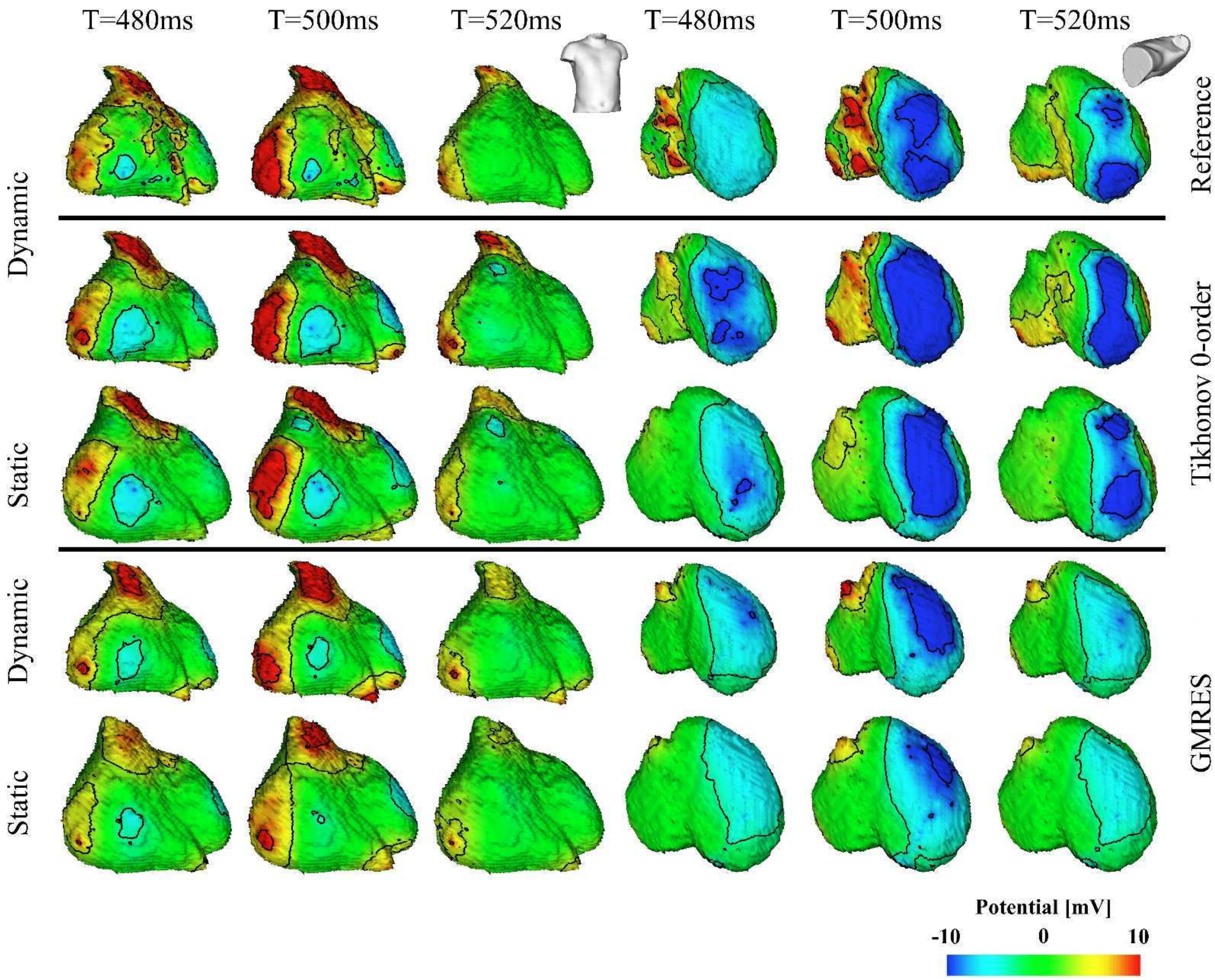

Figure 4. Comparison between the reconstructed distributions of epicardial potentials with dynamic (rows 2 and 4) and static (rows 3 and 5) heart model using the Tikhonov 0-order regularization (rows 2 and 3) and the GMRES method (rows 4 and 5) at 3 time instants during the T-wave. The simulation as the reference is shown on the dynamic heart model (row 1). The results are illustrated in two different views.

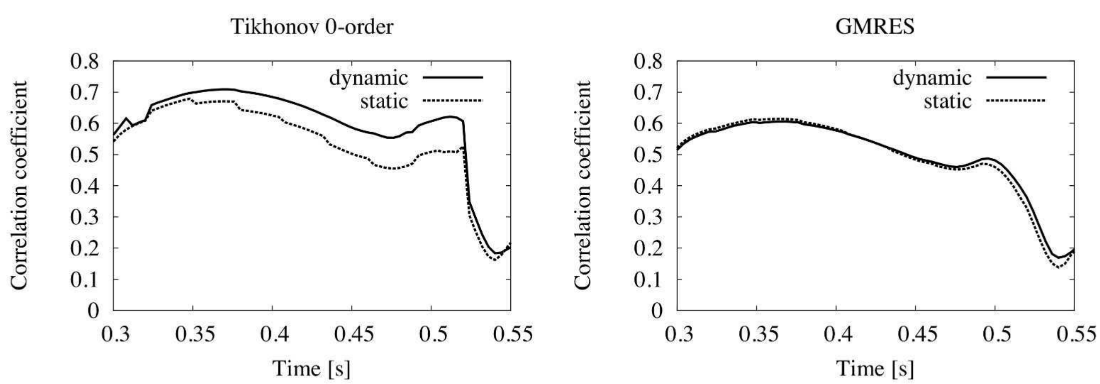

Figure 5. Correlation between the simulated reference and the inverse solutions obtained with dynamic and static heart model using the Tikhonov 0-order regularization (left) and the GMRES method (right).