Motion Estimation in X-Ray Rotational Angiography Using a 3-D Deformable Coronary Tree Model

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Abstract

This paper presents a two stages method for three-dimensional (3-D) coronary artery motion estimation from rotational X-ray projection. In the first step we estimate a 3-D skeleton of the coronary tree at each cardiac phase, using a deformable coronary tree model. The second stage proceeds with a motion estimation between the 3-D reconstructed coronary trees over the cardiac cycle. This estimation is based on a 3-D B-spline interpolation model.

1. Introduction

Coronary arterial diseases remain a major cause of mortality in Europe and in the US. Quantitative and accurate characterization of abnormal vascular patterns (location, shape) within the whole coronary network is of major importance for diagnosis and treatment. For a long time, only mono- and bi-plane X-ray techniques were available to deal with this problem. Pioneering attempts, mainly focused on static reconstruction, were thus based on computer vision approaches relying on epipolar techniques and feature matching (refer to [1] for a review). However, a first coupling with motion estimation was proposed in [2] and led to a fast and efficient scheme to recover the centrelines over the entire time sequence. Today, the recent availability of Rotational X (Rot-X) and Multi-detector CT devices opens new perspectives. If the latter can be seen basically as a diagnosis tool, the former offers joint pre-operative and intra-operative solutions. It provides during the rotation of the C-arm (over 120-220 degrees within 5-7 seconds) a higher number of projections (80-160). Unfortunately, due to the object motion (heart beating, breathing...) the full set of projections cannot be directly used: each projection corresponds to a different volume image. Thus, the estimation of the object deformation and its use in the tomographic reconstruction process are of major interest. In angiography for instance, Christophe Blondel [3] proposes a 3-D+t (time-dependant three-dimensional) reconstruction based on the estimation of a 3-D coronary centreline model at a reference time. The motion component is approximated by a 3-D+t B-spline model whose parameters are estimated such that the deformed skeleton fits the projections at each state instant. This paper deals with the reconstruction of the coronary tree motion through the Rot-X sequence, with assumptions similar to those made in [2, 3]. In other words, an initial exact static 3-D vascular tree described by centrelines is assumed to be known as well as the projective geometry and the centrelines in all projections. Moreover, the motion is supposed cyclic and the projections are synchronized through ECG-gating. We also assume that the vessel centrelines have been segmented on each projection. Our contribution departs from the previous approaches in the following aspect: we propose a deformable coronary tree centreline model that leads to an estimation of the motion field based on a quadratic minimization (section 2) instead of a non-quadratic formulation [3]. The results on simulated data are reported in section 3. Conclusions and perspectives are drawn in section 4.

2. Methods

2.1. Deformable tree model

Let $\Omega \subset \mathbb{R}^3$ be the coronary tree domain and let us denote $V_t \subset \Omega$ the coronary tree at time $t \in [0, NT]$. We assume that V_t is T-periodical: for all $t \in [0, (N-1)T]$, $V_{t+T} = V_t$, T denoting the duration of a cardiac cycle and N the number of observed periods. V_t is observed through a finite number of projections at regular instants $t_j = jT/S$, $j \in \{1, \ldots, NS\}$, where S is the number of projections in a period. Hence the sequence

$(V_{t_j})_j$ is S-periodical and we can rewrite $V_{t_j} = V_s$, with $s = j \mod(S)$. V_s now denotes the 3-D model at phase s of the cardiac cycle. Let $\Theta_s = \{\vartheta_{1,s}, \dots, \vartheta_{N,s}\}$ be the device positions (angles) corresponding to a given phase s of the cycle and $P(\vartheta_{1,s}), \dots, P(\vartheta_{N,s})$ be the corresponding projection planes. The proposed method considers that a first 3-D skeleton of the coronary tree has been reconstructed. This reconstruction may be based on the epipolar constraint and is performed using the projections acquired at phase $s = 1$ [3]. Let us denote $V_1 = \left\{v_1^1, \dots, v_L^1\right\}$ this initial tree, where $v_\ell^1 \in \mathbb{R}^3$ for all $\ell \in \{1, \dots, L\}$. Our aim is to estimate $V_s = \{v_1^s, \dots, v_L^s\}$ for each $s \in \{2, \dots, S\}$ using successive deformations of the 3-D model V_1. Proceeding this way allows each point v_ℓ^1 of V_1 to be tracked in time and to build a time-sequence $v_\ell^1 \in V_1, \dots, v_\ell^S \in V_S$ for each $\ell \in \{1, \dots, L\}$, which makes the estimation of a motion function $\varphi_s : \mathbb{R}^3 \to \mathbb{R}^3$ more simple, as we shall see further in this section. A topological structure on V_s is required to define a regularity cost to prevent its successive deformations to lead to a degenerated tree. A natural neighborhood structure is the following: assuming that V_1 can be separated[1] in J branches $B_1^1, \dots, B_J^1$ with $B_j^1 = \left\{v_{j,1}^1, \dots, v_{j,I_j}^1\right\}$, whose elements are listed in an order such that $v_{j,1}^1$ and v_{j,I_j}^1 are the border points of B_j^1, a clique is a set $\{(j,i),(j,i+1)\}$. The estimation of V_s from V_{s-1} is performed through the minimization of a cost function that is composed of a data fidelity term and a regularization term. For each projection angle ϑ, let $\mathcal{P}_\vartheta^{geom}(v)$ be the geometric cone-beam projection of the 3-D point v on the projection plane $P(\vartheta)$,

$$\mathcal{P}_\vartheta^{geom}(v) = \frac{a}{b + v_x\cos(\vartheta) + v_y\sin(\vartheta)} \begin{bmatrix} v_x\sin(\vartheta) - v_y\cos(\vartheta) \\ v_z \end{bmatrix},$$

where a (resp. b) is the distance of the X-ray source to the detector (resp. the volume center). Let $D_\vartheta : P(\vartheta) \to \mathbb{R}_+^*$ be a function such that $D_\vartheta(x)$ is small when x is located in the neighborhood of a projected vessel in $P(\vartheta)$ and large when x is in the background (see section 2.2 for the definition of the function). We can now define the data cost of a point v in Ω with respect to the projections at phase s:

$$E_s(v) = \frac{1}{N}\sum_{n=1}^{N} D_{\vartheta_{n,s}}(\mathcal{P}_{\vartheta_{n,s}}^{geom}(v)).$$

The data fidelity cost $\mathbf{E}_s(V)$ of a 3-D coronary tree model $V = \{v_1, \dots, v_L\}$ is thus given by:

$$\mathbf{E}_s(V) = \frac{1}{|V|}\sum_{v \in V} E_s(v).$$

[1] Bifurcation points belong to at least 3 branches

We consider then the regularization term $\mathbf{F}(V)$ (see section 2.3 for its definition), its objective being to constrain the deformation of the model to be smooth. Then for all $t \in \{2, \dots, T\}$, the global deformation energy $\mathbf{D}_s(V)$ for the estimation of V_s is given by

$$\mathbf{D}_s(V) = \mathbf{E}_s(V) + \kappa\mathbf{F}(V),$$

where κ is a parameter that controls the elasticity of V. For $s = 2, \dots, S$, a gradient based method is performed to estimate V_s from V_{s-1}:

1. Initialize $V^{(0)} = V_{s-1}$, $q = 0$
2. For $\ell = 1, \dots, L$
   Compute $\delta_{q,\ell} = E_s(v_\ell^{(q)})$
   Compute $v_\ell^{(q+1)} = v_\ell^{(q)} - \delta_{q,\ell}\lambda\nabla_\ell\mathbf{D}_s(V^{(q)})$
   End For
3. Compute $V^{(q+1)} = \left\{v_1^{(q+1)}, \dots, v_L^{(q+1)}\right\}$
4. If convergence then END For
5. Else $q = q + 1$, return to step 2.

Here $\nabla_\ell\mathbf{D}_s(V)$ denotes the gradient of $\mathbf{D}_s$ with respect to v_ℓ. The time step $\delta_{q,\ell}$ has been chosen to be equal to $E_s(v_\ell^{(q)})^{1/2}$ in order to slow down the motion as $V^{(q)}$ approaches the solution.

The cost function $\mathbf{D}_s$ is completed to take into account the length of the vessel. If we consider known the location of the branch extremities in each projection plane $P(\vartheta_{n,s})$, we can exploit this information for the energy function computation at these endpoints: let $v_{j,1}$ (resp. v_{j,I_j}) be the first (resp. the last) point in the branch B_j of a vessel 3-D skeleton V. The energy of $v_{j,1}$ and v_{j,I_j} at phase s we used are

$$E_{s,j,1}^{end}(v_{j,1}) = \sum_{n=1}^{N} \zeta_{j,1}^{n,s}\|\mathcal{P}_{\vartheta_{n,s}}^{geom}(v_{j,1}) - h_{j,1}^{n,s}\|^2$$

and

$$E_{s,j,I_j}^{end}(v_{j,I_j}) = \sum_{n=1}^{N} \zeta_{j,I_j}^{n,s}\|\mathcal{P}_{\vartheta_{n,s}}^{geom}(v_{j,I_j}) - h_{j,I_j}^{n,s}\|^2$$

where $h_{j,1}^{n,s}$ (resp. $h_{j,I_j}^{n,s}$) denotes an estimation of the first (resp. the last) point of the extracted branch j on projection plane $P(\vartheta_{n,s})$, and where $\zeta_{j,1}^{n,s}$ (resp. $\zeta_{j,I_j}^{n,s}$) is a non-negative normalized sequence (with respect to n) which takes large values if $h_{j,1}^{n,s}$ (resp. $h_{j,I_j}^{n,s}$) is likely to be a correct starting (resp. ending) point for the branch j on $P(\vartheta_{n,s})$ and small values otherwise.

2.2. Distance function

We define here the "distance" function D_ϑ used in 2.1. Let v_ℓ be a point of a 3-D coronary skeleton V, and

$h_{\vartheta,\ell} = \mathcal{P}_{\vartheta}^{geom}(v_\ell)$ be its projection on $P(\vartheta)$. Let us denote $\mathcal{H}_\vartheta = \mathcal{P}_{\vartheta}^{geom}(V)$ the projection of V on $P(\vartheta)$. Since we assumed that the two-dimensional (2-D) centrelines have been extracted on each projection plane $P(\vartheta)$, we can calculate a distance function between $h_{\vartheta,\ell}$ and the extracted centrelines. Let $\mathcal{C}_\vartheta \subset P(\vartheta)$ be the set of points of the extracted centrelines on projection plane $P(\vartheta)$. A natural way to define a distance between each point $h_{\vartheta,\ell} \in \mathcal{H}_\vartheta$ and $\mathcal{C}_\vartheta$ would be $D_\vartheta(h_{\vartheta,\ell}) = \min_{c\in\mathcal{C}_\vartheta} \|h_{\vartheta,\ell} - c\|^2$. However, this distance depends on the point $c \in \mathcal{C}_\vartheta$ that minimizes $\|h_{\vartheta,\ell} - c\|$ and c may belong to a wrong branch. To overcome this difficulty, we propose an alternative distance function. Let us assume we are estimating V_s from V_{s-1}. Let $v_\ell^{(q)}$ be a point of a 3-D coronary skeleton $V^{(q)}$ at iteration q of the deformation algorithm, and $h_{\vartheta,\ell}^{(q)} = \mathcal{P}_{\vartheta}^{geom}(v_\ell^{(q)})$. Remember that $v_\ell^{(q)}$ is a result of a displacement of $v_\ell^{s-1} \in V_{s-1}$, which is also the last element of the sequence $v_\ell^1 \in V_1, \ldots, v_\ell^{s-1} \in V_{s-1}$. Let $c_1(\vartheta,\ell,q), \ldots, c_{n_{\min}}(\vartheta,\ell,q)$ be the $n_{\min}$ closest points to $h_{\vartheta,\ell}^{(q)}$ in $\mathcal{C}_\vartheta$: we define $D_\vartheta(h_{\vartheta,\ell}^{(q)})$ by the weighted sum

$$D_\vartheta(h_{\vartheta,\ell}^{(q)}) = \frac{1}{\Gamma(\vartheta,\ell,q)} \sum_{i=1}^{n_{\min}} \gamma_i(\vartheta,\ell,q) \|h_{\vartheta,\ell}^{(q)} - c_i(\vartheta,\ell,q)\|^2,$$

where $\Gamma(\vartheta,\ell,q) = \sum_{i=1}^{n_{\min}} \gamma_i(\vartheta,\ell,q)$. A high $\gamma_i(\vartheta,\ell,q)$ value means that $c_i(\vartheta,\ell,q)$ is likely to be a good target for $h_{\vartheta,\ell}^{(q)}$. The weights $\gamma_i(\vartheta,\ell,q)$ may depend on local properties of $\mathcal{C}_\vartheta$ and $\mathcal{H}_\vartheta^1 = \mathcal{P}_\vartheta^{geom}(V_1)$, the projection of the initial coronary skeleton, such as the vessel local direction. Let $\overrightarrow{\mathrm{dir}}(h_{\vartheta,\ell}^1|\mathcal{H}_\vartheta^1)$ be the vessel direction of $\mathcal{H}_\vartheta^1$ at $h_{\vartheta,\ell}^1$ and $\overrightarrow{\mathrm{dir}}(c_i(\vartheta,\ell,q)|\mathcal{C}_\vartheta)$ be the direction of $\mathcal{C}_\vartheta$ at $c_i(\vartheta,\ell,q)$: we choose $\gamma_i(\vartheta,\ell,q) = 1/\left(\|\overrightarrow{\mathrm{dir}}(h_{\vartheta,\ell}^1|\mathcal{H}_\vartheta^1) - \overrightarrow{\mathrm{dir}}(c_i(\vartheta,\ell,q)|\mathcal{C}_\vartheta)\|_1 + 1\right)$.

2.3. Regularization cost

In this section we briefly introduce three regularity cost function $\mathbf{F}$. Let V be a coronary tree 3-D model, $\Upsilon(V)$ be the set of cliques in V, $\iota = \{\iota_1, \iota_2\}$ be an element of $\Upsilon(V)$, and $\psi_1, \psi_2, \psi_3 : \mathbb{R}^+ \to \mathbb{R}^+$ be three non-decreasing function. A first natural way to control the regularity of a 3-D coronary tree model V is to maintain a small distance between two neighbouring points. The regularity functional is therefore written as

$$\mathbf{F}_1(V) = \frac{1}{|\Upsilon(V)|} \sum_{\iota\in\Upsilon(V)} \psi_1(\|v_{\iota_1} - v_{\iota_2}\|).$$

Another way to control regularity is to maintain constant distance between neighbouring points. Let $\xi_V : \Upsilon(V) \to \mathbb{R}^+$ which maps a clique ι to the square distance between the 3-D points indexed by ι_1 and ι_2:

$$\forall \iota \in \Upsilon(V), \quad \xi_V(\iota) = \psi_2(\|v_{\iota_1} - v_{\iota_1}\|).$$

We can interpret ξ as a random variable, and then calculate its mean value and its variance:

$$\mathbb{E}(\xi_V) = \frac{1}{|\Upsilon(V)|} \sum_{\iota\in\Upsilon(V)} \xi_V(\iota)$$

and

$$\mathbb{V}(\xi_V) = \frac{1}{|\Upsilon(V)|} \sum_{\iota\in\Upsilon(V)} (\xi_V(\iota) - \mathbb{E}(\xi_V))^2 .$$

Thus we chose

$$\mathbf{F}_2(V) = \sqrt{\mathbb{V}(\xi_V)}.$$

Finally, the last solution consists of controlling the deformation magnitude from the initial 3-D model $V_1 = v_1^1, \ldots, v_L^1$. Assuming we apply our algorithm on the 3-D model $V = \{v_1, \ldots, v_L\}$, we define the local displacement function

$$\forall \ell \in \{1, \ldots, L\}, \quad \mathcal{D}_\ell(v_\ell) = v_\ell - v_\ell^1 \in \mathbb{R}^3,$$

the regularity score we used is

$$\mathbf{F}_3(V) = \frac{1}{|\Upsilon(V)|} \sum_{\iota\in\Upsilon(V)} \psi_3(\|\mathcal{D}_{\iota_1}(v_{\iota_1}) - \mathcal{D}_{\iota_2}(v_{\iota_2})\|).$$

2.4. Motion parameters estimation

Once we have an estimation of each 3-D coronary tree model V_s, the next step deals with the motion estimation between two successive 3-D trees. This motion function is computed using a B-spline based registration technique. For each $s \in \{2, \ldots, S\}$, we wish to build a function $\varphi_s : \mathbb{R}^3 \to \mathbb{R}^3$ such that for each $\ell \in \{1, \ldots, L\}$, $\varphi_s(v_\ell^s) \simeq v_\ell^1$. Let $\mathcal{M}$ be a grid of Ω. A B-spline parametric model is chosen to represent φ_s:

$$\varphi_s : \begin{bmatrix} x \\ y \\ z \end{bmatrix} \longmapsto \begin{bmatrix} x + \sum_{m=1}^{|\mathcal{M}|} \alpha_{X,s}^m b_m(x,y,z) \\ y + \sum_{m=1}^{|\mathcal{M}|} \alpha_{Y,s}^m b_m(x,y,z) \\ z + \sum_{m=1}^{|\mathcal{M}|} \alpha_{Z,s}^m b_m(x,y,z) \end{bmatrix},$$

where $b_m(x,y,z) = b(x - x_m)b(y - y_m)b(z - z_m)$ is a cubic B-spline function centered on (x_m, y_m, z_m). The estimation of $\alpha_s = \left\{(\alpha_{X,s}^m, \alpha_{Y,s}^m, \alpha_{Z,s}^m)\right\}_{m=1}^{|\mathcal{M}|}$ is carried out by minimizing a least square cost function:

$$\psi(\alpha_s) = \sum_{\ell=1}^{L} \|\varphi_{\alpha_s}(v_\ell^s) - v_\ell^1\|^2 + \mu \sum_{m\sim m'} \|\alpha_s^m - \alpha_s^{m'}\|^2 + \nu\|\alpha_s\|^2,$$

where the second sum is taken over the neighbouring points of $\mathcal{M}$, and where μ and ν are regularization parameters. By convention, $\alpha_1 = 0$.

Table 1. Scores for the three regularity cost functions ($\times 10^{-4}$).

regularity cost	$\mathbf{F}_1$	$\mathbf{F}_2$	$\mathbf{F}_3$
$\varepsilon(V, V^*)$	5.5	4.8	3.2

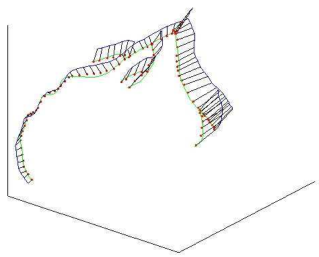

Figure 1. Result of the deformation algorithm between end of systole and end diastole, after 20 iterations

3. Results

We dispose of 20 3-D centrelines $V_1, \ldots, V_{20}$ at every 5% of the RR interval that had been previously extracted from a 3-D dynamic sequence acquired on a 64-slice GE LightSpeed CT coronary angiography [4]. The total number of projections is 80 and we simulated 4 cycles during one acquisition, which means each of the 20 phases is projected 4 times. The 2-D centrelines on each projection were simulated by performing geometric projections of the 3-D centrelines on the planes $P(\vartheta)$. We calculated a score function to evaluate the performance of our method: if $V^* = \{v_1^*, \ldots, v_L^*\}$ is the 3-D skeleton we wish to approximate and $V = \{v_1, \ldots, v_L\}$ is our deformable tree, we calculate the distance between deformable model V and the target V^*:

$$\varepsilon(V, V^*) = \frac{1}{L} \sum_{\ell=1}^{L} \|v_\ell^* - v_\ell\|^2.$$

We tried our method with the three proposed regularity cost functions $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$. The starting tree V_1 and the target tree V^* correspond respectively to the end diastolic and systolic phases in order to test the robustness of the method for the largest possible movements. The corresponding optimum regularity parameters κ_1, κ_2 and κ_3 were chosen empirically and were equal to 3.3, 0.22 and 3.8 respectively. We chose $\psi_i(x) = x^2$ for $i = 1, 2, 3$. Results can be seen in table 3 and Fig. 1. The motion parameter α was estimated with $\mu = 0.1$ and $\nu = 0.005$. The approximated motion field can be seen in Fig 2.

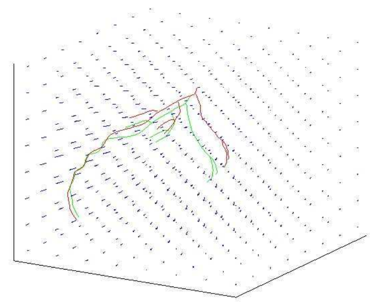

Figure 2. Estimation of the motion field φ_s

4. Discussion and conclusions

We have briefly presented a 3-D deformable method allowing the estimation of coronary tree motion over a cardiac cycle from a X-ray projection sequence. This motion field can be further introduced in a tomographic reconstruction algorithm to reconstruct the coronary tree using the set of available projections in order to improve the quality of the reconstruction. We tried our method with three different regularity cost functions. So far it appears that the third regularization cost (based on local regularization of the motion) gave the best results.

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