Long Memory and Volatility in HRV: An ARFIMA-GARCH Approach

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Abstract

Heart rate variability (HRV) data display non-stationary characteristics, exhibit long-range correlations (memory) and instantaneous variability (volatility). Recently, we have proposed fractionally integrated autoregressive moving average (ARFIMA) models for a parametric alternative to the widely-used technique detrended fluctuation analysis, for long memory estimation in HRV. Usually, the volatility in HRV studies is assessed by recursive least squares. In this work, we propose an alternative approach based on ARFIMA models with generalized autoregressive conditionally heteroscedastic (GARCH) innovations. ARFIMA-GARCH models, combined with selective adaptive segmentation, may be used to capture and remove long-range correlation and estimate the conditional volatility in 24 hour HRV recordings. The ARFIMA-GARCH approach is applied to 24 hour HRV recordings from the Noltisalis database allowing to discriminate between the different groups.

1. Introduction

The discrete series of successive RR intervals in the electrocardiogram (the tachogram) is the simplest signal that can be used to characterize heart rate variability (HRV) and has been applied in various clinical situations [1].

Ambulatory long-term HRV series correspond typically to 100000 beats in a 24-hour recording, exhibit non-stationary characteristics and can be described by time-variant autoregressive (AR) modelling. AR models are said short memory models since their autocorrelations (ACF) decay to zero exponentially. However, the sample autocorrelations (SACF) of HRV series show a very slow decay, indicating that the dependence between distant observations is not negligible, which is illustrated in Figure 1 (a) and (b). Correlations exhibiting this type of behaviour are called long-range correlations and the processes are denoted long-memory, correspond to a spectral density function obeying power law (1/f) in the very low frequencies [2]. In recent years, detrended fluctuation analysis (DFA) has become a widely-used technique for the detection of long-range correlations in non-stationary data [3,4]. An alternative approach, proposed by Leite *et al* [5], is to use fractional integrated autoregressive moving average (ARFIMA) models, which are an extension of the well-known autoregressive moving average (ARMA) models.

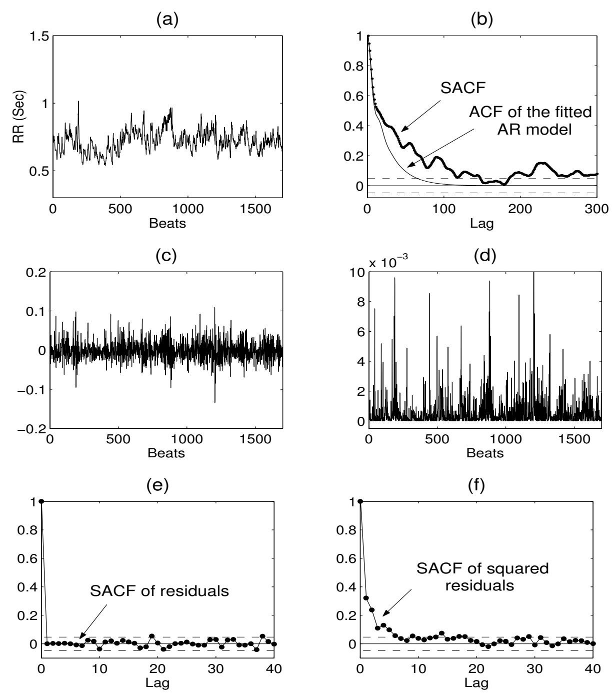

Figure 1. (a) Tachogram of a normal subject, (b) SACF and ACF of the AR(11) model (AIC criterion), (c) residuals of the fitted ARFIMA(7,0.47,0) model, (d) squared residuals, (e) and (f) SACF of residuals and squared residuals, respectively.

Another characteristic of HRV recordings is time-varying variance or volatility, clearly illustrated in 24 hour Holter HRV recording in Figure 3 (a). However, very little attention has been paid so far to the volatility characteristics of the HRV data. In fact, even though the residuals from ARFIMA modelling of the HRV series exhibit little correlation, indicating that the ARFIMA model is adequate, the squared residuals exhibit significant correlation, indicating time-varying variance [6,7], Figure 1 (c)-(f). Usually, the volatility in HRV studies is assessed by AR modelling basead upon recursive least squares.

In this work, we propose an alternative approach based on ARFIMA models with generalized autoregressive conditionally heteroscedastic innovations (ARFIMA-GARCH), which are an extension of the ARFIMA models. ARFIMA-GARCH models combined with selective adaptive segmentation may be used to capture and remove long-range correlation and estimate the condicional volatility, leading to an improved description of the components in 24 hour HRV recordings. This modelling can be used in reduced length segments of 512 beats. The ARFIMA-GARCH approach is applied to 24 hour HRV recordings of 30 subjects from Noltisalis database [8].

2. Long memory and volatility

A stationary process $x(t)_{t\in Z}$ is said to have long-range correlations if there exists a real number $\alpha \in]0,1[$ and a constant $c_f > 0$ such that

$$f(\omega) \sim c_f|\omega|^{-\alpha},\ \omega \to 0, \quad (1)$$

where $f(.)$ is the spectral density function. A class of processes with this property are the ARFIMA processes. These processes were introduced by Hosking [9] and have important applications since they are capable of modelling both the short- and the long-term behaviour of a time series. However, these processes assume that the condicional variance of the time series is constant over time.

Bollerslev [6] introduced GARCH processes to modelling time series with a time-varying condicional variance. These processes became central in field of financial and econometrics. Later, Baillie & Chung [10] proposed ARFIMA-GARCH models which are able to represent time series that exhibit both features: long memory and changing conditional variances.

2.1. ARFIMA-GARCH approach

A stochastic process $x(t)_{t\in Z}$ is an ARFIMA(p,d,q)-GARCH(r,s), $p,q,r,s \in N\cup\{0\}$ and $d \in R$, if it satisfies

$$\phi(B)\nabla^d x(t) = \theta(B)\epsilon(t), \quad (2)$$

with

$$\epsilon(t) = \sigma(t)z(t),$$

$$\sigma_\epsilon^2(t) = u_0 + \sum_{i=1}^{r} u_i\,\epsilon^2(t-i) + \sum_{j=1}^{s} v_j\,\sigma_\epsilon^2(t-i), \quad (3)$$

where $\phi(z) = 1 - \phi_1 z - ... - \phi_p z^p$ and $\theta(z) = 1 - \theta_1 z - ... - \theta_q z^q$ are polynomials such that $\phi(z) \neq 0$ and $\theta(z) \neq 0$ for $|z| \leq 1$, B is the backward-shift operator. ∇^d is the fractional difference operator defined by

$$\nabla^d = (1-B)^d = 1 + \sum_{j=1}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} B^j,$$

$\Gamma(.)$ is the gamma function, $z(t)$ is independent and identically distributed with zero mean and unit variance, $u_0 > 0$, $u_1, ..., u_r, v_1, ..., v_s \geq 0$, $\sum_{i=1}^{r} u_i + \sum_{j=1}^{s} v_j < 1$ and $\sigma_\epsilon^2(t)$ is the conditional volatility. In equation (2), the parameter d determines the long-term behaviour, whereas p, q and the coefficients in $\phi(B)$ and $\theta(B)$ allow the modelling of short-range properties. Equation (2) describes the mean of the process with serially uncorrelated residual, whereas equation (3) describes the volatility process as dependent on its own lagged values and on the squared residuals of the mean equation. For $-0.5 < d < 0.5$, the ARFIMA(p,d,q)-GARCH(r,s) is stationary and invertible and for $0 < d < 0.5$ the process has long-memory. Moreover, for $0.5 \leq d < 1$ the process is non-stationary and is mean reverting. For $d = q = s = r = 0$ ARFIMA(p,d,q)-GARCH(r,s) reduces to the classic short-memory AR(p) model. In this work, we consider ARFIMA($p,d,0$)-GARCH($1,1$) models, currently cited in literature [10,11] and give special attention to the parameters: d which characterizes the long memory and u_1 which characterizes the volatility. The spectral density function of ARFIMA($p,d,0$)-GARCH($1,1$) process is given by

$$f(t,\omega) = f_{SM}(t,\omega)|1-e^{-i\omega}|^{-2d}, \quad -\pi \leq \omega \leq \pi, \quad (4)$$

where $f_{SM}(t,\omega) = \frac{\sigma_\epsilon^2(t)}{|\phi(e^{-i\omega})|^2}$ is the spectral density of the corresponding short-range correlations, AR(p)-GARCH($1,1$) process and $\alpha = 2d$, equations (1) and (4).

Given a HRV series, $x(1), ..., x(N)$, the estimation of the parameters of the ARFIMA($p,d,0$)-GARCH($1,1$) models is as follows [2,10]: estimate d using the semi-parametric local Whittle estimator (LWE) and estimate the AR(p)-GARCH($1,1$) parameters in the filtered data $y(t) = (1-B)^d x(t)$. The LWE estimator is consistent for $-0.5 < d < 1$ [11,12] and the AR-GARCH parameters are estimated from the maximum likelihood estimation, with the order p determined by the Akaike Information Criterion (AIC).

2.2. ARFIMA-GARCH modelling of HRV

To illustrate the use of ARFIMA-GARCH models in short-term HRV data, the tachogram represented in Figure 1(a) is modelled with a ARFIMA(6,0.48,0)-GARCH(1,1). The estimated values for d and $\hat{u}_1$, ($\hat{d} = 0.48, \hat{u}_1 = 0.25$), indicate that the record has long memory and conditional volatility. The conditional volatility estimate $\hat{\sigma}_\epsilon^2(t)$, represented in Figure 2 (b), allows the identification of transient phenomena. These results indicate that ARFIMA($p, d, 0$)-GARCH(1,1) models are adequate in HRV recordings, allowing more parcimonious modelling that AR(p) modelling. Similar results were obtained in other recordings.

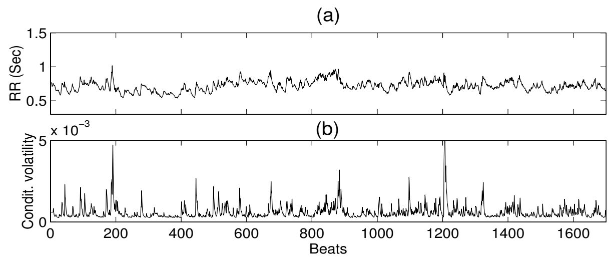

Figure 2. (a) Tachogram of Figure 1 and (b) conditional volatility estimate $\hat{\sigma}_\epsilon^2(t)$ from ARFIMA-GARCH modelling.

To describe long-range correlations and conditional volatility in the long-term HRV series (approximately 10^5 beats), ARFIMA-GARCH modelling combined with selective adaptive segmentation is used [2,5,10]: the long record is decomposed into short records of variable length and the break points, which mark the end of consecutive short records, are determined using the AIC criterion for ARFIMA models. The short records thus obtained have a minimum length 512 and are subsequently modelled using ARFIMA-GARCH models.

3. Results and discussion

The ARFIMA-GARCH approach is applied to 24 hour HRV recordings of 30 subjects from the Noltisalis database [8]: 10 healthy subjects (N, 34-56 years), 10 patients suffering from congestive heart failure (C, 36-68 years) and 10 heart transplanted patients (T, 18-60 years).

Figure 3 illustrates the results for a healthy subject-N6 (a), a patient affected by congestive heart failure-C10 (d) and an heart transplanted patient-T3 (g). The long memory estimates, $\hat{d}$, in (b), (e) and (h), change over time and the recordings present multifractality characteristics [3,5,7]. Moreover, these estimates present a circadian variation, with lowest values during the night periods. The volatility parameter estimate, $\hat{u}_1$, for healthy subjects, in (c), decreases during the night period and for sick subjects, in (f) and (i), is stable during the 24 hours.

The results for the three groups of patients during the 24 hours (P1), 6 hours of night (P2) and 6 hours of day (P3) periods are summarised in Figure 4 and Table 1. It is found that long memory increases for sick subjects, both

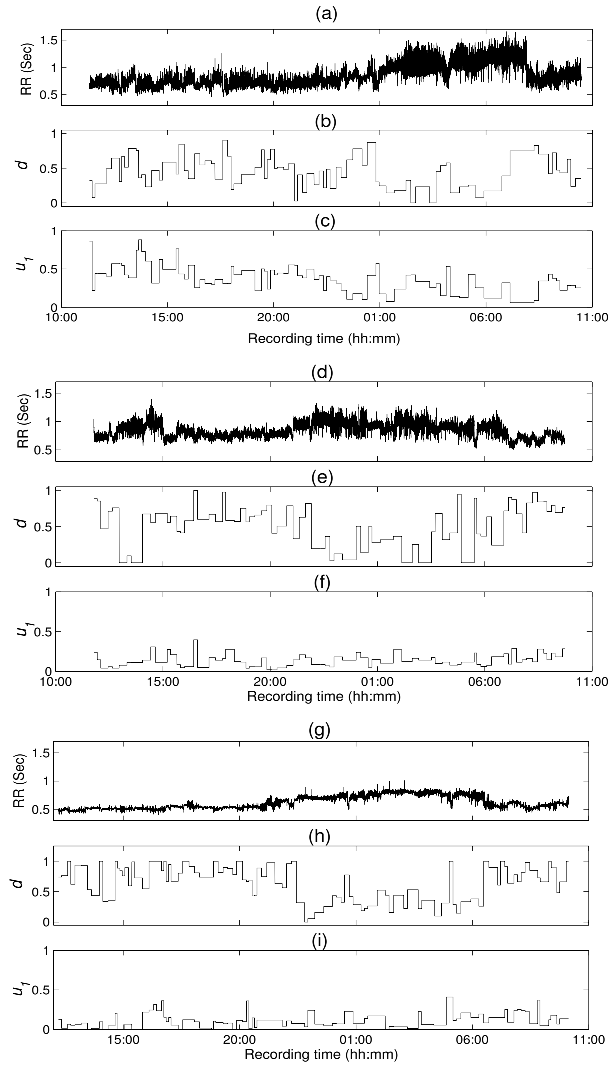

Figure 3. Tachograms of three subjects, 24 hours Holter recordings: (a) healthy subject-N6, (d) patient affected by congestive heart failure-C10 and (g) heart transplanted patient-T3. Evolution over 24 hours of $\hat{d}$ in (b), (e) and (h) and $\hat{u}_1$ in (c), (f) and (i), estimated using ARFIMA-GARCH models combined with selective adaptive segmentation.

during night and day periods, with the highest values for transplanted group. This is consistent with previous results reported in literature concerning the value of global scaling exponent calculated with different methods during the 24 hours [4]. However, ARFIMA-GARCH approach gives more information, namely the conditional volatility parameter u_1. This parameter decreases for sick subjects, both during night and day periods, with the lowest values for transplanted group. These results suggest that ARFIMA-GARCH modelling allows to discriminate between the different groups, more clearly during the day periods. Furthermore, the results of this modelling also suggest that long memory d increases and the conditional volatility u_1 decreases with age, as Figure 4 shows for the record N3

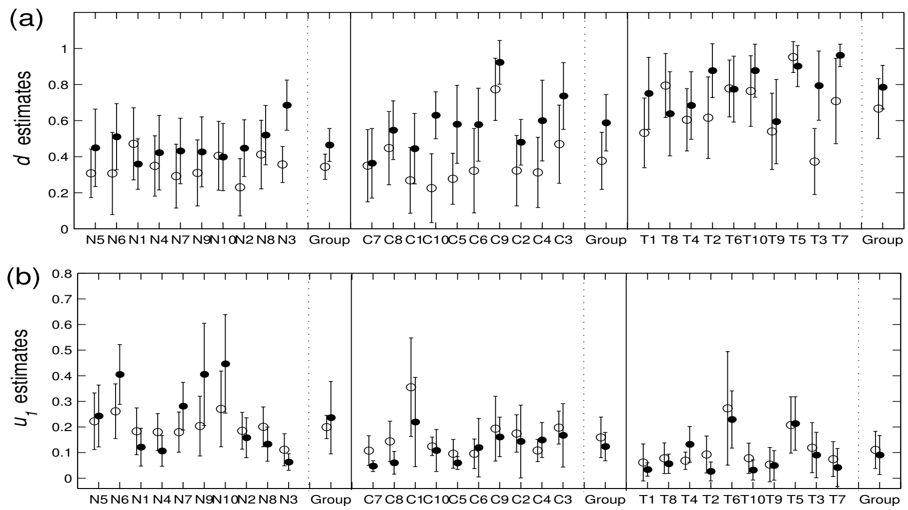

Figure 4. Average estimates and standard deviations of (a) $\hat{d}$ and (b) $\hat{u}_1$ for each Holter recording during (○) 6 hours of night and (●) 6 hours of day periods. The estimates are obtained using ARFIMA-GARCH modelling combined with selective adaptive segmentation. The subjects N-healthy, C-congestive heart failure and T-transplanted are ordered by age (ascending) and group estimates are presented on the right of each panel.

which belongs to the eldest subject of the healthy group.

Table 1. $\hat{d}$ and $\hat{u}_1$ values for the three groups of patients: healthy, subjects affected by congestive heart failure (CHF) and transplanted, during 24 hours (P1), 6 hours of night (P2) and 6 hours of day (P3) periods. For each case the average estimates ± standard deviations are presented.

Parameters		Healthy	CHF	Transplanted
Long memory	P1	0.44 ± 0.06	0.52 ± 0.14	0.76 ± 0.10
d	P2	0.34 ± 0.07	0.38 ± 0.16	0.67 ± 0.17
	P3	0.46 ± 0.09	0.59 ± 0.16	0.78 ± 0.12
Volatility	P1	0.23 ± 0.09	0.15 ± 0.08	0.11 ± 0.06
u_1	P2	0.20 ± 0.04	0.16 ± 0.08	0.11 ± 0.07
	P3	0.24 ± 0.14	0.12 ± 0.05	0.10 ± 0.07

4. Conclusion

In this study we focused our attention on the multifractality and conditional volatility characteristics of the HRV recordings. We employed ARFIMA-GARCH models which while being popular in other disciplines, namely economics and finance, are unknown in medical and clinical research. ARFIMA-GARCH modelling combined with selective adaptive segmentation of 24 hours HRV data shows that the long memory parameter has a circadian variation, with different regimes for night and day periods. Moreover, increased long memory, d, values and decreased conditional volatility, u_1, values for sick subjects, suggest that ARFIMA-GARCH modelling allows to discriminate between the different groups.

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