Individually Improved VCG Synthesis

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Abstract

Some prognostic electrocardiographic variables are to be measured in a vectorcardiogram (VCG). Normally, VCGs are synthesized from a standard 12-lead ECG by a transformation matrix. This occurs at the price of some information loss that can be defined as the error ε between the original ECG and reconstructed ECG (ECGr). Our assumption is that the larger ε, the less reliable the synthesized VCG. We attempted here to improve the VCG synthesis by reducing ε for each individual by using the Errors-In-Variables model. This technique minimizes ε by allowing 1-10% changes of the transformation matrix (M) that can be seen as the individualization of the conductive torso properties. We tested this procedure in 180 subjects, and used the squared correlation as a quality index for the resemblance of ECG and ECGr (R^2_{ECG}). On average, the R^2_{ECG} is markedly improved from 0.94 (inverse Dower) to 0.97 to 0.99 when we allow 1-10% adaptations of M. We conclude that improvement in the R^2_{ECG}, and thus individualizing M through our method, should give us a better and individualized VCG.

1. Introduction

Important prognostic electrocardiographic variables (e.g., spatial QRS-T angle, T-wave alternans) are defined / measured in the vectorcardiographic representation of the ECG. The standard electrocardiographic diagnostic tool in clinical practice is the 12-lead ECG, whereas a 3-lead X-Y-Z vectorcardiogram (VCG) is almost never directly recorded. Instead, commercial and experimental ECG analysis software synthesize the VCG mathematically. This is done by multiplying the 8 independent leads I, II, V1-V6 in the 12-lead ECG by an 8x3 transformation matrix M:

$$VCG=M*ECG.$$

It is reasonable to expect that the transformation of the 12-lead ECG to a lower-dimensional VCG will occur at the price of some information loss. We can assess this information loss by reconstructing the originally recorded ECG from the synthesized VCG by multiplying it with the inverse of M, a 3x8 matrix denoted by L:

$$ECGr=L*VCG.$$

Because M is not square, L is its pseudo-inverse:

$$L=inv(M'*M)*M',$$

and the reconstructed ECG

$$ECGr=L*VCG=L*M*ECG$$

is not identical to the original ECG. The use of the pseudo-inverse matrix for back-transformation of the ECG from the synthesized VCG leads to a reconstructed ECG for which the error ε, when expressed in the summed squared differences

$$\varepsilon = (ECGr-ECG)^2$$

is minimized.

A popular VCG synthesis matrix is the inverse Dower matrix [1], called "inverse", because of the earlier defined Dower matrix [2] that synthesized ECGs from VCGs. The original Dower matrix was derived from the tank experiments by Frank [3]. When we record an ECG in a given patient and synthesize a VCG from the recorded ECG by using the inverse Dower matrix, we tacitly assume that a number of prerequisites is fulfilled:

- that the electrical activity of the heart can be represented by a single dipole;
- that the human torso has homogeneous electrical conduction properties;
- that the shape of the torso of all patients is identical to the shape of the artificial Frank torso;
- that the location of the heart in the torso is for all patients identical to the location of the heart in the Frank torso;
- that all electrodes are correctly placed in their standard positions.

Any additional lack of compliance with these conditions results in an increase of ε.

The nice thing is that the value of ε can be computed for each combination of an individually recorded ECG and a given transformation matrix M. It is tempting to

assume that the larger ε, the less reliable the synthesized VCG. The unpleasant thing is that there is currently no methodology available that can improve VCG synthesis in case of unacceptable quality. In the following we explore a method that could be useful in this respect.

2. Methods

As a database we used a set of 180 ECGs made in our outpatient clinic. The ECGs were from 180 subjects (101/79 men/women), 54 ± 17 (19-87) years old, with body mass index (BMI) of 26 ± 4 (17-39) kg/m^2 and body surface area (BSA) of 1.94 ± 0.22 (1.37-2.60) m^2 [4]. The ECG diagnoses of these subjects were: 80 normal, 29 borderline, 18 abnormal frontal plane axis, 25 myocardial infarction, 26 ST-T changes, 12 ventricular hypertrophy, and 12 other diagnosis. The statements were generated by the University if Glasgow ECG Analysis Program. One ECG may have more than 1 diagnostic statement.

We have implemented the errors-in-variables method [5,6]. With this method, a reduction in ε can be achieved at the cost of an additional "error" in the transformation matrix. Obviously, our assumption is that "errors" in the transformation matrix that reduce ε are in fact not errors, but adaptations towards an individualized ECG-to-VCG transformation matrix that complies better with the aforementioned prerequisites than the "one-size-fits-all" inverse Dower matrix.

This errors-in-variables method can be used in such a way that the errors in the reconstructed ECG and in the transformation matrix are balanced; this trade-off is controlled by a parameter λ. Shifting λ from 0 to 1 leads from a situation where the individual transformation matrix Mi equals M to a situation were it deviates as much from M as is needed to reconstruct the ECG optimally. The corresponding synthesized VCG=Mi*ECG changes gradually from the VCG as computed by using the inverse Dower matrix to a version of the VCG corresponding to the singular value decomposition of the ECG. At the same time, the reconstruction error ε decreases.

We expressed the resemblance between the original matrix M and the individualized matrix Mi at a given value of λ in the squared correlation coefficient:

$$R^2_{Matrix}(\lambda) = \text{corr}\,(M, Mi(\lambda))^2.$$

Likewise, we adopted the squared correlation as quality index for the resemblance of ECG and ECGr:

$$R^2_{ECG}(\lambda) = \text{corr}\,(ECG, ECGr(\lambda))^2$$

Finally, we expressed the resemblance of a VCG that was synthesized by the individualized matrix Mi with the VCG synthesized by the fixed matrix M in a squared correlation coefficient:

$$R^2_{VCG}(\lambda) = \text{corr}\,(VCG, VCGi(\lambda))^2$$

For each of the 180 ECGs we have, by shifting λ, searched for situations where $R^2_{Matrix}(\lambda)$ assumed the following values: 100% (Mi=M, the inverse Dower matrix), 99%, 98%, 95%, 90%, and the percentage that corresponded to the singular value decomposition. Additionally, we computed $R^2_{ECG}(\lambda)$ and $R^2_{VCG}(\lambda)$ for all these situations.

All computations in this study were done in the MATLAB (The MathWorks, Natick, Mass; version 7.5.0.342 (2007b)) programming environment.

3. Results

An example of the trade-off between $R^2_{ECG}(\lambda)$ and $R^2_{Matrix}(\lambda)$ is shown in the upper panel of Figure 1. This patient (subject 50) had the lowest value of R^2_{ECG} (0.81) with the unmodified inverse Dower matrix. When tolerating the maximal deviation in the transformation matrix, R^2_{ECG} increased to 0.99; however Mi did not resemble M very well anymore (R^2_{Matrix} = 0.85). This is the singular value decomposition (SVD) state. The SVD VCG of this patient differs dramatically from the original VCG (R^2_{VCG} = 0.09) (Figure 3).

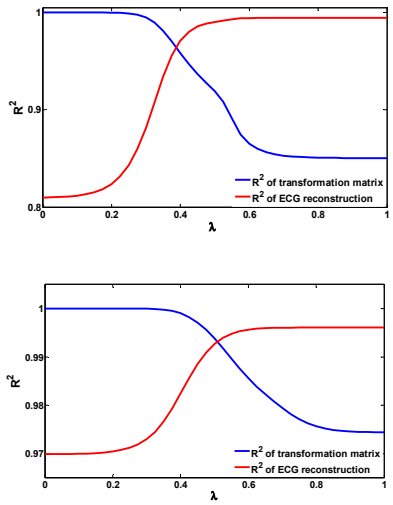

Figure 1. Trade-off between $R^2_{ECG}(\lambda)$ and $R^2_{Matrix}(\lambda)$ as function of λ in subject 50 (upper panel) and 69 (lower panel).

In Figure 1, the lower panel shows similar data, now from the patient (subject 69) with a relatively high value of R^2_{ECG} (0.97) with the unmodified inverse Dower matrix; in the SVD situation R^2_{ECG} has become 0.99, at the cost of a slight reduction in R^2_{Matrix} (till 0.97), while R^2_{VCG} is 0.99 in the SVD situation.

An example of the changes in ECGr and in the synthesized VCG at different levels of tolerated alterations in the ECG-to-VCG transformation matrix in subject 50 is given in Figure 2. Obviously, when the transformation matrix Mi deviates more from the inverse Dower matrix M, R-amplitude differences between the original and the reconstructed ECG gradually disappear, while the VCG shows progressive changes in the QRS and T axes. In the SVD situation, the synthesized VCG differs dramatically from the VCG as generated by the inverse Dower matrix. The progressive changes in M when λ moves from 0 to 1 can be regarded as a change in the electrode positions (along and inside or outside the thorax). For reasons of a visual illustration, we have computed how the electrode positions of subject 50 changed during progressive alterations in the transformation matrix (Figure 3)

Figure 4 gives an overview of the improvements in R^2_{ECG} when the R^2_{Matrix} in the 180 subjects is progressively lowered. It can be seen that substantial reconstruction improvements can already be achieved when R^2_{Matrix} is only slightly lowered to, e.g., 0.99 or 0.98. In the SVD situation, the R^2_{Matrix} varies from 0.64-0.98. As already illustrated by the examples in Figures 1 and 2, it is clear that there are patients where the SVD corresponds to dramatic changes in the transformation matrix, while there are also subjects in which the transformation matrix is only slightly affected.

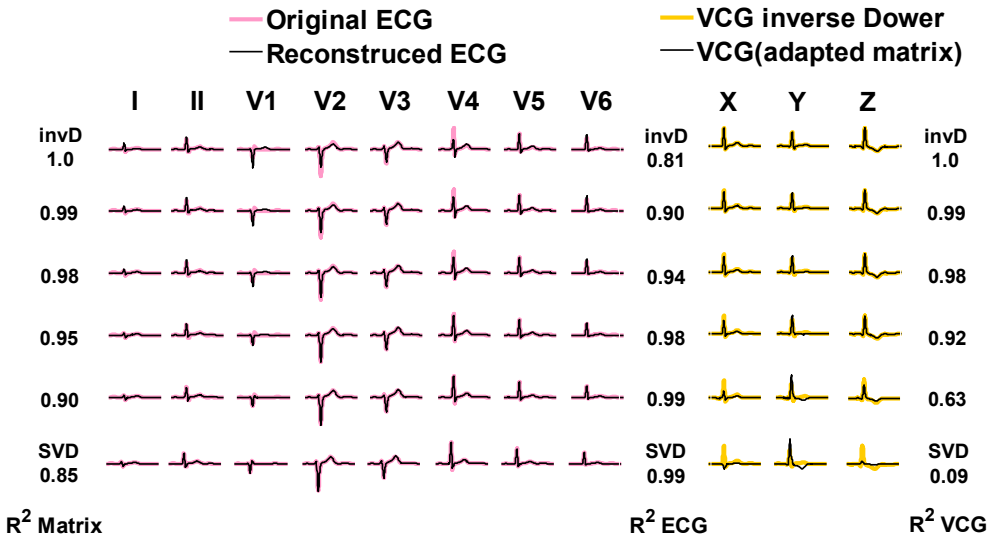

Figure 2. Changes in ECGr and in the synthesized VCG at different levels of tolerated alterations in the ECG-to-VCG transformation matrix in subject 50

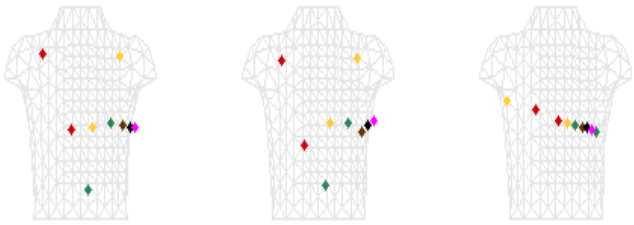

Figure 3. Electrode locations by varying λ in subject 50. From left to right, we see the familiar locations corresponding to the Dower matrix, a slight modification thereof and a complete alteration which is needed to get the optimal ECG reconstruction (singular value decomposition). We have constructed this figure by noting that the transformation matrix M (or Mi) essentially represents the electrode locations in image space. To map these back to the physical space, we have assumed a homogeneous torso with a fixed location dipole. We have also assumed that the Wilson Central Terminal is located infinitely far from the cardiac source.

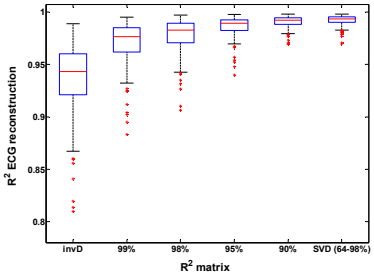

Figure 4. An overview of the improvements in R^2_{ECG} as function of the R^2_{Matrix} in the 180 subjects.

4. Discussion

Our study intends to investigate how we can improve the quality of VCGs synthesized from 12-lead ECGs. We assume that a synthesized VCG is to be mistrusted if the 12-lead ECG that can be reconstructed from the synthesized VCG by using the quasi-inverse of the matrix that was used for VCG synthesis deviates much from the originally recorded ECG.

We have shown that the reconstruction quality can be improved considerably by tolerating a relatively small change in the ECG-to-VCG transformation matrix, and may be improved even much further when we tolerate the maximum change in the transformation matrix in the SVD situation. The question is if, and under which conditions, we can trust the thus modified synthesized VCG more than the VCG as synthesized by the unmodified inverse Dower matrix. While there are good general arguments that an individual ECG-to-VCG transformation matrix is to be preferred above a general ECG-to-VCG transformation matrix [7], it is not clear under which conditions our method provides such an improvement.

Regarding extreme situations, particularly the SVD situation, it is obvious that the VCG produced in this situation cannot always be trusted. The VCGs in Figure 3 show dramatic changes in the QRS and T axes in unlikely directions, while this is a relatively normal electrocardiogram. However, when we impose restrictions on the amount of change in the transformation matrix (e.g., no further reduction of R^2_{Matrix} than 0.98 or 0.95) we remain relatively close to the original matrix and the changes in the matrix more likely represent adaptations to compensate for individual differences in thorax size, conductive properties and electrode placements.

Currently, our research is directed towards algorithms that assess the position of the heart in the thorax and individualize the transformation matrix accordingly. This method imposes more restrictions on the changes in the transformation matrix, which keeps the matrix still closer to the original matrix, thus yielding more certainty about the validity of this individualized matrix.

5. Conclusion

An important measure of the quality of a synthesized VCG=M*ECG, is the reconstruction error (ε) of the ECG. We listed a number of reasons why ε may be quite large. Most importantly, the use of a fixed matrix M does not allow for the anatomic differences that are known to exist between individuals.

We have proposed the errors-in-variables approach to reduce ε by modifying the matrix M for a given individual. However, there is a trade-off: the more we want to improve the reconstruction, the more we must be willing to deviate from M. Too much deviation is unrealistic, but we find that only a slight modification of M (maintaining a correlation of 0.99) already provides a considerable improvement of the reconstruction error. Such a small modification is physically plausible, and we believe that the reduction in ε is a strong indication that the VCG is improved.

References

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