Optimal Design of Neural Stimulation Current Waveforms

Mark Halpern, Member, IEEE

Abstract— This paper contains results on the design of electrical signals for delivering charge through electrodes to achieve neural stimulation. A generalization of the usual constant current stimulation phase to a stepped current waveform is presented. The electrode current design is then formulated as the calculation of the current step sizes to minimize the peak electrode voltage while delivering a specified charge in a given number of time steps. This design problem can be formulated as a finite linear program, or alternatively by using techniques for discrete-time linear system design.

I. INTRODUCTION

Many implanted medical devices rely on electronic circuits providing electric stimulation of nerves. These devices include retinal implants and cochlear implants, where the electrical stimulation is used for transferring information as well as devices for applications where the stimulation is used for motor control. For chronic use in patients, it is necessary to have zero nett charge and the use of charge-balanced rectangular biphasic current pulses for neural stimulation is well established. These pulses usually comprise a constant current stimulating cathodic phase followed by an interphase gap and a constant current charge-balancing anodic phase.

Various kinds of stimulation performance improvement have been sought by varying the waveform of the signal. For instance variations from the basic symmetric rectangular biphasic current pulse have been investigated for their effect on threshold [1], for selective recruitment of different sized fibres [2] and for increasing charge delivery capacity of electrodes [3], [4].

This paper presents an analytical approach for designing an electrode stimulation current waveform to reduce the maximum electrode voltage while delivering a given charge in a specified time. Reducing the maximum electrode voltage is desirable for several reasons. Firstly, it allows the supply voltage to the electronics to be reduced, thereby reducing power loss in the stimulation circuitry. Other approaches for reducing the power dissipated in neural stimulation circuitry are through careful design of current sources [5] or by using a voltage drive waveform designed to match approximately the electrode voltage under constant current drive [10]. Secondly, devices using small feature semiconductor technologies face limits on the allowed size of supply voltage. Existing approaches to enable devices using small feature technologies to be used with larger voltages are through increased circuit complexity [6] or by using a technology which allows both high and low voltage transistors on the same die [7]. A third reason for limiting the maximum electrode voltage is to prevent the formation of undesirable chemical products [8].

This paper uses an approach based on optimization and linear dynamic systems for the design of a stepped current waveform for the stimulation phase, where the step sizes are chosen to minimize the maximum electrode voltage, while transferring a designated quantity of charge in a specified time. A known linear dynamic model of the electrode-tissue interface is assumed. The electrode current design problem can then be tackled with techniques used for control system design since zero-order-hold sampled signals are widely used [9] in digital control systems where a digital computer is used to compute signals to drive an analog device in order to achieve desired performance. With the framework used here, the problem of delivering given charge with minimum electrode voltage is closely related to that of designing a current waveform which maximizes charge delivered under the constraint of not exceeding a designated voltage level.

The layout of this paper is as follows. Section II contains the problem formulation and notation used. The main results of the paper are in Section III which contains the presentation of the current parametrization as well as two approaches for obtaining the optimal design. The first of these uses a finite linear program while the second is a direct approach based on features of the optimal solution and uses elements from the theory of linear dynamic systems. That direct approach leads to a closed-form solution for the minimum peak electrode voltage. Section III also contains more general results concerning how the solution changes when the transferred charge is altered and looks at the problem of maximizing transferred charge without exceeding a given bound on the electrode voltage. Section IV contains a numerical example designed to illustrate the results of the paper.

II. PROBLEM SETUP AND NOTATION

The problem considered in this paper is the design of the stimulation current phase, the first part of a biphasic waveform, for the delivery through a pair of electrodes of a specified charge Q coulombs over a specified stimulation phase duration T seconds. For convenience of presentation, the charge is specified positive ie Q > 0, while it is known that the stimulation phase is usually negative. The actual negative stimulation phase would be obtained by changing the signs of the currents from the calculated values.

M. Halpern is with National Information and Communication Technologies Australia (NICTA) Victoria Research laboratory (VRL), Department of Electrical and Electronic Engineering, University of Melbourne, Parkville, Victoria 3010, Australia mark.halpern@nicta.com.au.NICTA is funded by the Australian Government as represented by the Department of Broadband, Communications and the Digital Economy and the Australian Research Council through its ICT Centre of Excellence program.



Fig. 1: Circuit used to model the electrode-tissue interface.

A. Notation

Voltages and currents which are functions of time t are denoted by lower case letters such as v(t). Samples of v(t) taken at time intervals $t = kT_s$ where k = 0, 1, 2, ... are denoted v_k , shorthand for $v(kT_s)$. The z-transform of a sequence $h = \{h_k\}_{k=0}^{\infty}$ is denoted $\hat{h}(z)$ and is given by

$$\hat{h}(z) = \sum_{k=0}^{\infty} h_k z^k.$$
(1)

With this convention, a stable transfer function has all its poles at values of z : |z| > 1. Also the symbol z denotes the unit delay.

III. STIMULATION SIGNAL DESIGN

A. Model of the Electrode-Tissue Interface

The electrode-tissue interface is modelled with the circuit [11] of Fig. 1 comprising access resistance R_a ohms, doublelayer capacitance C Farads and Faradaic resistance R ohms. It is assumed the values of these three parameters are known.

With constant electrode current $i(t) = i_0$ applied for t > 0, the internal voltage w(t) and the electrode voltage v(t), both for t > 0 are given by

$$w(t) = w(0)e^{-t/RC} + i_0 R(1 - e^{-t/RC}), \qquad (2)$$

$$v(t) = w(t) + i_0 R_a.$$
 (3)

B. Parametrization of Stimulation Current

Firstly, the stimulation phase duration T is broken up into a whole number, n, of discretization time intervals each of duration T_s . Thus $T = nT_s$, where n is a positive integer. Setting n = 1 specifies the standard constant current stimulation phase. The current i(t) is parametrized to be piecewise constant over time intervals T_s as follows:

$$i(t) = \begin{cases} 0; & t \le 0, \\ i_k; & kT_s < t \le (k+1)T_s; \ k = 0, 1, ..., n-1, \\ 0; & t > nT_s. \end{cases}$$
(4)

The desired charge Q of the stimulation phase is obtained by setting

$$\sum_{k=0}^{k-1} i_k = \frac{Q}{T_s}.$$
 (5)

From (4), i(t) takes on n values i_k , which are constrained to satisfy (5). If the current has the form (4), then over each time interval given by $kT_s < t \leq (k+1)T_s$ where $k = 0, 1, \ldots, n-1$, voltage w(t) is given by

$$w(t) = w(kT_s)e^{-(t-kT_s)/RC} + i_k R(1 - e^{-(t-kT_s)/RC})$$
(6)

and v(t) is given by

$$v(t) = w(t) + i_k R_a. \tag{7}$$

C. Linear Programming Approach to Minimizing Peak Electrode Voltage

From (2) and (3), the maximum value of v(t) in response to a current step occurs either at the beginning or the end of that current step. Thus the problem of minimizing the peak electrode voltage in response to a current of the form (4) requires consideration of v(t) only at sample times $t = kT_s$. This allows the electrode-tissue dynamics to be represented by discrete-time versions of (2) and (3) namely

$$w_k = \alpha w_{k-1} + i_{k-1} R(1 - \alpha), \tag{8}$$

$$_{k} = w_{k} + i_{k-1}R_{a} \tag{9}$$

where k = 1, 2, ..., n and

v

$$\alpha = e^{-T_s/RC}.$$
 (10)

Denoting the minimum peak value of v(t) by J, the problem of calculating J can be formulated as the following finite linear program where γ is a variable introduced to bound v_k .

$$J = \min_{i_k, w_k, v_k} \gamma \tag{11}$$

subject to

$$i_k \geq 0; \ k = 0, 1, \dots, n-1,$$
 (12)

$$\sum_{k=0}^{i-1} i_k = \frac{Q}{T_s},$$
(13)

$$w_0 = 0, \qquad (14)$$

$$w_k = \alpha w_{k-1} + i_{k-1} R(1-\alpha);$$

$$k = 1, 2, \dots, n, \tag{15}$$

$$v_k = w_k + i_{k-1}R_a; \ k = 1, 2, \dots, n, \ (16)$$

$$v_k \leq \gamma; \ k = 1, 2, \dots, n. \tag{17}$$

This optimization problem can be solved numerically to determine the current step sizes and the minimized peak electrode voltage. The voltages w(t) and v(t) between the sample values can be calculated from (6) and (7). There is scope to modify the problem by the addition of further inequality or equality constraints on variables i_k, v_k, w_k . For example bounds could be placed on the values of some of the i_k or on their rate of change.

D. Direct Calculation of Stimulation Current

In this section, an alternative approach for obtaining the solution to the problem (11)–(17) without solving a numerical optimization is presented. It can be shown that the solution to the optimization problem (11)–(17) has the property that the electrode voltage satisfies $v_1 = v_2 =$ $\cdots = v_n > 0$. This enables the values of i_0, \ldots, i_{n-1} and v_1, \ldots, v_n to be constructed directly. The procedure involves three steps.

Firstly, the discrete-time transfer function h(z) of the electrode-tissue equivalent circuit voltage response at times $t = kT_s$ to a unit step current applied over one sample time $0 < t \leq T_s$ is determined. Then the relation between the electrode voltage samples and the current values is given by

$$\hat{v}(z) = \hat{h}(z)\hat{i}(z). \tag{18}$$

Eliminating w_k from (8) and (9) gives

$$\hat{h}(z) = \frac{b_1 z + b_2 z^2}{1 - \alpha z}$$
(19)

where

$$b_1 = R_a + R(1 - \alpha),$$
 (20)

$$b_2 = -\alpha R_a. \tag{21}$$

Secondly, a stepped electrode current denoted $\hat{f}(z)$ with the form of (4) which would give an electrode voltage satisfying

$$v_0 = 0, v_k = 1; k = 1, 2, \dots$$
 (22)

or equivalently

$$\hat{v}(z) = \frac{z}{(1-z)} \tag{23}$$

is calculated. Now $\hat{f}(z)$ such that

$$\hat{v}(z) = \hat{f}(z)\hat{h}(z) = \frac{z}{(1-z)}$$
 (24)

is given by

$$\hat{f}(z) = \frac{1}{(1-z)} \frac{z}{\hat{h}(z)}$$
 (25)

$$= \frac{1}{(1-z)} \frac{1/b_1(1-\alpha z)}{(1+(b_2/b_1)z)}$$
(26)

Thirdly $\hat{f}(z)$ is truncated to *n* terms and then scaled to give a current $\hat{i}(z)$ which satisfies the charge constraint (5):

$$\hat{i}(z) = \frac{Q}{T_s} \frac{\sum_{j=0}^{n-1} f_j z^j}{\sum_{j=0}^{n-1} f_j}.$$
(27)

Moreover the value of v_1, v_2, \ldots, v_n is given by the scaling factor above, so that

$$J = \frac{Q}{T_s} \frac{1}{\sum_{j=0}^{n-1} f_j}.$$
 (28)

This approach can be used to calculate numerical solutions, identical to those from (11)–(17). Furthermore a closed-form solution for the minimum value of the peak electrode voltage can be obtained by resolving (26) into two partial fractions,



Fig. 2: Electrode current with n = 1.



Fig. 3: Electrode voltage with n = 1.

followed by truncating the individual series expansions and scaling to obtain:

$$J = \frac{Q}{T_s} \frac{(R_a + R)^2 (1 - \alpha)}{(R_a + R)(1 - \alpha)n + R_a (1 - (\frac{R_a \alpha}{R_a + R(1 - \alpha)})^n)}.$$
(29)

E. More General Results

Suppose the solution to (11)–(17) for given parameters and charge Q_0 has a minimum peak electrode voltage of value $J(Q_0)$. Then the following hold:

1) Scaling with charge: If only the charge is changed to $Q = cQ_0$ where c > 0, the solution to (11)–(17) becomes

$$J(cQ_0) = cJ(Q_0); \quad c > 0,$$
(30)

2) Maximizing charge with given bound on electrode voltage: Given $\beta > 0$

$$\max_{v_k \le \beta} Q = \frac{Q_0}{J(Q_0)}\beta.$$
(31)

IV. EXAMPLE

This example uses electrode-tissue interface parameter values $R_a = 1100\Omega$, $C = 0.98\mu$ F, $R = 10k\Omega$, loosely adapted from those in [10], with charge parameters Q =



Fig. 4: Optimized electrode current with n = 5.



Fig. 5: Optimized electrode voltage with n = 5.

 1μ C, T = 5ms, used in [10]. A constant-current stimulation phase is obtained by setting n = 1. For this case, there is no scope for optimization. The electrode voltage is obtained by evaluating (6) and (7) with initial condition w(0) = 0 and current i_0 given by (5). Plots are shown in Figs. 2 and 3.

To illustrate the approach shown in this paper, a five-step current waveform is obtained by setting $T_s = 1$ ms and n = 5. Solving (11)–(17) gives the sampled voltages w_k and v_k and the currents i_k . The voltage values between time samples are obtained from (6) and (7). Electrode current and voltage plots are in Figs. 4 and 5.

Results for various values of n and T_s chosen to keep the stimulation phase duration $T = nT_s$ constant at 5ms are shown in Table I, obtained using (29). For this example, the maximum electrode voltage can be reduced by approximately 21% through the use of this approach. Most of the performance improvement is achieved with 5–10 steps.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

A neural stimulation current design approach using a current waveform comprising piecewise constant segments with regular time intervals between transitions in place of the often used single constant current has been presented.

TABLE I: Values of minimized peak electrode voltage for a 5ms stimulation phase.

n	T_s (ms)	J (volts)	Voltage reduction (%)
1	5	1.019	0
2	2.5	0.906	11
5	1	0.843	17
10	0.1	0.823	19
100	0.05	0.807	21
1000	0.005	0.806	21

The use of numerical optimization using a finite linear program to compute the current step sizes to minimize peak electrode voltage has been demonstrated. A direct approach for synthesizing the optimal current steps is also given.

B. Future Works

Experimental work is required to verify how effectively the approach works. As it stands, direct application of the results in this paper to an *in-vivo* situation would require estimates of the parameters of the circuit model for the electrode-tissue interface. This may not be convenient and the approach can be modified to use a more directly identified dynamic model of the impedance of the electrode-tissue interface.

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