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ECG Denoising Using Modulus Maxima of Wavelet Transform

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Abstract— **ECG denoising has always been an important issue in medical engineering. The purposes of denoising are reducing noise level and improving signal to noise ratio (SNR) without distorting the signal. This paper proposes a method for removing white Gaussian noise from ECG signals. The concepts of singularity and local maxima of the wavelet transform modulus were used for analyzing singularity and reconstructing the ECG signal. Adaptive thresholding was used to remove white Gaussian noise modulus maximum of wavelet transform and then reconstruct the signal.**

Keywords— ECG Denoising, Wavelet Transform, singular points, Lipschitz exponents.

I. INTRODUCTION

ECG signals are originated from the electrical activities and stimulate the heart muscles. Signals generated from these activities distribute in body and are picked up by especial electrodes placed on skin. On the route to the pick up points, the signal passes a variety of tissues with different characteristics. In addition to deformations due to inherent non-linear model of the paths, the ECG signal receives contamination from electrical disturbances of alternate sources; including but not limited to EMG, thermal noise and radio frequency interference (RFI). It is common practice to model the unwanted noise by white Gaussian noise.

ECG denoising has always been an important issue in medical signal processing. The goal of denoising is decreasing the noise and increasing SNR; while preserving as much of the information in the signal as possible [1]

Various approaches have been used to remove Gaussian white noise [1-3]. We used singular point properties based on wavelet transform to remove the noise. Singular points carry much of the information content in any signal [4].

We used wavelet to detect singular point. Adaptive thresholding removes the singular points of noise and then the signal is reconstructed.

The organization of this paper is as follows: In section II we briefly introduce ECG signal. Methodology is the subject of section III, in this section, biorthogonal quadratic spline wavelet and its usage for proposed method have been concisely introduced. Readers can refer to [8][9] for more details. In section IV we present our method and compare it with other wavelet based method. Finally, summary and conclusion are provided in section V.

II. THE HEART'S ELECTRICAL CONDUCTION SYSTEM

Heart is a muscle tissue that pumps blood into body. This multi-cavity compound pump needs a source of stimulation to trigger the involved cavities. An inborn electrical conduction system issues these stimulations.

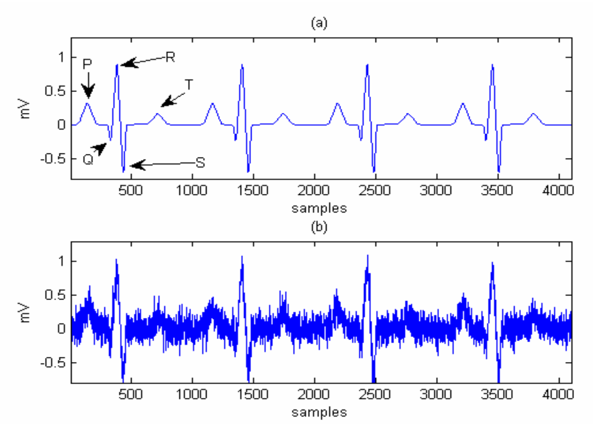

Fig. 1.Synthetic ECG data[5], a) Noise free ECG b) noisy ECG ($SNR = 4db$)

Each beat of the heart generates a series of deflections from the baseline on the ECG. These deflections show the electrical activity in the heart causing muscle contraction. There are some turning points in each cycle of ECG labeled sequentially with the letters P, Q, R, S, and T[5] (Fig. 1,a).

III. MATERIALS AND METHODS

A. Wavelet Transform

The wavelet transform of $x \in L^2(R)$ at time u and scale s is defined as:

$$W_x(u,s) = \langle x, \psi_{u,s} \rangle = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{s}} \psi^*(\frac{t-u}{s}) dt$$

(*) denotes the complex conjugate. $\psi(t)$ is known as mother wavelet or wavelet function. For discrete application the scale factor is selected from dyadic sequence $s = 2^j$ ($j \in Z$, Z is the integers set). Depending on the application, particular initial conditions and specific properties are used to create suitable mother wavelets. The wavelet we used is categorized as a biorthogonal quadratic spline wavelet with compact support and one vanishing momentum. This wavelet is suitable for detecting singular points. The Fourier transform of this wavelet is:

$$\psi(\omega) = j\omega \left(\frac{\sin(\frac{\omega}{4})}{\frac{\omega}{4}} \right)^4 \qquad (j)^2 = -1$$

The dyadic wavelet of a digital signal $x(n)$ can be calculated with two analyzing and synthesizing filter banks. These two filter banks can be implemented using three discrete filters h, g and k with Fourier transforms[6-7]:

$$H(\omega) = e^{j\frac{\omega}{2}} \left(\cos(\frac{\omega}{2}) \right)^3$$

$$G(\omega) = 4je^{j\frac{\omega}{2}} \sin(\frac{\omega}{2})$$

And

$$K(\omega) = \frac{1 - |H(\omega)|^2}{G(\omega)}$$

The analyzing filter bank F_a^j consists of J+1 filters, defined as:

$$F_a^j = \begin{cases} G(\omega) & j = 1 \\ G(2\omega)H(\omega) & j = 2 \\ G(2^{j-1}\omega)H(2^{j-2}\omega)...H(\omega) & 3 \le j \le J \\ H(2^{j-1}\omega)H(2^{j-2}\omega)...H(\omega) & j = J+1 \end{cases}$$

The wavelet coefficients of signal $x(n)$ are the outputs of these filters. With these filters we can decompose $x(n)$ as follows:

$$x(n) = f_s^{J+1} * S_x(2^J, n) + \sum_{j=1}^{J} f_s^j * W_x(2^j, n)$$

where (*) denotes convolution operator and $W_x(2^j, n), S_x(2^j, n)$ can be calculated by convolving $x(n)$ and time domain coefficient of the filter F_a^j (denotes by f_a^j)

$$W_x(2^j, n) = f_a^j * x(n) \quad 1 \le j \le J$$
$$S_x(2^J, n) = f_a^{J+1} * x(n)$$

Synthesizing filter bank is defined as follows:

$$F_s^j(\omega) = \begin{cases} K(\omega) & j = 1 \\ K(\omega)K^*(\omega) & j = 2 \\ K(2^{j-1}\omega)H^*(2^{j-2}\omega)\cdots H^*(\omega) & 2 < j \le J \\ H^*(2^{j-1}\omega)H^*(2^{j-2}\omega)\cdots H^*(\omega) & j = J+1 \end{cases}$$

B. Singular points

Reference [4] shows that most of the signal information is carried by its irregular structures and singular points. In mathematics, singularity is often consider as opposite of smoothness and can be measured by Lipschitz exponent α .

A function x is point wise Lipschitz $\alpha \ge 0$ at v, if there exists $K > 0$, and polynomial p_v of degree $m = \lfloor \alpha \rfloor$ ($\lfloor \alpha \rfloor$ defines largest integer $m \le \alpha$) such that

$$\forall t \in R, \ | x(t) - p_v(t) | \le K | t - v |^\alpha . \qquad (1)$$

A function is uniformly Lipschitz α over $[a,b]$ if it satisfies (1) for all $v \in [a,b]$ with a constant K that is independent of v. The Lipschitz regularity of x at v or over $[a,b]$ is sup of the α such that f is Lipschitz α . If f is uniformly Lipschitz $\alpha > m$ in the neighborhood of v, it can be verified that x, is m times continuously differentiable in this neighborhood and if $\alpha < 1$ at v, then x is not differentiable and v is a singular point [8]. With wavelet this singularity can be detected.

C. Detection and Reconstruction

The relation between the singular points of signal and wavelet is explained below:

Let n be a strictly positive integer. Let $\psi(t)$ be a wavelet with compact support, n vanishing moments and n times continuously differentiable. Let $x(t) \in L^1([a,b])(\int_a^b |x(t)| dt < +\infty)$.

If there exists a scale $s_0 > 0$ such that for all scales $s<s_0$ and $x \in]a,b[$. $|W_x(s,t)|$ has no local maxima, then for any $\varepsilon>0$ and $\alpha<n$, x(t) is uniformly Lipschitz α in $]a+\varepsilon,b-\varepsilon[$.

Therefore, assuming that $\psi(t)$ is the n-th derivative of a so-called smoothing function, it can be shown that singular points can be detected by modulus maxima of the wavelet (see [8]). Once these singular points are known, straight forward optimization techniques can be employed to reconstruct the signal.

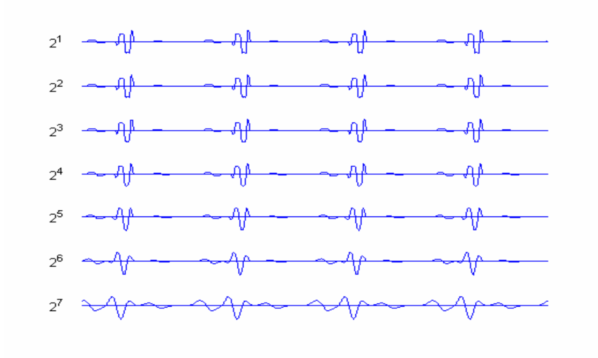

Fig. 2. Wavelet coefficient of ECG signal in different scale.

IV. PROPOSED METHOD

An important step in wavelet denoising is the comparison of the wavelet coefficients of the signal transform with thresholds. Based on this comparison, these coefficients will be adjusted to obtain the wavelet coefficients of transform of a less noisy signal.

Let the noisy signal be $NECG=ECG+\sigma N$, where N is white noise with zero mean and variance σ^2 . Normally either hard or soft thresholding on coefficients of the discrete wavelet transform (WT) is used to suppress the noise. Assume that $d_{j,k}$ represent the coefficients of WT in scale j . Appropriate hard or soft thresholding can give us $\hat{d}_{j,k}$ which are an approximation of the coefficients of de-noised transform.

In hard thresholding

$$\hat{d}_{j,k} = \begin{cases} d_{j,k} & \text{if} \quad |d_{j,k}| \geq t \\ 0 & \quad otherwise \end{cases}$$

In soft thresholding

$$\hat{d}_{j,k} = \begin{cases} sign(d_{j,k})(|d_{j,k}| - t) & \text{if} \quad |d_{j,k}| \geq t \\ 0 & \quad otherwise \end{cases}$$

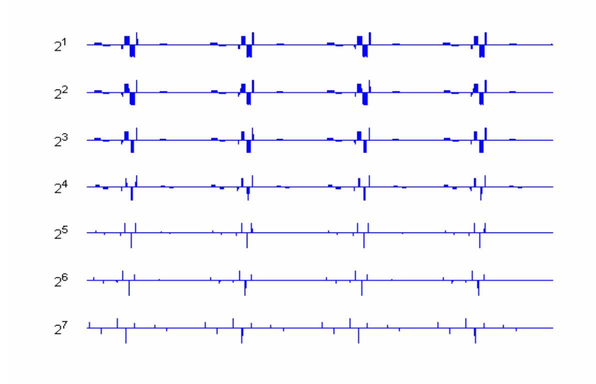

Fig. 3. Modulus maxima of Wavelet coefficient of ECG

where threshold t is given by $\sigma\sqrt{2\log N}$ and N is the number of samples and $\sigma = median(d_{1,k}) / 0.6745$ [10]. Inverse WT of $\hat{d}_{j,k}$ recovers a less noisy ECG.

In our proposed method this thresholding is applied on individual modulus maxima of wavelet transform and thus develop a method to reduce white noise from ECG signals.

The ECG signal is first decomposed [9] in various scales (see figure 2, 3, 4 and 5). Figure 5 shows that Noise modulus Maxima is only visible on Lower scale and amplitudes of these modulus maxima is smaller than ECG modulus Maxima so if we remove modulus maxima smaller than a threshold (see figure 6) and reconstruct signal (figure 9), white noise can be suppressed.

In order to evaluate our method and compare it with conventional wavelet method, we have calculated signal to noise ratio (SNR) of our results, with different white noises in -6db to 6db range. After denoising with the proposed method, output SNR was evaluated. Input SNR was

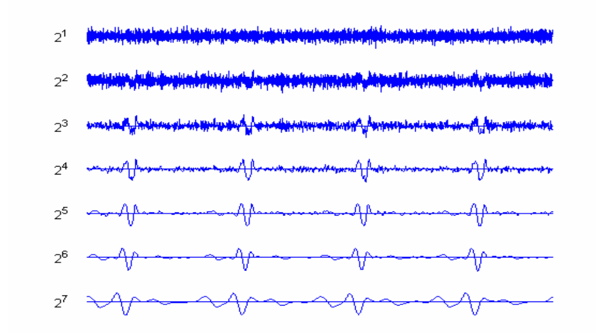

Fig. 4. Wavelet coefficient of noisy ECG signal in different

evaluated as:

$$SNR_{input} = 10\log_{10}\frac{||ECG_{original}||}{||N||}$$

$||ECG_{original}||$ is power of original ECG signal and $||N||$ is power of white noise. The output SNR is calculated by equation:

$$SNR_{output} = 10\log_{10}\frac{\|ECG_{output}\|}{\|ECG_{original} - ECG_{output}\|}$$

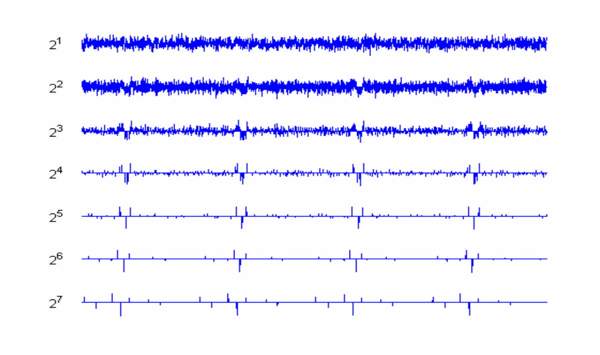

Fig. 5. modulus maxima of Wavelet coefficient of noisy ECG signal

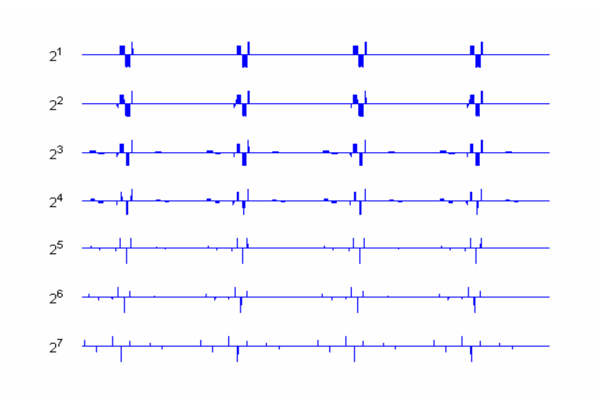

Fig. 6. Modulus maxima of Wavelet coefficient of noisy ECG signal after soft thresholding. after thresholding

In figure 7, we have compared our method with common wavelet method. We have use Daubechies 6 in common wavelet method which has best result among other wavelets. In this figure, the output SNR vs. input SNR is plotted and, the results show the effectiveness of our proposed method objectively.

Figure 8 shows that this method does not change the morphology of ECG for visual assessment.

V. CONCLUSION

Based on wavelet transform and singular points a new method was developed to reduce white noise from ECG signal. Quadratic spline wavelet was used to detect singular points of ECG and thresholds were set to remove noise. Finally the SNR of output signal was evaluated. Objective and visual assessment of the result have shown effectiveness of propose method.

Proposed method have been used for denoising ECG signal, However it can be used as general method in other areas of signal processing.

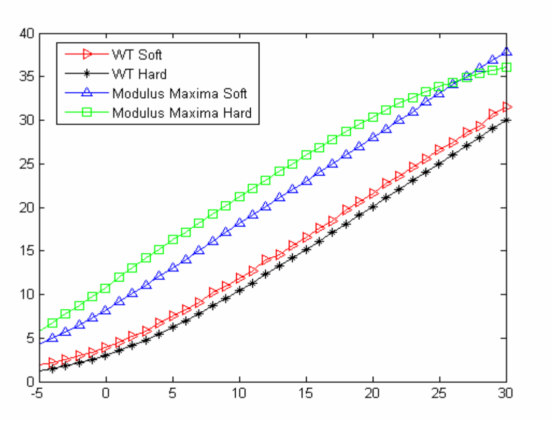

Fig. 7. SNR_{out} Versus SNR_{input} of WT and proposed

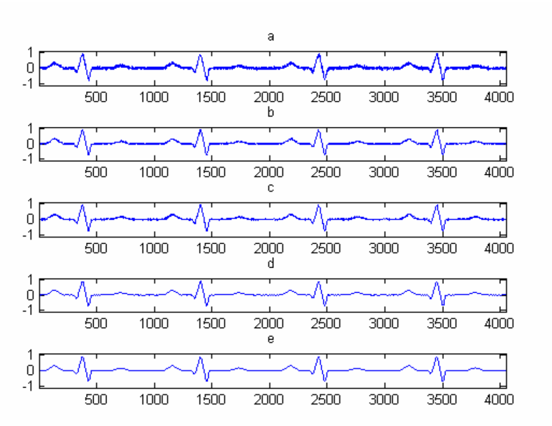

Fig. 8. a) Noisy ECG, b) De-noising by WT with soft thresholding, c) Denoising by WT with hard thresholding, d) Denoising by modulus maxima with soft thresholding, e) Denoising by modulus maxima with soft thresholding.

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