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Application of Polarimetry Group Theory for Characterization of Biological Tissues via Mueller Coherency Matrix Analysis

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***Abstract*—Optical characterization of biological tissues provides advantages like the non-invasive or non-contact characters, or an increased image resolution. The use of the polarization information, apart from the intensity, leads to new data for a better diagnosis. In this work, we use the Group Theory applied to polarimetry to analyse the polarization behaviour of samples. The SU(4)-O+(6) homomorphism allows us to obtain the Mueller Coherency matrix from the Mueller matrix, and applying the target decomposition theorem, which provides information on tissue structure and separates different polarization effects by means mainly of the eigenvalues and eigenvectors, tissue imaging contrast can be increased. The analysis is applied to glucose suspensions of polystyrene spheres of different concentrations, whose behaviour can be modelled by means of single or multiple scattering depending on the concentration, either in the Rayleigh or Mie regimes. The results could be applied to cell cultures, where cancerous cells grow without control, or even to some anemia pathologies, where the number of erythrocytes in blood decreases.**

I. Introduction

Optical techniques in characterization of biological tissues present advantages like being harmless, non-invasive, without contact and with a very good resolution [1]. Optical characterization of tissues is usually related with intensity measurements, that allows the achievement of partial optical information from tissues. The fact that most tissues have intrinsic and/or structural anisotropy, makes that polarization parameters can add important information to the images acquired, in such a way that hidden compounds or structures, significant from the point of view of diagnosis, may appear. Furthermore, scattering from tissues, that can also be anisotropic, changes the degree of polarization of light and this is reflected in polarization parameters [2]. For instance, blood or adipose tissues present no significant anisotropy but scattering due to the particles involved in their composition. On the other hand, collagen fibers like tendons show anisotropy as a consequence of their structural orientation. Polarimetry is an optical technique focused on the measurement of polarization properties of samples including the properties of depolarizing optical media. Polarimetric techniques are specially appropriate for biological tissues, due to the fact that their properties show dependence with the polarization of light. They usually exhibit a depolarising behaviour. Methods of analysis that do not take into account tissue depolarisation, like Jones matrix, produce limited results. The extension of these characterization techniques to Mueller matrix measurement [3] adds data to the image obtained, but further information can be extracted.

In this work, the Group Theory is applied to polarimetry in order to obtain more information of the polarization behaviour of biological tissues. By means of the SU(4)-O+(6) homomorphism, a complex Mueller Coherency matrix can be extracted from the usual Mueller matrix [4]. The Mueller Coherency matrix analysis is applied to glucose suspensions of polystyrene spheres of different concentrations. These suspensions could simulate cell cultures, where healthy ones proliferate to some extent, whereas cancerous cells grow without control, or even some anemia pathologies, where the number of erythrocytes in blood decreases [5,6]. The concentration of the scatterers determines if the radiative process can be modelled by a simple or a multiple scattering approach [7]. The relative relation between the size of the scatterers and that of the applied wavelength shows if we are in the Rayleigh or the Mie regimes [8]. Depolarisation of radiation is strictly related with this multiple scattering process, and a relationship between their concentration and the degree of polarization can be established.

Next section shows the theoretical model of single and multiple scattering, and also the implications in tissue depolarisation behaviour, depending on whether Rayleigh or Mie regimes apply. Afterwards the Mueller Coherency Matrix method is presented. In section 4 the experiment and Mueller matrices measurement of suspensions of different concentrations are exposed. Finally, the Mueller Coherency matrix analysis is applied and results are discussed.

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II. POLARIZATION ANALYSIS OF SINGLE AND MULTIPLE SCATTERING DEVICES

Scattering is produced by inhomogeneities in the sample, whose different refraction index makes radiation being deviated by a particular angle. The distribution of radiation depends mainly on the relationship between the scatterer size and the wavelength of the radiation [8]. According to this parameter, Rayleigh, Mie and geometrical regimes are applied. The density of scatterers has also its importance in the study, due to the mutual interaction of radiation coming from different particles. Under this point of view, there are mainly two approaches, single and multiple scattering [7]. In the first case, radiation is considered to be detected after only suffering one scattering event, and so only its state but not its degree of polarization is affected. With multiple scattering, however, the fact that radiation can undergo a great number of scattering events, and subsequent interaction among light coming from different particles after being scattered a random number of times, make that the electric field rotation randomizes and so not only the state but also the degree of polarization change.

Our interest now is in the Mueller matrix resulting from spatially random media. If the scatterers size is minor than the radiation wavelength, then the Rayleigh approach applies, and taking into account that we are in a medium with specific symmetries, the Mueller matrix would have the following form [7]:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & m_{11} & 0 & 0 \\ 0 & 0 & m_{22} & 0 \\ 0 & 0 & 0 & m_{33} \end{bmatrix} \quad (1)$$

The expression of each one of the elements of Mueller matrix of equation 1 can be calculated from Belthe-Salpeter equation as a function of the number of scattering events n+1:

$$m_{11}(n) = m_{22}(n) = \frac{3\cdot\left(\frac{7}{10}\right)^n}{2+\left(\frac{7}{10}\right)^n} \quad (2)$$

$$m_{33}(n) = \frac{3\cdot\left(\frac{1}{2}\right)^n}{2+\left(\frac{7}{10}\right)^n} \quad (3)$$

It is clear from the expressions that the output degree of polarization will be in general different from the input degree of polarization, and so the medium can depolarize the incident radiation as it was expected for multiple scattering. The degree of polarization goes down as the number of scatterers increases, as a consequence of the randomisation of the electric field rotation after combining radiation scattered a random number of times.

III. MUELLER COHERENCY MATRIX METHOD

There are techniques that take advantage of the Mueller matrix measurements [9], but the information obtained is limited because the direct interpretation of the Mueller matrix elements is difficult to relate with tissue structure or composition. Group Theory is a mathematical method widely applied to fields such as quantum mechanics [10]. Its use in polarimetry allows the extraction of useful information by means of different transformations, like homomorphisms, in which a relationship between complex and real sets is established [4]. One of the best known homomorphisms is the SU(2)-O+(3), that applied to polarimetry allows the calculation of the real Stokes vector from the complex Coherency matrix, and the useful mapping of this vector on the Poincaré sphere. It also decomposes the coherent scattering matrix or Jones matrix in a base of 2x2 matrices like Pauli matrices. The complex coefficients or Quaternions of this decomposition allow the construction of a complex target vector. However, here we pay attention to the SU(4)-O+(6) homomorphism, which has more to do with polarization devices rather than polarization radiation. By means of this mathematical transformation, the Mueller Coherency matrix is obtained from the Mueller matrix, and the decomposition of the former in a base of 4x4 matrices like Dirac matrices provides four target vectors, which further represent a scattering matrix, and four eigenvalues, which show the importance of each one in the general behaviour of the device [4].

The great potentiality of the Mueller Coherency matrix can be obtained from the eigenvalue analysis [11,12]. A maximum of four non-zero eigenvalues λ_i, with their corresponding eigenvectors or target vectors C_i, can be extracted from its decomposition:

$$\mathbf{C}_{4x4} = \lambda_1 \mathbf{C}_1 + \lambda_2 \mathbf{C}_2 + \lambda_3 \mathbf{C}_3 + \lambda_4 \mathbf{C}_4 \quad (4)$$

The number of significant (non-zero) eigenvalues and their values are directly related to the depolarizing characteristics of the optical media. In polarization maintaining media, there is only one significant eigenvalue, and then a direct correspondence between the Jones and Mueller matrices can be established [7,13]. The target vector corresponding to the dominant eigenvalue can be transformed in a coherent scattering matrix or Jones matrix, and the polarization behaviour of the sample can be interpreted according to simple polarization devices, like polarizers or retarders. However, in depolarizing devices, there is more than one significant eigenvalue, and the information contained in the eigenvalues and eigenvectors coming from the Mueller Coherency matrix decomposition after applying the SU(4)-O+(6) homomorphism is much more complete, and can provide further information about tissue behaviour. Depolarizing characteristics, strongly related with tissue structure, and also behaviour paralelisms

with usual optical non-depolarizing devices can be deduced from this analysis, depending on the strength of the depolarizing characteristics of the sample. Further polarization parameters can be obtained, like the crosstalks or rotation angles, coming from the associated Jones matrix of this equivalent optical device [4,13].

IV. Mueller Coherency Matrix Analysis of Varying Concentration Suspensions

We will now apply these results to glucose suspensions of polystyrene spheres of different concentrations, as a way of showing how this method can provide additional information to the optical characterization.

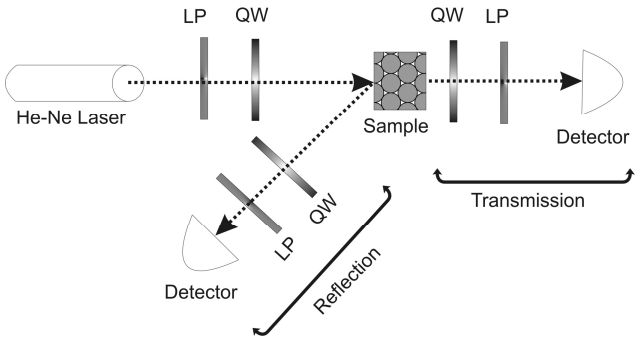

Fig. 1. Polarimeter configuration in transmission and reflection, where LP is a Linear Polarizer, and QW is a Quarter Wave Plate.

A polarimeter composed by linear polarizers and quarter wave plates, with an He-Ne optical source (632.8 nm), was used to measure Mueller matrices of suspensions [14]. The configuration can be seen in Figure 1.

TABLE 1
Measured Mueller Matrices of Different Samples

Sample	Mueller matrix
Linear retarder + glucose suspension	$M_1 = \begin{bmatrix} 1 & 0.09 & -0.093 & -0.2 \\ 0.155 & 0.874 & 0.119 & -0.435 \\ -0.179 & 0.303 & 0.487 & 0.804 \\ 0.029 & 0.310 & -0.837 & 0.383 \end{bmatrix}$
Glucose suspension of spheres (μ_s=0.6 mm^{-1}) in transmission	$M_2 = \begin{bmatrix} 1 & 0.026 & 0.044 & -0.039 \\ 0.029 & 0.962 & -0.144 & -0.047 \\ 0.002 & 0.126 & 0.975 & 0.026 \\ -0.039 & 0.019 & 0.115 & 0.936 \end{bmatrix}$
Glucose suspension of spheres (μ_s=5 mm^{-1}) in transmission	$M_3 = \begin{bmatrix} 1 & -0.009 & -0.021 & -0.041 \\ -0.002 & 0.256 & -0.029 & -0.003 \\ 0.024 & 0.045 & 0.235 & -0.032 \\ 0.041 & 0.024 & 0.017 & 0.538 \end{bmatrix}$
Glucose suspension of spheres (μ_s=0.6 mm^{-1}) in backscattering	$M_4 = \begin{bmatrix} 1 & -0.115 & -0.066 & 0.023 \\ -0.111 & 0.759 & -0.061 & -0.001 \\ -0.018 & 0.151 & -0.435 & -0.139 \\ -0.046 & 0.006 & 0.128 & -0.334 \end{bmatrix}$

The samples were composed by aqueous glucose 5M suspensions of polystyrene spheres with a mean diameter of 2 μm, and a related anisotropy of scattering parameter of g=0.91, and they were kept in a 10 mm cuvette. The relationship between wavelength and scatterers size locates the process in the Mie regime, as discussed in previous section. Measurements of transmission and backscattering configurations were performed. In the former, the exact forward measurement direction was taken, while in the latter an angle of around 30º was kept to avoid direct Fresnel reflection.

Table 1 shows the Mueller matrices measured for suspensions of different concentrations of scatterers and different measurements configurations, and also one of a linear retarder and a glucose suspension with no scatterers [14].

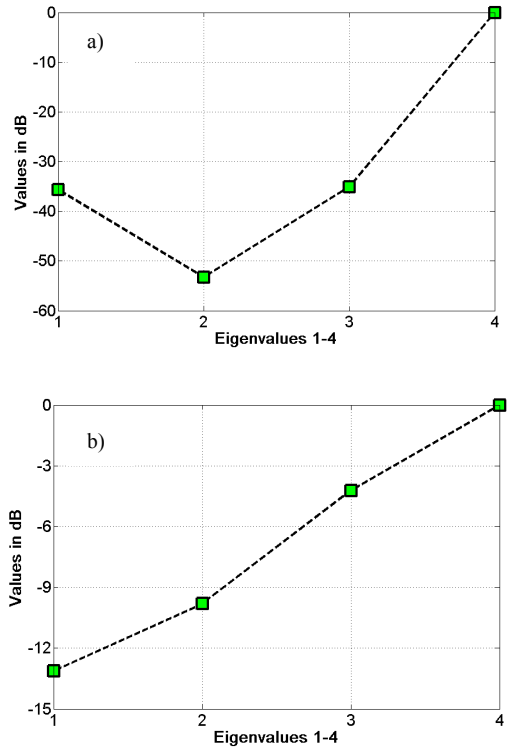

Fig. 2. Eigenvalues representation of samples corresponding to Mueller matrices M_1 (a), and M_4 (b), with its relative importance in dBs.

With all these Mueller matrices, it is possible to apply the Group Theory SU(4)-O+(6) homomorphism, and to obtain the Mueller Coherency matrices. From them, and following the procedure detailed in section 3, the eigenvalues and target vectors can be calculated. Graphs for the eigenvalues relative significance for each case are showed in Figure 2.

V. Discussion

First of all, let's have a look at the case with the linear retarder and the glucose suspension with no scatterers, whose eigenvalues appear in Figure 2 a). Here a dominant eigenvalue is clearly stated, because the fourth is 35.11 dB over the second more significant one. This implies that the behaviour of the sample can be approximated quite well with

a non-depolarizing optical device modelled via the coherent scattering or Jones matrix associated with the fourth target vector corresponding to the dominant eigenvalue:

$$J = \begin{pmatrix} 1 & 0.2098e^{j1.7365} \\ 0.2741e^{j1.1767} & 0.8768e^{-j1.0834} \end{pmatrix} \quad (5)$$

As the linear retarder has a retardance of approximately 1.27 radians at 632.8 nm, its ideal Jones matrix would be:

$$J_{lretarder} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-j1.27} \end{pmatrix} \quad (6)$$

The similarity between both matrices in equations 5 and 6 can be seen, and the influence of the optical activity of the glucose appears evidently in the nonzero values of the elements out of the main diagonal. This response is expected, due to the fact that a linear retarder is a non-depolarizing basic device, and the glucose suspension provokes no scattering but simple optical rotation due to optical activity, which is also a non-depolarizing process. In the second suspension, that of glucose with microspheres and a scattering coefficient of 0.6 mm^{-1} measured in transmission, matrix M_2 in Table 1, the difference between the dominant eigenvalue and the next one is around 13 dB. The interpretation according to the Group Theory in polarimetry says that the eigenvalues take closer quantities, and this means that the behaviour of the sample cannot be strictly approximated by a non-depolarizing device. The effect is even more accused if the concentration of scatterers goes up like in matrix M_3, where this difference of eigenvalues is only 2.893 dB. If the measurement configuration is changed to reflection, and so backscattering radiation is obtained, the results are contained in Figure 2 b). Now the difference between eigenvalues is only 4.217 dB, and not 13 dB like in the previous case. The fact that the same sample gives such different results under transmission or reflection configurations is assumable if we think that the backscattering process implies interactions with more scatterers as the optical path increases with the forward and back ways, and this can be interpreted like a transmission measurement in which the concentration of scatterers increases.

VI. Conclusions

In this article, the application of Group Theory to polarimetry as a way of improving contrast in biological images for tissue characterization has been stated. The SU(4)-O+(6) homomorphism has been presented in this field, and so the Mueller Coherency matrix that can be obtained from the Mueller matrix. The subsequent analysis of the Mueller Coherency matrix by means of its decomposition in eigenvalues and target vectors has been shown. The relative weight of these eigenvalues is very interesting from the point of view of tissue characterization, because a dominant value implies a non-depolarizing behaviour, as long as a coherent scattering matrix that describes its polarization effects.

The method was applied to glucose suspensions of polystyrene spheres of different concentrations. The Group Theory applied to measured Mueller matrices, either in transmission or reflection, allows us to conclude that analyzing the eigenvalues and target vectors it is possible to perfectly distinguish different concentrations of the scatterers, as long as the structure and behaviour of the sample. In this particular case, the immediate medical application could be the diagnosis of cell cultures of possible cancerous tissues, or some anemia pathologies, where the number of erythrocytes in blood decreases.

The potentiality of the Group Theory in tissue polarization characterization has been stated as a way of trying to evaluate its structure by comparing with usual optical devices (rotators, retarders,…), as long as their depolarising characteristics. This could be used to increase contrast in biological images, and in this way a better diagnosis could be made. Also it could provide a guide to eliminate secondary effects that hide the principal ones from the point of view of diagnosis, by means for instance of optical clearing to reduce scattering.

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