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Analysis of image-reconstruction algorithms for circular, cone-beam CT by Hotelling observer performance on a detection task

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Abstract— **We apply task-based evaluation methods to image-reconstruction in circular cone-beam CT. The particular task considered is that of detection of a micro-calcification of known location in a known background; namely a signal-known-exactly/background-known-exactly (SKE/BKE) detection task. The image-reconstruction algorithms are evaluated based on efficiency for detection. This task-based metric can be applied to any linear image-reconstruction algorithm, such as the FDK algorithm or other more recent approximate methods for image-reconstruction in circular, cone-beam CT.**

I. Introduction

Evaluation of CT image-reconstruction algorithms is a difficult task. For example, resolution properties are difficult to quantify in a meaningful way, because they tend to be non-uniform in the imaging volume. On the other hand, complete analysis methods, such as the cross-talk matrix, are unwieldy due to the large size of volume images (for a typical 500^3 voxel image the cross-talk matrix has 500^6 entries). Evaluation metrics assessing the noise properties of cone-beam CT image-reconstruction run into similar issues.

Though difficult, objective assessment of image-reconstruction in CT is becoming an important issue as CT is being developed for screening and for dedicated imaging systems such as breast CT. For screening applications, CT would be employed repeatedly on a nominally healthy, yet high-risk population; thus x-ray dose should be kept to a bare minimum. For dedicated scanning systems, a limited set of imaging tasks will be performed with the imaging device; thus it makes sense to evaluate the imaging system components based on these tasks and tailor the system to perform well on its specific use. To this end, we have been working to apply task-based metrics, as described in chapters 13 and 14 of "Foundations of Image Science" by Barrett and Myers [1], to the evaluation of cone-beam CT image reconstruction algorithms.

As an example for the application of task-based assessment, we consider circular, cone-beam CT in dedicated breast scanning. We are interested in evaluating and optimizing system components, specifically image-reconstruction, thus we model a task that tests the limits of the system. For breast CT, a logical choice of task is micro-calcification detection. This task challenges resolution and noise properties of the breast CT system. The task-based metric is based on the performance of the linear "ideal-observer's", or Hotelling observer's, ability to distinguish between two situations: signal-present and signal-absent. To evaluate image-reconstruction we use the efficiency [2], [3], which takes the ratio of Hotelling performance in the reconstructed images to that of the pre-processed projection data. By this metric, the best performance for a linear image-reconstruction algorithm is an efficiency of 1.0, but in general the efficiency will be less than 1.0. In other words, linear operations, such as image-reconstruction algorithms considered, can only degrade the Hotelling observer's ability to distinguish between the signal-present and signal-absent hypotheses.

In this work, we present the method for evaluating Hotelling observer efficiency in cone-beam CT. Its application to evaluating various algorithms in circular, cone-beam CT will be presented at the meeting.

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II. Cone-beam CT data model and Hotelling observer computation

We introduce the data model for cone-beam CT, describing both hypotheses for the computation of the Hotelling observer on the SKE/BKE detection task. We then discuss how to evaluate the Hotelling observer.

The data model for the CT system relates the data vector g to the object function $f(\vec{r})$:

$$\mathbf{g_i} = \int_{-\infty}^{\infty} \mathbf{dlf}\left(\tilde{\mathbf{s}}_\mathbf{i} + \mathbf{l}\hat{\theta}_\mathbf{i}\right) \; + \; \mathbf{n_i} \text{ where } \mathbf{i} \in [\mathbf{1}, \mathbf{N}], \tag{1}$$

where $\mathbf{g_i}$ and $\mathbf{n_i}$ are random variables. The data measurement $\mathbf{g_i}$ is the line integral over $f(\vec{r})$ along the ith ray defined by the x-ray source location s_i and ray direction $\hat{\theta}_i$ with noise $\mathbf{n_i}$ added. A reasonable approximation of the CT noise model is that the measurements are independent, and they follow a Gaussian distribution with variance:

$$(K_g)_{i,i} = \alpha < \mathbf{g_i} > +\beta. \tag{2}$$

The covariance of the sinogram, K_g, is assumed diagonal, where the diagonal elements have a constant background noise level β plus a component proportional to the data mean $< \mathbf{g_i} >$. To generate data corresponding to the signal-absent hypothesis the object function $f(\vec{r})$ is taken to be the background b_g. For example, in breast CT the object function could be computer-simulated breast model. For the signal-present hypothesis, the object function, $b_g + s_g$, is the same background plus a model of a micro-calcification with known shape and location. In evaluating the Hotelling observer, we assume the weak-signal limit, where the noise model is assumed to be the same for both hypothesis.

The data model corresponding to each hypothesis include a random component, giving only a probability distribution for each case. Given a particular data realization, the observer is charged with the task of determining which hypothesis is correct based on the one data set. The Hotelling observer multiplies the data set by a template, $w_g^{(hot)}$, that maximizes separability between the two probability distributions. For the above data model, the Hotelling template is straight-forward to compute:

$$w_g^{(hot)} = s_g \cdot (K_g)^{-1}. \tag{3}$$

Because the noise model includes only a diagonal covariance matrix, Eq. (3) can be computed directly. The Hotelling observer performance is evaluated by computing the signal-to-noise (SNR) ratio

$$SNR_g^2 = w_g^{(hot)} \cdot s_g. \tag{4}$$

In order to obtain the efficiency e, we next need to find the Hotelling observer performance in the reconstructed images.

Because the data model is random, so will be the reconstructed images. Again, the Hotelling observer will employ a template that maximizes separability between the reconstruction of both hypothesis. As we are considering only linear algorithms and, in practice, the algorithms transform a discrete data set to a discrete image, the image-reconstruction operator can be view as a large matrix A. The reconstructed image y is obtained by

$$y = A \cdot g.$$

Linear transforms applied to Gaussian distributions yield another Gaussian distribution. The means for the signal-absent and signal-present distributions are, respectively $A \cdot b_g$ and $A \cdot (b_g + s_g)$. The covariance matrix for both hypothesis will again be the same for the reconstructed images because of the weak signal limit, and it is given by

$$K_y = A \cdot K_g \cdot A^T. \tag{5}$$

Direct computation of this matrix is impractical for CT, because it is most certainly non-diagonal and it can be as large as $10^9 \times 10^9$. The Hotelling template $w_y^{(hot)}$ can nevertheless be accurately computed by using iterative algorithms such as conjugate gradients to solve the following linear system

$$\begin{aligned} K_y \cdot w_y^{(hot)} &= s_y \\ \left(A \cdot K_g \cdot A^T\right) \cdot w_y^{(hot)} &= A s_g \end{aligned} \tag{6}$$

for the template. After computing this template an SNR in the reconstructed images can be computed by

$$SNR_y^2 = w_y^{(hot)} \cdot s_y. \tag{7}$$

With both SNRs in hand the efficiency of the image reconstruction algorithm becomes

$$e = SNR_y^2 / SNR_g^2. \tag{8}$$

III. Application to image-reconstruction algorithms for cone-beam CT

In general, the efficiency will be less than one because the reconstruction operator A is not invertible. If the size of the reconstructed image is smaller than that of the data, then it is clear that A is not invertible. Even when the dimensions are not reduced in going to the reconstructed image, A is not in general invertible. The reason for this is that only a subset of possible data sets correspond to the projection of an actual object, and data with inconsistencies due to, for example, noise will not correspond to any object function. Any image reconstruction algorithm is then implicitly projecting a given data set to a data set consistent with the X-ray transform. Different algorithms will perform this projection differently. As a result, one can expect also different efficiencies.

As described above the efficiency depends on a particular background and signal. Thus, to characterize the image-reconstruction efficiency for a given algorithm a realistic background should be modeled in terms of geometry and attenuation value. The modeled micro-calcification signal should have dimensions near the limit of what the imaging system can resolve. An efficiency map can then be generated by moving the signal to various places in the imaging volume.

IV. Summary

We have developed a task-based image quality metric for circular cone-beam CT that can objectively assess image-reconstruction algorithms in circular, cone-beam CT. The application of this metric to various algorithms such as FDK will be shown at the meeting.

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