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SENSE Reconstruction with Nonlocal TV Regularization

Dong Liang, Haifeng Wang and Leslie Ying

***Abstract*—Ill-conditioning is serious problem in SENSE reconstruction, especially when large acceleration factors are employed. For Cartesian SENSE, Tikhonov regularization and total variation have been commonly used. However, the Tikhonov regularized image usually tends to blur edges and total variation regularization has a blocky effect. In this paper, we propose a new SENSE regularization technique that is based on nonlocal total variation with Bregman iteration. It penalizes highly oscillatory noise and allows sharp edges and fine textures in reconstruction. The method is shown to be able to significantly reduce the artifacts in SENSE reconstruction.**

Index Terms- SENSE, nonlocal total variation regularization, Bregman iteration

I. Introduction

Parallel MRI improves imaging speed by reducing the number of samples simultaneously acquired from multiple channels. Standard reconstruction methods include SENSE, SMASH, GRAPPA, etc. [1-3]. Among them, SENSE (SENSitivity Encoding) [1] is known to theoretically be able to give the exact reconstruction of the imaged object in the absence of noise. However, in practice, a well-known problem of SENSE is the amplification of data noise due to the ill-conditioned nature of the inverse problem. And it is especially serious when a high reduction factor is employed. General solutions include optimization of the coil geometry [4,5] and application of mathematical regularizations to reconstruction [6-8]. Two common choices for regularization are Tikhonov regularization [6-8] primarily due to the existence of a closed-form solution, and total variation (TV) regularization with edge-preserving property [9-10]. The regularization image, usually of poor quality (e.g. low resolution), introduces bias in Tikhonov regularization, which is seen as imparted image blurring, or residual aliasing artifacts at high reduction factors. TV regularization replaces the smoothness prior in Tikhonov regularization by an edge preserving prior that imposes equal-weight relationship between local neighboring pixels. A drawback of TV regularization is the possible loss of fine structures and textures. It is due to the limitation that the cost function of TV regularization is based solely on derivatives which are local features of the image.

Recently, nonlocal TV regularization attracts a lot of attention due to allowing much more flexibility in the regularization. The nonlocal TV energy was proposed by Gilboa et al. [11] in the continuous setting. It is used in the discrete setting in order to perform denoising [12-13] and inverse problem [14-15].

This regularization prior is described as non-local since pixels belonging to the whole image are used when calculating the gradient, instead of only the nearest pixels as used in TV regularization. In addition, when calculating the gradient there is a weighted graph between the current pixel and all pixels. With the addition of Bregman iteration, the nonlocal TV regularization has been shown superior performance than TV regularization on denoising and inverse problem. [12-15]. In this paper, we apply nonlocal TV regularization to SENSE reconstruction and demonstrate its superior performance to other competing regularization methods.

II. Tikhonov and TV Regularization On Sense

Parallel MRI [1-3] is a new technique to improve on conventional Fourier encoding for fast imaging. In parallel imaging, *k*-space data are acquired from multiple channels simultaneously such that they can be sampled with a rate lower than the Nyquist sampling rate. The imaging equation for SENSE can be written in matrix form as

$$\mathbf{Ef} = \mathbf{d} \tag{1}$$

where $\mathbf{d}$ is a concatenation of data from all channels, and $\mathbf{f}$ the desired image to be reconstructed with length n. $\mathbf{E}$ is the sensitivity encoding matrix comprising Fourier encoding and sensitivity weighting. The image $\mathbf{f}$ can be reconstructed by the least-squares solution to Eq. (1).

$$\mathbf{f} = \left(\mathbf{E}^{H}\mathbf{\Psi}^{-1}\mathbf{E}\right)^{-1}\mathbf{E}^{H}\mathbf{\Psi}^{-1}\mathbf{d} \tag{2}$$

where the superscript H indicates transposed complex conjugate, $\mathbf{\Psi}$ is the receiver noise covariance [1].

When the linear equation in Eq. (1) is ill-conditioned, the data noise can be amplified which leads to poor reconstruction. Tikhonov and TV regularization have been used to address the ill-conditioning problem. In Tikhonov framework, the reconstruction is given by:

$$\mathbf{f}_{\text{reg}} = \arg\min_{\mathbf{f}}\left\{\left\|\mathbf{d}-\mathbf{Ef}\right\|_2^2 + \lambda\left\|\mathbf{A}(\mathbf{f}-\mathbf{f}_r)\right\|_2^2\right\} \tag{3}$$

where the regularization parameter λ is chosen to balance the data fitting error and the penalty (or regularization) term, which is formed from the difference between the expected solution and the prior image $\mathbf{f}_r$ known as the regularization image. A closed-form solution for $\mathbf{f}_{\text{reg}}$ exists and is given by

$$\mathbf{f}_{\text{reg}} = \mathbf{f}_r + \left(\mathbf{E}^{H}\mathbf{\Psi}^{-1}\mathbf{E} + \lambda\mathbf{A}^{H}\mathbf{A}\right)^{-1}\mathbf{E}^{H}\mathbf{\Psi}^{-1}\left(\mathbf{d}-\mathbf{Ef}_r\right) \tag{4}$$

The Tikhonov regularized reconstruction usually suffers from blurring effects due to L_2 norm in Eq. (4). To alleviate the problem, a low-resolution image generated from several

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Dong Liang, Haifeng Wang, and Leslie Ying are all with the Department of Electrical Engineering and Computer Science, University of Wisconsin, Milwaukee, WI 53201 USA (e-mail: liangd, haifeng, leiying@ uwm.edu).

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additional lines acquired at the center of the *k*-space is used as the regularization image $\mathbf{f}_r$, but at an expense of prolonged imaging time. Due to the low quality of this regularization image, the reconstruction still suffers from aliasing artifacts at high reduction factor *R*.

TV regularization has been proved to be able to recover piecewise smooth functions without smoothing sharp discontinuities. The reconstructed image using TV regularization is given by

$$\mathbf{f}_{\text{reg}} = \arg\min_{\mathbf{f}} \left\{ \|\mathbf{d} - \mathbf{E}\mathbf{f}\|_2 + \lambda \|\mathbf{f}\|_{\text{TV}} \right\} \quad (5)$$

where the regularization term is TV norm of the image defined as

$$\|\mathbf{f}\|_{\text{TV}} = \sum \sqrt{\left|\nabla_{-} f\right|^2 + \left|\nabla_{|} f\right|^2} \quad (6)$$

∇_{-} and $\nabla_{|}$ denote the gradient along horizontal and vertical directions respectively [16], and $|\cdot|$ denotes the complex modulus. The TV prior is based on the fact that the highly oscillatory noise usually increases the TV of an image [17]. TV regularization is known to be edge-preserving, but may result in loss of texture due to the assumption that the image TV is small [16]. The detail textures usually have large local variations, which lead to increased TV value. As a result, the minimization of Eq.(5) with TV regularization reduces fine structures at the same time when suppressing noise in reconstruction.

III. PROPOSED REGULARIZATION METHOD

In order to utilize the benefit of TV regularization with edge-preserving and overcome the blocky effect, the nonlocal TV regularization is applied to SENSE. Then the reconstructed image is given by

$$\mathbf{f}_{\mathbf{reg}} = \arg\min_{\mathbf{f}} \left\{ \|\mathbf{d} - \mathbf{E}\mathbf{f}\|^2 + \lambda \|\mathbf{f}\|_{\mathbf{NLTV}} \right\}, \quad (7)$$

$\|\mathbf{f}\|_{\mathbf{NLTV}} = \sum_x \|\nabla_x^w \mathbf{f}\|_2$ is the L_1 norm of L_2 norm of the weighted graph gradient $\nabla_x^w \mathbf{f}$ across all image pixels. The weighted graph gradient is defined as [14-15]:

$$\nabla_x^w \mathbf{f} = \left(\sqrt{w(x,y)} \left(\mathbf{f}(y) - \mathbf{f}(x) \right) \right)_y \in \mathbb{R}^n \quad (8)$$

For a given pixel *x*, this operator defines a gradient vector. Its adjoint operator is the divergence operator $div^w = \left(\nabla^w\right)^T$. For a gradient field $F_x \in \mathbb{R}^n$, the divergence is

$$\left(div^w(F) \right)(x) = \sum_y \sqrt{w(x,y)} \left(F_x(y) - F_y(x) \right) \quad (9)$$

Here, $w(x,y)$ is an adapted graph calculated for a given image so that the regularization by nonlocal TV can efficiently removes noise without destroying the salient features of the image.

$$w(x,y) = \frac{\tilde{w}(x,y)}{Z_x}$$
$$\tilde{w}(x,y) = \begin{cases} e^{-\frac{\|p_x(f) - p_y(f)\|_2}{2\sigma^2}} & if \ \|x - y\|_2 \le \frac{\delta}{2} \\ 0 & otherwise \end{cases} \quad (10)$$

where $Z_x = \sum_y w(x,y)$ is the normalization constant. The parameter $\delta > 0$ restricts the non-locality of the method and also allows to speed-up computation. The parameter σ controls how many patch are taken into account to perform the averaging. This parameter is difficult to set and ideally it should also be adapted to the noise level. In our work, we use the method proposed in ref [18] to estimate this parameter. Basically, after SENSE reconstruction without regularization, the noise level is estimated as the median absolute deviation of the wavelet coefficients at the finest scale divided by .6745.

Please note this nonlocal graph is constructed by comparing patches around each pixel. A patch $p_x(f)$ of size $m \times m$ (*m* being an odd integer) around a pixel position *x* is

$$\forall t \in \left\{ -(m-1)/2 + 1, \ldots, (m-1)/2 \right\}^2$$
$$p_x(f)(t) = f(x+t) \quad (11)$$

Here, a patch $p_x(f)$ is cascaded as a one dimensional vector of size m^2 and *t* is the index.

Compared with the definition of TV in Eq. (6), it is clear that all pixels are used in nonlocal TV. In order to reduce the computation time in practice, the seek of the neighborhood is limited to a window search around the pixel to be estimated. Even in this case, the number of used pixels is still larger than TV norm which usually use the nearest pixels only. In addition, weights are different between different pair of pixels.

To further recover the fine details in reconstruction, we use an iterative regularization procedure. Instead of stopping at the solution $\mathbf{f}_0$ to Eq. (7), we use it to iteratively refined nonlocal TV regularization:

$$\mathbf{f}_k = \arg\min_{\mathbf{f}} \{ \| \mathbf{d} - \mathbf{E}\mathbf{f} \|_2^2 + \lambda D(\mathbf{f}, \mathbf{f}_{k-1}) \}, \text{ for } k > 1. \quad (12)$$

where $D(\mathbf{f}, \mathbf{f}_k)$ is the Bregman distance between $\mathbf{f}$ and $\mathbf{f}_k$ associated with the nonlocal TV norm, defined as

$$D(\mathbf{f}, \mathbf{f}_k) = \| \mathbf{f} \|_{\text{NLTV}} - \| \mathbf{f}_k \|_{\text{NLTV}} - < \mathbf{f} - \mathbf{f}_k, \partial(\| \mathbf{f}_k \|_{\text{NLTV}}) >$$

where $< \cdot, \cdot >$ denotes the inner product and $\partial(\| \mathbf{f}_k \|_{\text{NLTV}})$ is an element of the sub-gradient of the nonlocal TV norm at point $\mathbf{f}_k$. The Bregman distance is an indication of the increase in $\| \mathbf{f} \|_{\text{NLTV}}$ over $\| \mathbf{f}_k \|_{\text{NLTV}}$ above linear growth with slope $\partial(\| \mathbf{f}_k \|_{\text{NLTV}})$.

Using nonlocal TV regularization with Bregman iteration, it has been shown [15] that the sequence $\mathbf{E}\mathbf{f}_k$ monotonically converges to the acquired noisy data $\mathbf{d}$ in L_2 sense, i.e., the reconstruction $\mathbf{f}_k$ approaches the unregularized basic SENSE reconstruction. For λ sufficiently large, the sequence also monotonically gets closer to the noise free data $\mathbf{E}\mathbf{f}_{\text{true}}$, whose convergence is faster than noise. Therefore, if a good stopping rule is applied, Bregman iteration can recover fine details of the image before the noise. The discrepancy principle [19] is used here, which stops the iterative procedure when the data inconsistency residual is reduced to below the measurement noise level for the first time. When the noise level is not available, another strategy is to iterate until the reconstruction is visually noisier than the one from the previous iteration. According to [15], the minimization in Eq. (7) is equivalent to

$$\mathbf{f}_k = \arg\min_{\mathbf{f}} \left\{ \left\| \mathbf{d} + \mathbf{v}_{k-1} - \mathbf{E}\mathbf{f} \right\|_2^2 + \lambda \left\| \mathbf{f} \right\|_{\text{NLTV}} \right\} \quad (13)$$

where $\mathbf{v}_k = \mathbf{v}_{k-1} + \mathbf{d} - \mathbf{E}\mathbf{f}_k$ for $k > 1, \mathbf{v}_1 = \mathbf{0}$. It is seen that the above minimization in each Bregman iteration is the same as nonlocal TV regularization formulation Eq. (7) except that $\mathbf{d}$ is replaced by $\mathbf{d} + \mathbf{v}_{k-1}$ in the first term. It becomes nonlocal TV regularization when the iteration index is $k = 1$.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A. Experiments Settings

The feasibility of the proposed method was validated on a set of in vivo brain data, which were acquired on a 3T commercial scanner (GE Healthcare, Waukesha, WI) with an 8-channel head coil (Invivo, Gainesville, FL). A healthy volunteer was scanned with a 2D T_1-weighted spin echo protocol (axial plane, TE/TR = 11/700 ms, 22cm FOV, 10 slices, 256×256 matrix). Informed consent was obtained from the volunteer in accordance with the institutional review board policy. The sensitivity information of each coil was obtained by pre-scanning.

The proposed method is compared with iterative conjugate gradient (CG) SENSE reconstruction due to its inherent regularization capability [19], TV regularization and Tikhonov regularization with a reduction factor 4. All methods are implemented in MATLAB (Mathworks, Natick, MA). The sum-of-square (SoS) reconstructions from the fully sampled data of all channels are shown in the top left of Fig. 1as the reference for comparison. In nonlocal regularization, the width of searching window is $\delta = 11$, which means when calculating the weighted graph, we only consider the neighbors within an 11 × 11 window central the current pixel. The patch size is $m = 5$, which means when calculating the gradient between two pixels, we not only use the pixel itself but also the patch of this pixel. These two parameters are chose with an appropriate tradeoff between computation time and image quality. All images are normalized and shown in the same scale. We label the method on the top-left corner of each reconstructed image.

From Fig.1, we can see that NLTV can preserve most details in human brain images and only presents slight artifacts. It is due to calculating the nonlocal graph from 11 × 11 neighboring pixels and each with a 5 × 5 patch. While TV regularization and Tikhonov regularization both exhibit obvious residual aliasing artifacts. And nonlocal TV regularization is able to suppress more noise than Tikhonov regularization. Additionally, some high intensity details are lost due to the piecewise smooth constraint in TV regularization, which makes the image look blocky. The reason is that it only calculates the gradient from the nearest few neighboring pixels and each only with itself. CG reconstructions have most severe artifacts compared with the other methods because the inherent regularization is not sufficient.

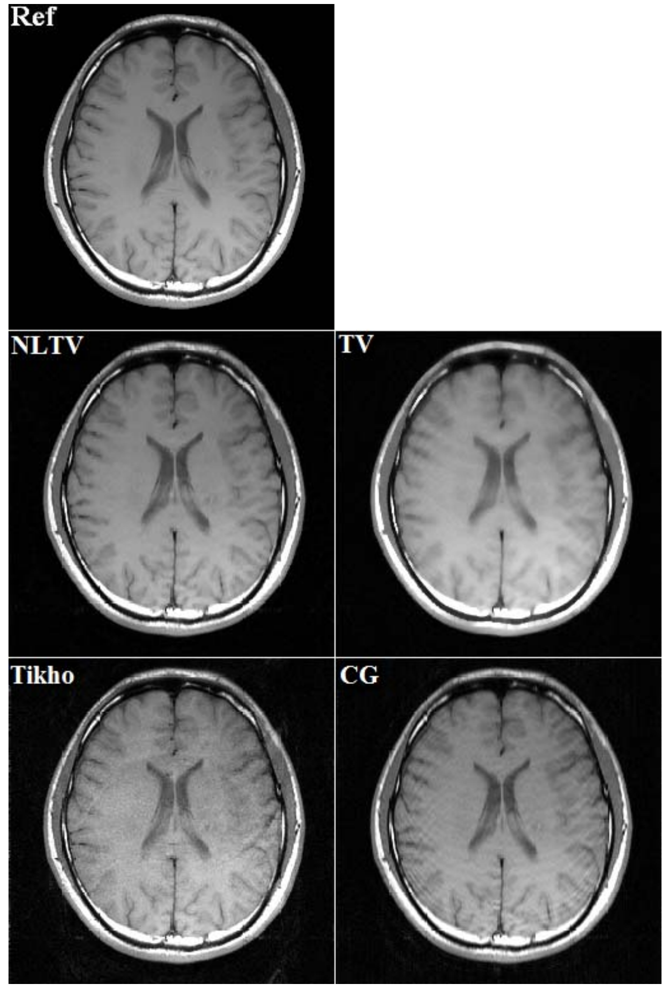

Fig. 1. The reconstructions using four different methods with reduction factor 4 from an 8 channel scanned human brain data. NLTV regularization can preserve most details and only presents slight artifacts. TV and Tikhonov regularization both exhibit obvious residual aliasing artifacts. CG reconstructions have most severe artifacts.

Figure 2 shows the corresponding error images for the above reconstructions with four different regularization methods. The improvement of nonlocal TV regularization over the existing regularization methods in reducing artifacts is clearly seen in the error images.

Table 1. Comparison of NMSEs

NMSE ($\times 10^{-002}$)	NLTV	TV	Tikho	CG
R=4	0.41	1.04	0.99	1.06

The normalized mean square error (NMSE) provides a combined metric for both image noise and artifacts. The

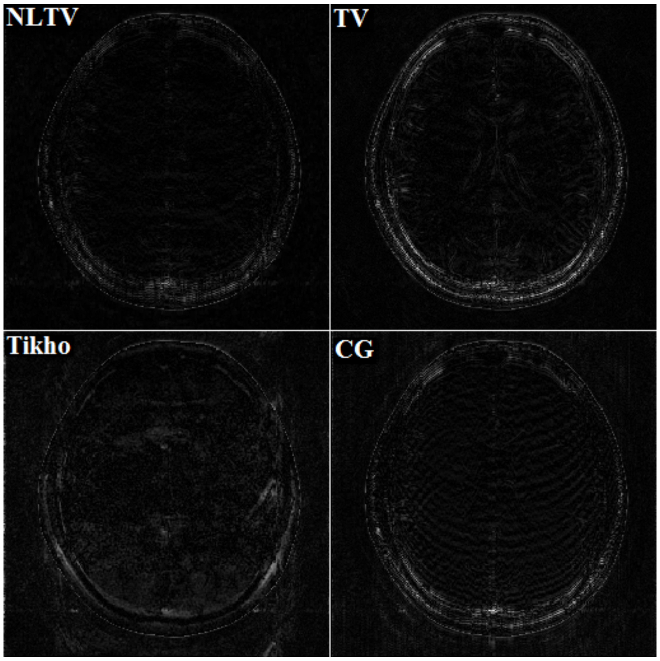

Fig. 2 The corresponding error images of the reconstructions in Fig.1.

NMSE between the reference and reconstructed images for human brain data were also computed to evaluate the reconstruction performance with different regularization methods, given in Table 1. In terms of NMSE, the nonlocal TV regularization is superior to the other regularization methods. This may be due to calculating the gradient using a patch of nonlocal pixels.

V. CONCLUSION

In this paper, the nonlocal TV regularization with Bregman iteration is used to solve the ill-conditioning problem encountered in parallel MR imaging. This regularization method uses a nonlocal weighted graph to present the similarity of different pixels instead of direct subtraction. The experimental results demonstrate the nonlocal TV regularization with Bregman iteration is able to preserve more details and fine structures than some commonly used regularization methods.

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