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A new time-frequency approach to estimate single joint upper limb impedance

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Abstract—**This paper proposes a new technique to estimate single joint impedance during postural tasks. The method is based on a reassigned spectrogram and can track the frequency modulation of biomechanical system after perturbations.**

Compared to the existing techniques, this procedure successfully estimated rapidly time varying impedance parameters in a faster and equally accurate way. For this reason it can be an optimum tool to easily estimate limb impedance of stroke patients, before, during, and after robot therapy sessions, without interfering with the delivered treatment.

I. Introduction

Mechanical impedance is considered one of the main parameters for controlling human movement both during posture and reaching tasks. Monitoring the impedance on impaired subjects, can quantitatively identify the interaction between the patient and the environment to monitor the progress in rehabilitation treatment. To estimate impedance, several methods have been proposed; however the majority of these techniques imply the measure of impedance during steady state conditions [1-5]. When the condition is transitory, the assumption of ergodicity is often used [7-9]; thus numerous repetitions of the movement are necessary for estimation. Depending upon the level of impairment, the number of movement repetitions could result in fatigue and exceed the maximal performance of the subject.

Here, a new technique is proposed for the estimation of impedance, both during steady and transitory states. This technique is based on the principles of modal testing [10] and uses a force pulse to estimate the mechanical behavior of the upper limb by means of a reassigned spectrogram operating in a joint time-frequency domain [11].

This paper presents a set of postural task simulations where both damping and stiffness are non-linearly time-varying. The technique presented neither requires the assumption of steady state system nor of ergodicity. Therefore, the estimation of impedance can be performed on the same set of data at different time instants, which quickens the experimental procedure. Ideally, only one force impulse is necessary to obtain the stiffness estimation for different instants. As a result, this procedure is quite fast and suitable for stiffness and damping estimation of impaired subjects, which otherwise could be problematic because of the long time required by other methods for the acquisition of a complete data set.

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II. Methods

A. The inverse problem

The mechanical features of the forearm are supposedly nonlinear and higher than second-order; however, a good approximation of the system behavior is given by a linear time-varying harmonic oscillator, namely:

$$I(\theta,t)\ddot{\theta}(t)+B(\dot{\theta},\theta,t)\dot{\theta}(t)+K(\ddot{\theta},\dot{\theta},\theta,t)\theta(t)=0 \qquad (1)$$

whose solution in the time domain can be expressed in terms of instantaneous amplitude and instantaneous phase:

$$\theta(t)=A(t)\cdot\cos(\varphi(t)) . \qquad (2)$$

TABLE I
GEOMETRY AND INERTIAL PARAMETERS

Symbol	Denomination	Value
l_1	Upper arm length	0.4 [m]
r_1	Upper arm center of mass	0.17 [m]
m_1	Upper arm mass	2.5 [kg]
I_1	Upper arm moment of inertia about the center of mass	0.02 [kg m^2]
I	Upper arm moment of inertia about the center of rotation	0.2 [kg m^2]

Inertial and geometrical parameters used in the simulation. Parameters were obtained from a subject using a regression equation proposed in [6].

In (1), $\theta(t)$ is the variation of elbow angular displacement, and I, B, and K are the coefficients of inertia, damping and stiffness respectively.

The instantaneous pulsation of the system is defined as the derivative with respect to time of the instantaneous phase:

$$\omega_i(t)=\dot{\varphi}(t) \qquad (3)$$

To estimate stiffness and damping with the proposed procedure, the inertial parameters have to be known prior the application of the method. Under the hypothesis of an under-damped system, the values of the coefficients B and K are determined from the instantaneous pulsation $\omega_i(t)$ during an impulse response. This process is known as the inverse vibration problem.

B. Biomechanical model

We implemented Equation 1 within a Simmechanics (Mathworks) simulation where a model of the forearm was defined as one link with known geometry and inertial parameters (Table I). Considering the elbow as the origin of the reference system, the x-axis was directed laterally and the y-axis ventrally. At time $t_I = 0s$ the longitudinal axis of the forearm was aligned with the y-axis, and a force pulse (5N, 20ms) was applied along the x-axis. The system was tested for the period $t_F - t_I = 1.25s$ with a sampling frequency $F_s = 4000Hz$ in three operating conditions. First, we studied the system during steady state condition with constant stiffness and damping; hence, the two parameters were represented as time-varying sigmoidal profiles. A generic sigmoidal function is as follows:

$$y(t) = \frac{y(t_F) - y(t_I)}{1 + e^{-\xi\left(t - \frac{t_F}{2}\right)}} + y(t_I) \qquad (4)$$

The profile parameters for damping and stiffness can be found in Table II.

C. Impedance Estimation

To estimate the impedance parameters of the proposed system from the impulse response signal, Equation 1 can be written in the following convenient forms:

$$\ddot{\theta}(t) + \frac{B}{I}\dot{\theta}(t) + \frac{K(t)}{I}\theta(t) = 0 \qquad (5a)$$

$$\ddot{\theta}(t) + 2\Gamma(t)\dot{\theta}(t) + \eta^2(t)\theta(t) = 0 \qquad (5b)$$

If the instantaneous pulsation given by (3) is not constant, then the normalized stiffness and damping $\eta^2(t)$ and $\Gamma(t)$ are time-varying and can be estimated as follows [12]:

$$\Gamma(t) = -\sigma - \frac{\dot{\omega}_i}{2\omega_i} \qquad (4)$$

$$\eta^2(t) = \omega_i^2 + \sigma^2 + \frac{\sigma\dot{\omega}_i}{\omega_i} - \dot{\sigma} \qquad (5)$$

where

TABLE II
STIFFNESS AND DAMPING TIME PARAMETERS

Condition		Stiffness [Nm/rad]		Damping [Nms/rad]	
I		30		0.3	
II	$\xi = 7$	$K(t_I) = 35$	$K(t_F) = 100$	$B(t_I) = 0.35$	$B(t_F) = 1.35$
III	$\xi = 30$	$K(t_I) = 35$	$K(t_F) = 100$	$B(t_I) = 0.35$	$B(t_F) = 1.35$

Stiffness and damping parameters to be inserted in Equation (4) to determine the time profile imposed in the simulation as a function of time [s].

$$\sigma(t) = \frac{d}{dt}\ln A(t) = \frac{\dot{A}(t)}{A(t)} \qquad (6)$$

Recalling that the transformation between instantaneous pulsation and instantaneous frequency is

$$\omega_i(t) = 2 \cdot \pi \cdot f_i(t), \qquad (7)$$

analyzing the impulse response of the system in the time-frequency domain allows us to identify the variation of instant amplitude $A(t)$ and instantaneous frequency $f_i(t)$ and therefore the estimation of stiffness and damping in the time domain (Fig.1).

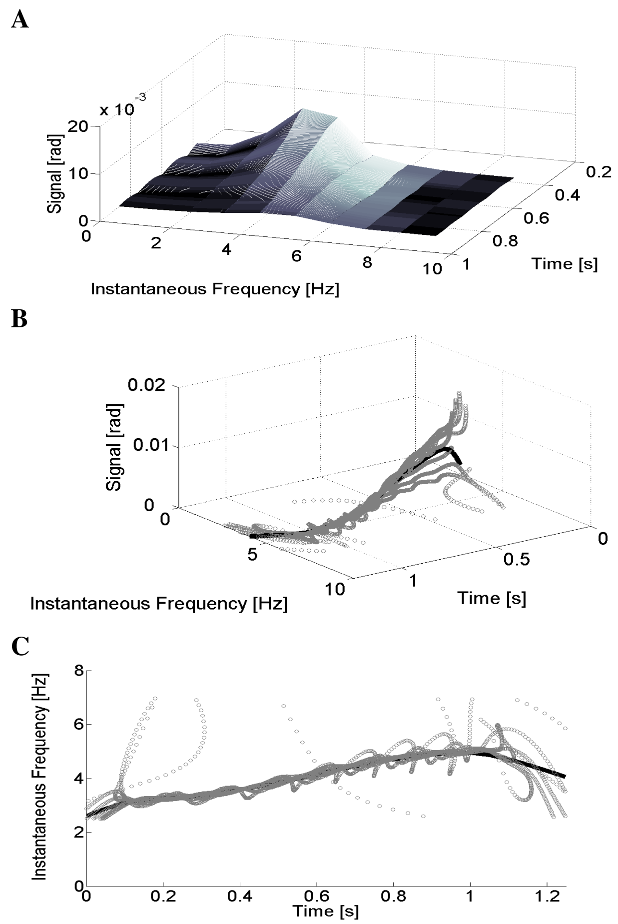

Fig. 1. Spectrogram (A), and reassigned spectrogram (B) of condition II in table II. The thick black line in B and C represents the average of the RS signal obtained using a bidirectional averaging filter with a 0.14s window.

D. *Time-Frequency approach*

Equation (1) is a model for a time-variant system whose solutions can be found both in time and frequency domain. When assuming the system is stationary, classical Fourier transform can be used to approach the problem in the frequency domain; nevertheless, such transformation does not associate with any time instant. If the system is not stationary, signals shall be convolved with a set of elementary functions whose feature is to be simultaneously localized both in the time and frequency domains. These elementary functions are commonly known as "windows", and a convolution of these functions along a time series is called "Short Term Fourier Transform" (STFT). A spectrogram is a representation of the signal convolution with the same type of window functions positioned in subsequent instants in the time series. The spectrum of the signal at one instant is calculated as the average of all STFTs enclosing that instant in the window functions.

To calculate the spectrogram, we used a 0.75s Kaiser window, with β=3. Convolving the window every 2.5ms, we obtained a resolution in frequency and time of 1.33Hz and 0.0025s. Notice that because of a notorious characteristic of STFTs known as the "uncertainty principle", both time and frequency domains of a spectrogram have lower resolution than the homologous originally sampled signal [11]. To overcome this drawback, an innovative method known as reassigned spectrogram (RS) was used to track the variation of $f_i(t)$ after the perturbation. This method is able to track the frequency peaks of existing spectrograms calculating the partial derivatives of the STFT phase, with respect to time and frequency. In contrast to other commonly used methods, locally stationary approximations of the signal were not assumed. Based on a re-mapping algorithm, RS methods can provide "super-resolution" in both time and frequency, giving much better accuracy than the Heisenberg-like uncertainty of STFT [11]. As depicted in Figure 1, the use of RS allows us to clearly identify the time variation of the system's natural frequency despite some identifiable computational artifacts.

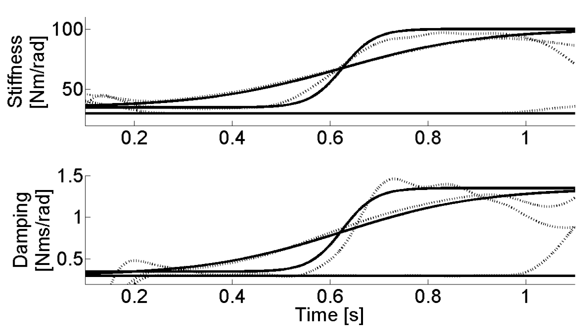

Fig. 2. Comparison between imposed and estimated stiffness and damping profiles. Considering the stiffness profile of Table II, the curve with the sharpest variation corresponds to condition III. Dotted line refers to the estimations, while solid lines represent the imposed profiles.

III. RESULTS

The stiffness and damping time-profiles were estimated for the three conditions in Table I. Typical results of the estimations are represented in Figure 2. Profiles present bigger errors at the beginning and final part of the time series because of the procedure necessary to compute the STFT spectrogram. The convolution of the signal discharged an amount of data, at the beginning and at the end of the signal, equal to the duration of half a window.

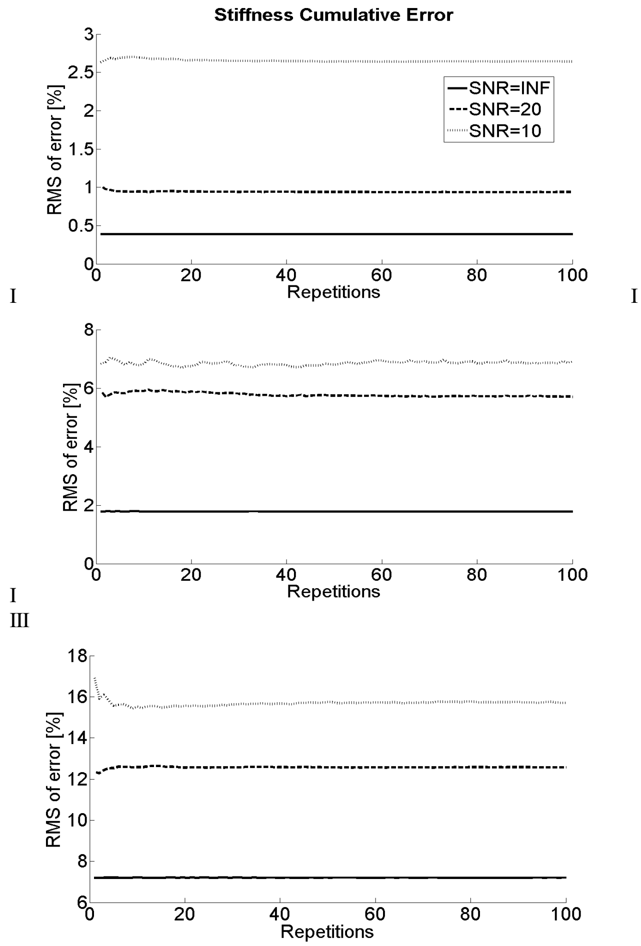

Fig. 3. Cumulative RMS error of for the stiffness profile. From top to bottom, the roman numerals on the top-left corner refer to the different stiffness profiles in Table II. For each repetition of the simulation, the percentage error along the stiffness profile was calculated; hence, the cumulative RMS of the percentage error was calculated after each repetition.

To test the accuracy of the estimation technique and the robustness to external disturbances, the simulated data were corrupted with zero-mean Gaussian noise. The signal-to noise ratios (SNR) in terms of the root mean square (RMS) of the signal were 20 dB and 10 dB respectively.

We calculated the RMS of the percentage error along the stiffness and damping profile with respect to the imposed parameters of the simulation. To test the robustness of the method, each simulation was performed 100 times, and the cumulative RMS error was calculated.

Figure 3 depicts the cumulative RMS error for the stiffness estimation. For case I (stationary) the error for SNR=inf is below 0.4%. The technique is robust with respect to noise, since the power of the noise is distributed in the whole time-frequency plane (Fig.1C). Stiffness cumulative error in case III was slightly above 7%, even though the variation of stiffness form $K(t_I) = 35$ to $K(t_F) = 100$ occurred in approximately 100ms.

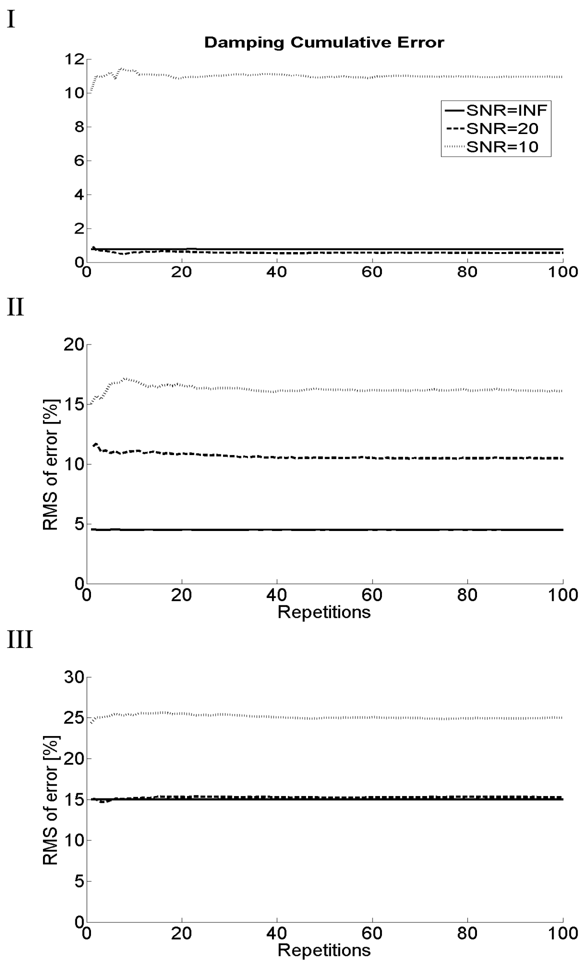

Fig. 4. Cumulative RMS error of for the damping profile

The estimation of the damping parameter was slightly less accurate since the evaluation of $\sigma(t)$ was performed using a logarithmic scale, and was therefore less sensitive to small variation.

IV. Conclusion

This work presented a new method for the estimation of stiffness and damping in non-stationary mechanical systems. Specifically, an application to determine the impedance of a single degree of freedom forearm posture with time-varying stiffness has been proposed. Results of simulations show that this method is appropriate to estimate rapidly varying parameters as the modulation of mechanical properties in the human limbs.

A second order system was considered, for comparison with the literature. Furthermore, the procedure is very robust when using noise corrupted signals.

In the ideal case only one perturbation allows estimation of the entire profile of impedance. Thus, this procedure is fast and easy to use with impaired subjects. Specifically, this method can be used before, after, and during sessions of robot therapy for estimating the stiffness variation without interferences with the treatment.

Further development of this technique will include the estimation of impedance parameters in multiple degrees of freedom cases such as a double pendulum model of the whole arm. Additionally, taking advantage of the capability to study non-stationary system, the technique could be employed to study impedance profiles during reaching movements.

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