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# Convex Hull Based Neuro-Retinal Optic Cup Ellipse Optimization in Glaucoma Diagnosis

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*Abstract* — **Glaucoma is the second leading cause of blindness. Glaucoma can be diagnosed through measurement of neuro-retinal optic cup-to-disc ratio (CDR). Automatic calculation of optic cup boundary is challenging due to the interweavement of blood vessels with the surrounding tissues around the cup. A Convex Hull based Neuro-Retinal Optic Cup Ellipse Optimization algorithm improves the accuracy of the boundary estimation. The algorithm's effectiveness is demonstrated on 70 clinical patient's data set collected from Singapore Eye Research Institute. The root mean squared error of the new algorithm is 43% better than the ARGALI system which is the state-of-the-art. This further leads to a large clinical evaluation of the algorithm involving 15 thousand patients from Australia and Singapore.**

## I. INTRODUCTION

Glaucoma is a chronic and irreversible neurodegenerative disease in which the neuro-retinal nerve that connects the eye to the brain (optic nerve) is progressively damaged and patients suffer from vision loss and blindness. Patients with early glaucoma do not usually have any visual signs or symptoms. Progression of the disease results in loss of peripheral vision and patients may complain of "tunnel vision" (being only able to see centrally). Advanced glaucoma is associated with total blindness.

According to World Health Organization, glaucoma is the second leading cause of blindness; it is responsible for approximately 5.2 million cases of blindness (15% of the total burden of world blindness) [1] and will affect 60 million people by 2010 [2].

The disease is mostly caused due to increased intraocular pressure (IOP) resulting from a malfunction or malformation of the eye's drainage structures. If left untreated, it would lead to degeneration of optic nerve and retinal fibers. Early diagnosis of glaucoma through analysis of the neuro-retinal optic disc and cup (in short, "optic disc/cup" or "disc/cup") area is crucial. The damage caused is irreversible, but treatment (e.g., lowering the intraocular or eye pressure) can prevent progression of the disease if detected early.

One of the characteristic features of glaucoma atrophy is the appearance of the Optic Nerve Head (ONH), which includes cupping or excavation of the optic disc, with loss of the neuro-retinal rim typically seen as an enlargement of the optic cup-to-disc ratio (CDR) as shown in Figure 1. Clinically, the diagnosis of Glaucoma can be done through measurement of CDR, defined as the ratio of the vertical height of the optic cup to the vertical height of the optic disc. An increment in the cupping of ONH corresponds to increased ganglion cell death and hence CDR can be used to measure the probability of developing the disease. A CDR value that is greater than 0.65 indicates high glaucoma risk [3]. ONH assessment is currently performed by a trained specialist (ophthalmologist, usually glaucoma specialist), or using specialized expensive equipment such as the Heidelberg Retinal Tomography (HRT) system. However, ONH assessment by an ophthalmologist is subjective and the availability of HRT is very limited because of the cost involved. Thus, there remains a lack of cost effective, sensitive and precise method to screen for glaucoma. An automatic CDR measurement system is in strong demand using a cost effective method for fast, reliable and efficient diagnosis of glaucoma. Ideally, the system can make use of the 2D retinal fundus images obtained from the fundus cameras, which are widely used nowadays in clinics.

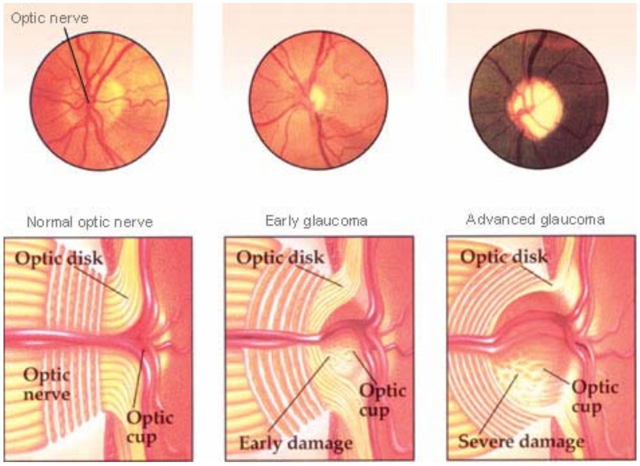

Figure 1. Stages of progressive cupping in Neuro-retinal optic disc and cup area for Glaucoma patient

In order to calculate the CDR automatically, the cup and disc boundaries are to be segmented. Many approaches have been proposed in the past to segment the disc boundary and extract the optic disc region from the fundus images [4, 5]; however, cup segmentation is more challenging due to the interweavement of blood vessels with the surrounding tissues

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around the cup, and very few approaches have been proposed [6].

Due to the complexity of the cup boundary, the segmented cup is normally smoothened and an ellipse fit is generated to better estimate the cup boundary. Neuro-retinal Optic Cup Ellipse Optimization is critical in cup estimation and thus the calculation of accurate CDR.

Convex hull [7] is the smallest convex region enclosing a specified group of points. A convex hull based algorithm is developed to better estimate the neuro-retinal optic cup boundary. The experiment done on 70 clinical data sets collected from Singapore Eye Research Institute shows the new algorithm outperforms ARGALI [6] system by 43%. This promising results further leads to a large clinical evaluation of the algorithm involving 15 thousand patients from Center for Eye Research Australia as well as Singapore Eye Research Institute and Singapore National Eye Center.

## II. METHODOLOGY

### A. *Neuro-Retinal Optic Disc and Cup boundary detection and CDR calculation in Glaucoma Diagnosis*

Clinically, Neuro-Retinal Optic Disc and Cup boundaries are measured in order to calculate the CDR value. Figure 2 shows a simplified workflow of computer-aided glaucoma diagnosis through cup-to-disc ratio measurement.

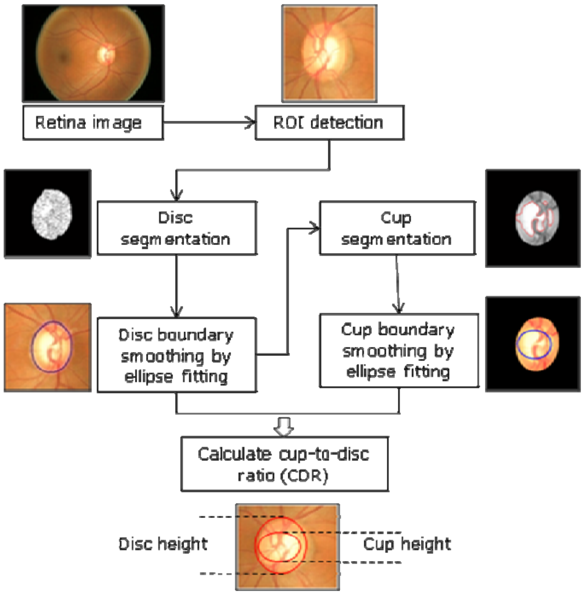

Figure 2. CDR calculation in Glaucoma Diagnosis

*Disc segmentation* – The Neuro-retinal image is processed in order to detect the disc boundary using optimal color channel as determined by the color histogram analysis and edge analysis

*Disc boundary smoothing* – The disc boundary detected from the above step may not represent the actual shape of the disc since the boundary can be affected by a large number of blood vessels entering the disc. Therefore, ellipse fitting is performed to reshape the obtained disc boundary.

*Cup segmentation* – As there are more amounts of blood vessels and noises intersecting this region and also the transition between the cup and the rim is often not too prominent as that of disc boundary, more robust image processing techniques are normally used to segment the cup.

*Cup boundary smoothing* – After the cup boundary has been detected, ellipse fitting is again employed to eliminate some of the cup boundary's sudden changes in curvature. Ellipse fitting becomes especially useful when portions of the blood vessels in the neuro-retinal rim outside the cup are included within the detected boundary. The CDR is consequentially obtained based on the height of detected cup and disc.

### B. *Ellipse Optimization (Fitting) for optic disc and cup*

Ellipse fitting algorithm can be used to smooth the disc and cup boundary. Ellipse fitting is usually based on least square fitting algorithm which assumes that the best-fit curve of a given type is the curve that has the minimal sum of the deviations squared from a given data points (least square error).

B2AC (*Direct Least Square Fitting Algorithm)* [8] is chosen to fit the optic and cup over other popular ellipse fitting algorithms like *Bookstein Algorithm* [9], *Taubin Algorithm* [10]. Instead of fitting general conics or being computationally expensive, B2AC minimizes the algebraic distance subject to a constraint, and incorporates the ellipticity constraint into the normalization factor. It is ellipse-specific, so that effect of noise (ocular blood vessel, hemorrhage, drusens, etc.) around the cup area can be minimized while forming the ellipse. It can also be easily solved naturally by a generalized eigensystem.

In B2AC, a quadratic constraint is set on the parameters to avoid trivial and unwanted solutions. The goal of B2AC is to search a vector parameter which contains the six coefficients of the standard form of a conic. Minimizing the sum of the squared algebraic distance $\mathbf{Da}$, can be solved by considering rank-deficient generalized eigenvalue system,

$$\mathbf{D}^T\mathbf{Da} = \lambda\mathbf{Ca} \qquad (1)$$

where $\mathbf{D} = [\mathbf{x}_1\ \mathbf{x}_2 \dots \mathbf{x}_n]^T$ is the $\mathbf{n} \times \mathbf{6}$ *design matrix* for $\mathbf{n}$ data points $\mathbf{x}_i$ and $\mathbf{C}$ is the *constraint matrix*.

B2AC method further constrains the parameter vector '$\mathbf{a}$' in such a way that it forces the conic to be an ellipse through imposing the equality constraint

$$\mathbf{4ac - b^2 = 1} \qquad (2)$$

Where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are the first three coefficients of the conic. This quadratic constraint can be expressed in matrix form $\mathbf{a}^T\mathbf{Ca} = \mathbf{1}$. The constrained ellipse fitting problem reduces to minimize $||\mathbf{Da}||^2$ subjected to the constraint $\mathbf{a}^T\mathbf{Ca} = \mathbf{1}$. It is possible to rewrite eq. (1) as

$$\mathbf{Sa} = \lambda\mathbf{Ca} \qquad (3)$$

where $\mathbf{S}$ is the scatter matrix, $\mathbf{D}^T\mathbf{D}$ and this system can readily be solved by considering the generalized eigenvectors of eq. (3). The solution of the eigensystem (eq. 3) gives six eigen-value-eigenvector pairs $(\lambda_i, \mathbf{u}_i)$ but by considering the minimization $||\mathbf{Da}||^2$, subjected to the constraint (2) would yield only one solution, which corresponds, by virtue of constraint, to an ellipse.

### C. Convex Hull Based Ellipse Optimization

A convex hull [7] of a set of points is the smallest convex polygon that contains every one of the points. It is defined by a subset of all the points in the original set, as shown in figure 3.

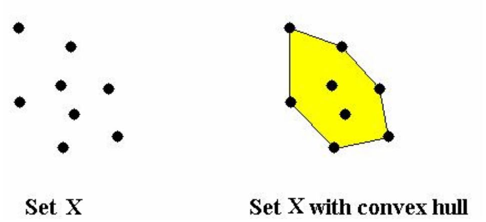

Figure 3. Example of a Convex Hull

The convex hull of **X** can be described constructively as the set of convex combinations of finite subsets of points from X: that is, the set of points of the form $\sum_{j=1}^{n} t_j x_j$, where **n** is an arbitrary natural number, the numbers $\mathbf{t_j}$ are non-negative and sum to 1, and the points $\mathbf{X_j}$ are in **X**. It is simple to check that this set satisfies either of the two definitions above. So the convex hull **Hconvex(X)** of set **X** is:

$$H_{\text{convex}}(X) = \left\{ \sum_{i=1}^{k} \alpha_i x_i \;\middle|\; x_i \in X,\, \alpha_i \in \mathbb{R},\, \alpha_i \geq 0,\, \sum_{i=1}^{k} \alpha_i = 1,\, k = 1, 2, \ldots \right\}.$$

If X is a subset of an N-dimensional vector space, convex combinations of at most **N+1** points are sufficient in the definition above. This is equivalent to saying that the convex hull of X is the union of all simplexes with at most **N+1** vertex from X.

In this paper we proposed convex hull based ellipse optimization for optical cup detection. Figure 4 shows how convex hull was applied in our system in selecting feature points around the optic cup region. The pixel set obtained from level set method for cup region (see Fig.4.a) were usually segmented with the influence of the interweavement of surrounding ocular blood vessels, hemorrhage, drusens and other noises. If all those pixels were fed to ellipse fitting algorithm, they could yield an unreal cup boundary.

## III. IMAGE ANALYSIS

To evaluate the performance of our approach, we obtained 70 patients' Neuro-retinal images from Singapore Eye Research Institute. The cup-to-disc ratio (CDR) for each neuro-retinal image was provided by ophthalmologist using stereographic viewers and was used as "ground truth" against which the performance of our proposed method was evaluated. We compared our method's performance with ARGALI system [4], which uses B2AC without convex hull for cup ellipse fitting. The following 5 steps were performed to calculate the CDR value.

***RIO detection:*** Region of Interest (ROI) localization was performed in order to reduce the computational requirements by only focusing on an appropriate region.

***Disc segmentation****:* Variational level-set algorithm [11] was performed on the Neuro-retinal image in order to detect the disc boundary using optimal color channel as determined by the color histogram analysis and edge analysis. This algorithm was chosen to avoid shocks during level-set process by the fact that it introduced an energy term to keep evolving level-set function close to a signed distance function. In this way, we avoided errors in segmentation due to shocks and discontinuities from the re-initialization process.

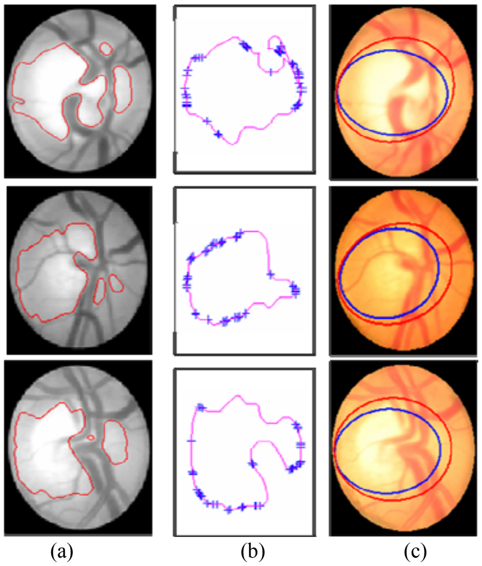
(a) (b) (c)

Figure 4. (a) Cup region pixel set detected by level set method; (b) Feature points selected by convex hull, to feed ellipse fitting; (c) ellipse optimization by convex hull. Blue ellipses were generated without convex hull, red ellipse were generated using convex hull based ellipse optimization.

***Disc boundary smoothing***: B2AC ellipse fitting was performed to reshape the obtained disc boundary.

***Cup segmentation***: An approach (Threshold -initialization based level-set), different from the disc segmentation method used above, was used for the cup segmentation. In threshold-initialization-based level-set method, the green channel of the extracted optic disc was processed using histogram analysis to determine a threshold value, which segments out the pixels corresponding to the top 1/3 of the grayscale intensity, was used to define the initial contour in the ROI.

***Cup boundary smoothing*** – Different from the ARGALI system [4], which feeds all boundary points to B2AC; the new Convex Hull Based Neuro-Retinal Optic Cup Ellipse Optimization algorithm used only those feature points selected by Convex Hull to feed B2AC. This eliminated the effect of the unwanted inner points.

## IV. EXPERIMENT AND RESULT

### A. Analysis

Figure 4 shows an example of the improvement in calculating the Cup to Disc ratio based on the new approach. Fig.4.a shows cup region pixel set detected by level set method; Fig.4.b shows the feature points selected by convex hull, which are used to feed ellipse fitting; and Fig.4.c illustrates the improvement achieved for convex-based ellipse optimization as compare to ARGALI method. For the top image, the clinical CDR for it is 0.6381 while the one generated by ARGALI and the new algorithm are 0.564, 0.6612 respectively. The error rates for the both cases are 10.9% and 3.6%. We can see that using feature points selected from the pixel set using convex hull to fit the ellipse cup (see Fig.4.b), the system could generate more realistic neuro-retinal optic cup (As shown in Fig.4.c)

### B. *Error Distribution*

The error ranges of our new algorithm and ARGALI approach are measured in the experiment. It is found that the error range of convex hull based algorithm is smaller (between 0.15 and -0.3) comparatively. After calculating the root mean squared error (RMS) for the ARGALI and our new method, a significant difference of 0.0753 is observed. To compare the spread of the errors for the ARGALI and our new method, standard deviations for the approaches are calculated. As shown in Fig 5, the spread of CDR errors with respect to clinical values is more spaced out for ARGALI while those based on the new algorithm cluster around the 'zero error line'.

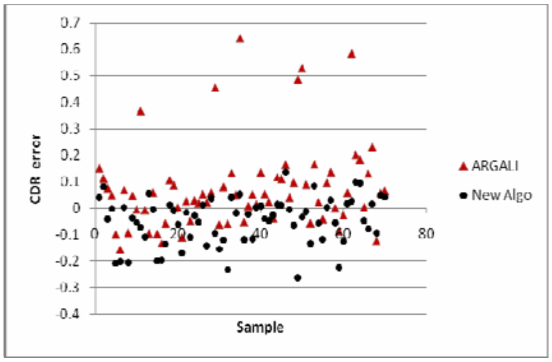

Figure 5. Error distribution over ground truth

Table 1 shows the RMS error and Standard deviation calculated for both ARGALI and our new algorithm. The smaller RMS error value indicates a smaller variation of the errors; smaller standard deviation also shows more stable the detection result. The improvement of RMS error as compare to ARGALI is (0.1761-0.1008)/0.1761* 100 = 42.8%.

TABLE I
ROOT MEAN SQUARED ERROR AND STANDARD DEVIATIONS

| Method | Error Range | RMS | Standard Deviation |
|---|---|---|---|
| ARGALI | 0.7 - -0.2 | 0.1761 | 0.1239 |
| New Algorithm | 0.15 - 0.252 | 0.1008 | 0.0614 |

### C. *Glaucoma Diagnosis*

Figure 6 shows the detection of glaucoma based on ARGALI and our new method, using clinical CDR value 0.65 as the guideline. It is found that the new algorithm doubles the chances of correct diagnosis. The correlation coefficients of *ARGALI-GT* and *new algorithm-GT* are found to be 0.0907, 0.2304 respectively.

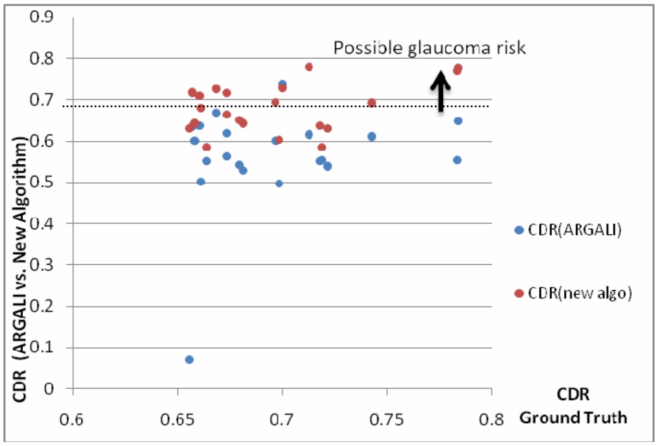

Figure 6. Detection of Glaucoma cases using ARGALI and new algorithm

## V. CONCLUSION

In the paper, we presented a convex hull based ellipse optimization algorithm for a more accurate detection of neuro-retinal optical cup. Comparing with the state-of-the-art ARGALI system, the new approach achieves a better CDR value calculation, which results to more accurate Glaucoma Diagnosis. The good performance of the new approach leads to a large scale clinical evaluation involving 15 thousand patients from Australia and Singapore. We will be able to report large clinical findings in the future.

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