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Curve Fitting of Spikes in Neural Signals

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***Abstract—Purpose*: Find an optimal function model of spikes of high signal-to-noise ratio (SNR) spontaneous signals in the spinal cord of a rat, and use it to recognize the patterns of spikes of low SNR signals in the sciatic nerve of the rat. *Method*: Firstly, several function models of spikes of high SNR spontaneous signals in the spinal cord of a rat are calculated under the rule of least square. By choosing an optimal function model based on minimum standard deviation (SD) of error of fitting, it is contrasted with the waveform of classical action potential (AP). Then, this model is used as a pattern to recognize spikes of low SNR signals in the sciatic nerve of the rat. *Result*: The optimal function model of spikes of high SNR spontaneous signals in the spinal cord of a rat is a proportional model whose numerator is a 5-order polynomial while the denominator is a 4-order polynomial. The waveform of a typical AP can be obtained from this model. It can also achieve good performance by recognizing the pattern of spikes of signals whose SNR is lower than 8 dB in sciatic nerve of the rat.**

I. Introduction

The nervous system is the way for human to apperceive inside and outside environment, and also is the way for the brain to control the movement of human body. The basic unit of the nervous system is the neuron which generates, transmits and processes neural information. There are two kinds of neural information: neural electrical signals and neural chemical signals. Since it is easier to record neural electrical signals by electric devices than neural chemical signals, neural signals are usually represented by neural electrical signals.

Thanks to the rapid development of microelectronics, we now can easily record neural signals by microelectronic devices [1]. Neurons use APs to code the information. The waveforms of neural signals recorded by microelectrodes are stochastic spikes without stimulation or regular clusters of spikes with stimulation [2], and the noise can also be seen from them.

For the patterns of APs generated by different neurons are not the same in any case, spikes which are generated by different neurons are usually classified into different templates [3]. The typical waveform of an AP recorded by a single-electrode or by multi-electrode can be obtained from neuroscience textbooks and literatures [4-5]. However, it is just the waveforms whose mathematical expressions can hardly be obtained from books, web and literatures. Besides that, the SNR of a neural signal is usually low, which baffles classification of spikes.

Nowadays, researching reports about spikes from single neurons, peripheral nerve and some parts of central nervous system (CNS) such as visual nerve, auditory nerve, olfactory nerve and cortical cluster of neurons can hardly be found. However, it is rare to find researching reports of spikes in neural signals recorded from a spinal cord. Generally speaking, the neural functions can not be reconstructed after spinal cord injury. Besides that, hitherto we can not reconstruct neural functions by surgical operation [6]. In order to help patients who suffer from paralysis, our researching group has been doing exploratory research of neural channel bridging and signal regeneration by using a micro-electronics embedded system [7-8]. Up to now, many animal experiments with rats, rabbits and toads have been carried out, and a series of nerve signals have been successfully detected from spinal cords and sciatic nerves.

Data used in this paper were obtained from one of these animal experiments with rats. The experiment was done in the Key Lab of Neural Regeneration of Jiangsu Province in Nantong University, Nantong, China. In these experiments, different types of cuff micro-electrodes made by Fraunhofer Institute of Biomedical Engineering, Sant Ingeburg, Germany, were used. The neural signal detecting circuit was developed by ourselve and an oscillograph of Agilent 33220A was used to observe the waveforms and to make signal recording. Since the spectrum of neural signal is mainly lower than 4 kHz [9], the sampling frequency was higher than 8 kHz.

In this paper, the curve fitting of spikes in a high SNR spontaneous signal from the spinal cords of rats based on least-squares will be done firstly and the optimal function model under the rule of minimization of SD of fitting will be found. From this function model, the waveform of typical AP given in textbooks can be obtained. It will be shown that the patterns can be recognized from the nervous signals of the sciatic nerve of a rat even though the SNR is lower than 8 dB.

II. METHODS

A. Rules and Tool of Curve Fitting

The tool of curve fitting used in this paper is Curve Fitting Toolbox 1.1.3 in MATLAB7.0.4, a product of Mathworks. In MATLAB, the parameter or non-parameter model can be used to fit. When considering the purpose of this paper is to find the mathematic expressions of spikes, we chose the parameter model.

After the function model was chosen, parameters of the model will be determined based on least-squares. That means, if we find a smooth curve y(ti, k) (k is the unbeknown parameter vector) from given data (ti, xi) (i=1, 2, …M, ti is

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the time coordinate of each sample and x_i is the value of each sample) and these parameters make $\sum_{i=1}^{M}[x_i - y(t_i, \mathbf{k})]^2$ minimal, $y(t_i, k)$ can be called the function model of data (t_i, x_i) under the rule of least-squares.

B. *Assumptions of Curve Fitting*

The assumptions of curve fitting are as follows: the error of fitting is stochastic, satisfies normalized Gaussian distribution and has zero mean and invariable variance. That is error ~N(0, σ2), where σ2 represents the variance. Since we did not restrict the statistical characteristic of data before curve fitting based on least-squares, the least-squares is a kind of adaptive rule for estimation without any priori knowledge of data.

C. *Parameter Models for Curve Fitting*

Five parameter models in common use are as follows:

1) Polynomial Model. The general formula of s order polynomial model is

$$f(t) = \sum_{i=0}^{s} a_i t^{s-i} \qquad (1)$$

where ai is the parameter, and we use "s-P" to denote s-order polynomial model infra.

2) Exponential Model. The general formula of p-order exponential model is

$$f(t) = \sum_{i=1}^{p} a_i e^{(b_i t)} \qquad (2)$$

where ai and bi are both parameters. Generally speaking, it can bring large SD of curve fitting of neural spikes when this model is used. Thus, it is not used it in this paper.

3) Fourier Model. The general formula of p-order Fourier model is

$$f(t) = a_0 + \sum_{i=1}^{p}[a_i \cos(i\omega t) + b_i \sin(i\omega t)] \qquad (3)$$

where a0, ai and bi are parameters, and we use "F-p" to denote p order Fourier model infra.

4) Gaussian Model. The general formula of p-order Gaussian model is

$$f(t) = \sum_{i=1}^{p} a_i e^{-\left(\frac{t-b_i}{c_i}\right)^2} \qquad (4)$$

where ai, bi and ci are parameters, and we use "G-p" to denote p-order Gaussian model infra.

5) Rational Fraction Model. The general formula of rational fraction model whose numerator is m-order polynomial and whose denominator is n-order polynomial is

$$f(t) = \frac{\sum_{i=0}^{m} a_i t^{m-i}}{t^n + \sum_{i=0}^{n-1} b_i t^{n-1-i}} \qquad (5)$$

where ai and bi are both parameters, and we use "R-m/n" to denote this kind of model infra.

Certainly we can design some other models for the usage of curve fitting, yet we can obtain good performance of fitting by using one of 5 models above, so it is restricted in these 5 models in this paper.

D. *Guideline of Choosing Optimal Function Model*

The minimization of SD can be used to choose optimal function model from umpty function models.

III. RESULTS

A. *Curve Fitting of Spikes in A High SNR Neural Signal*

A high-SNR spontaneous signal in the spinal cord of a rat, recorded by a cuff triple-electrode, is shown in Fig. 1(a). The formula for the SNR calculation is

$$SNR = 20\log_{10}\left(\frac{V_{usefulness}}{V_{noise}}\right) \qquad (6)$$

where $V_{usefulness}$ is the average amplitude, and V_{noise} is the gap with the average. The SNR is an estimate of average amplitude based on sliding window, calculate the greatest difference between amplitude and average, and then calculate the statistical average of difference in all the windows, which estimate the SNR. The SNR of this signal is about 20 dB. Four clusters of spikes and a single spike can be easily discovered from Fig. 1. That means the neurons use single and cluster release modes to generate APs and there are a different numbers of spikes in each cluster. We used the segment of signal around 80 ms and then did fast Fourier transformation (FFT) to obtain the spectrum which was shown in Fig. 1(b). The highest peak of spectrum envelope is near 1100 Hz and the second highest peak of spectrum envelop is near 1900 Hz, and these results are similar to the characteristic of spectrum of neural signal recorded by cuff electrodes in literature [9].

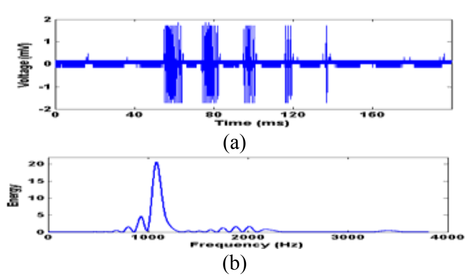

Fig. 1 A spontaneous signal with high SNR in the spinal cord of a rat recorded by cuff triple-electrode and its spectrum. (a) A spontaneous signal with high SNR in the spinal cord of a rat recorded by cuff triple-electrode. (b) Spectrum of the segment of the signal around 80 ms.

For the regulation of waveform of this signal, we arbitrarily selected 3 spikes (around 54.94 ms, 78.94 ms and 98.31 ms) to make curve fitting, and then plotted all the function models whose fitting error is less than 0.2 in each subgraph of Fig. 2.

In Fig. 2, the black solid dots denote samples of original signal, and each curve with different color denotes the result by using a given model for fitting.

The goodness-of-fit of 3 spikes above are listed in Table 1, 2 and 3. Since the abscissa of original signal is time, the independent variable of fitting model is t. The criteria for evaluation is the minimization of SD. The more nearness to zero of SD, the better is goodness-of-fit.

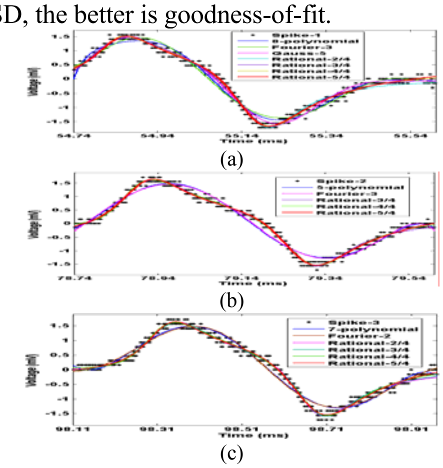

Fig. 2 The results of selecting arbitrarily 3 spikes (around 54.94ms, 78.94ms and 98.31ms) to make curve fitting, and to plot all the function models whose fitting error is less than 0.2 in each sub-graph. (a) Curve fitting of spike around 54.94ms. (b) Curve fitting of spike around 78.94ms. (c) Curve fitting of spike around 98.31ms. Black solid dots denote samples of original signal, and each curve with different color denotes the result by using a given model for fitting.

Table 1 Goodness-of-fit of spike around 54.94 ms in the signal shown in Fig. 1(a)

Model	Function Expression	SD
8-P	$f(t)=-0.13t^8+0.13t^7+0.97t^6-0.77t^5-2.8t^4+1.7t^3+3.8t^2-1.9t-1.2$	0.165
F-3	$f(t)=1.4\times10^8-2.1\times10^8\cos(0.056t)-1.1\times10^6\sin(0.056t)+8.5\times10^7\cos(0.11t)+9\times10^5\sin(0.11t)-1.4\times10^7\cos(0.17t)-2.3\times10^5\sin(0.17t)$	0.183
G-5	$f(t)=1.1e^{-\left(\frac{t-90.7}{93.0}\right)^2}-1.7e^{-\left(\frac{t-239.1}{67.3}\right)^2}+0.4e^{-\left(\frac{t-71.7}{19.8}\right)^2}+0.4e^{-\left(\frac{t-44.7}{17.7}\right)^2}-5.6\times10^{11}e^{-\left(\frac{t-3967}{653.4}\right)^2}$	0.131
R-2/4	$f(t)=\frac{-0.4t^2-1.3t-0.4}{t^4+2.5t^3+1.8t^2+0.2t+0.3}$	0.126
R-3/4	$f(t)=\frac{0.5t^3+0.1t^2-2t-0.7}{t^4+2.8t^3+2.2t^2+0.2t+0.5}$	0.116
R-4/4	$f(t)=\frac{0.3t^4+0.7t^3-0.7t^2-2t-0.6}{t^4+2.7t^3+2.1t^2+0.5t+0.4}$	0.107
R-5/4	$f(t)=\frac{0.2t^5+0.6t^4+0.3t^3-1.1t^2-1.7t-0.5}{t^4+2.6t^3+2.1t^2+0.5t+0.3}$	0.106

Table 2 Goodness-of-fit of spike around 78.94 ms in the signal shown in Fig. 1(a)

Model	Function Expression	SD
5-P	$f(t)=-0.3t^5-0.1t^4+1.9t^3+0.4t^2-2.8t-0.1$	0.193
F-3	$f(t)=-4\times10^7+6\times10^7\cos(0.05t)-3.6\times10^6\sin(0.05t)-2.4\times10^7\cos(0.1t)+2.9\times10^7\sin(0.1t)+4\times10^6\cos(0.15t)-7.2\times10^5\sin(0.15t)$	0.192
R-3/4	$f(t)=\frac{0.3t^3-0.4t^2-1.5t+0.1}{t^4+1.2t^3-0.5t^2-0.6t+0.7}$	0.108
R-4/4	$f(t)=\frac{0.2t^4+0.5t^3-0.9t^2-1.7t+0.1}{t^4+1.3t^3-0.5t^2-0.4t+0.7}$	0.103
R-5/4	$f(t)=\frac{0.2t^5+0.3t^4-0.3t^3-t^2-1.2t-0.1}{t^4+1.2t^3-0.3t^2-0.3t+0.5}$	0.099

Table 3 Goodness-of-fit of spike around 98.31ms in the signal shown in Fig. 1(a)

Model	Function Expression	SD
7-P	$f(t)=0.02t^7-0.06t^6-0.55t^5+0.4t^4+2.5t^3-0.82t^2-3t+0.4$	0.182
F-2	$f(t)=0.076+0.16\cos(1.46t)-0.93\sin(1.46t)+0.17\cos(2.91t)-0.63\sin(2.91t)$	0.189
R-2/4	$f(t)=\frac{-0.5t^2-0.8t+0.2}{t^4+0.6t^3-0.6t^2-0.2t+0.4}$	0.110
R-3/4	$f(t)=\frac{0.1t^3-0.5t^2-0.9t+0.2}{t^4+0.6t^3-0.6t^2-0.3t+0.4}$	0.109
R-4/4	$f(t)=\frac{0.2t^4+0.2t^3-t^2-t+0.3}{t^4+0.7t^3-0.6t^2-0.2t+0.5}$	0.103
R-5/4	$f(t)=\frac{0.2t^5+0.3t^4-0.3t^3-t^2-0.8t+0.2}{t^4+0.6t^3-0.5t^2-0.1t+0.4}$	0.100

From Fig. 2 and Table 1, 2 and 3 we can easily find that, for each spike, R-5/4 model has the least SD, so R-5/4 model is more suitable for curve fitting of spikes in spontaneous signals with high SNR in spinal cord of rats.

B. *Comparation Between Waveform of Classical AP and R-5/4 Model*

The general formula of R-5/4 model is

$$f(t)=\frac{a_1t^5+a_2t^4+a_3t^3+a_4t^2+a_5t+a_6}{t^4+b_1t^3+b_2t^2+b_3t+b_4} \qquad (7)$$

where a1, a2, a3, a4, a5, a6, b1, b2, b3 and b4 are fitting parameters.

Suppose a1=-0.0345, a2=–0.021, a3=0.025, a4=0.023, a5=0.002, a6=–0.0009, b1=1.255, b2=0.573, b3=0.145 and b4=0.024, f(t)(t∈[-1.5,1]) can be plotted in Fig. 3. We can clearly find that the functional image of f(t) is very similar to the waveform of a typical AP. Five parts in functional image corresponding to resting potential, depolarization, repolarization, hyperpolarization and comes back to resting potential respectively.

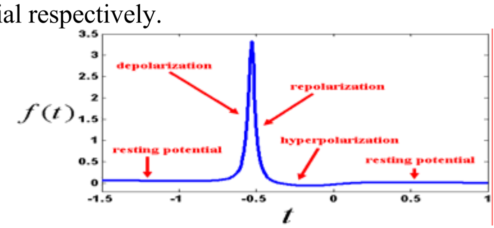

Fig. 3. The functional image of $f(t)(t\in[-1.5,1])$ of R-5/4 model when choosing a_1=-0.0345, a_2=-0.021, a_3=0.025, a_4=0.023, a_5=0.002, a_6=-0.0009, b_1=1.255, b_2=0.573, b_3=0.145 and b_4=0.024. Five parts in functional image corresponding to resting potential, depolarization, repolarization, hyperpolarization and comes back to resting potential respectively.

Thus, it can be seen, the R-5/4 model can be used not only to fit spikes in neural signals recorded in experiments to get good performance, but also to generate the waveform of a typical AP.

C. *Pattern Recognition of Sciatic Signal with Low SNR of Rat*

A low-SNR nervous signal recorded by a cuff triple-electrode from the sciatic nerve of a rat, while the head of the rat was rapped, is shown in Fig. 4(a). The SNR of the signal is about 8 dB which is much lower than that in Fig. 1(a). The result of pattern recognition of the signal is shown in the lower Fig. 4(b). The bars in 4 rows in top-down direction denote R-5/4, R-2/4, R-3/2, R-4/4, and G-3 models, respectively.

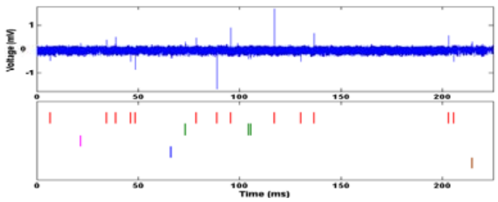

Fig. 4 (a) A low-SNR nervous signal recorded by cuff triple-electrode from the sciatic nerve of a rat, while the head of the rat was rapped. (b) The result of pattern recognition of the signal

From Fig. 4 we can easily find the pattern recognition functions. 13 patterns are recognized from 19 spikes, by using R-5/4 model even if SNR is lower than 8 dB. The recognition ratio is 68.4%. There are two likelihoods for invalidation of R-5/4 model of other spikes: interference or other modes of generation of APs of neurons.

On one hand, a spike can be generated by some kind of interference; on the other hand, sciatic nerve contains tens of thousands of nerve fibers, and these fibers belong to different kinds of neurons. It is well known that the modes of generating APs of different neurons are not the same, and superposition of these APs can produce a distinctive spike, so R-5/4 model is unsuitable at this point.

IV. DISCUSSION

A. Analysis of Sensitivity of R-5/4 Model to Independent Variable

It is not a contingency to obtain the waveform of a typical AP from Eq. (7). It is determined by the property of R-5/4 model itself. Using a long division, the identical transformation of Eq. (7) is

$$f(t) = \frac{a_1t^5 + a_2t^4 + a_3t^3 + a_4t^2 + a_5t + a_6}{t^4 + b_1t^3 + b_2t^2 + b_3t + b_4} = a_1t + A + \frac{Bt^3 + Ct^2 + Dt + E}{t^4 + b_1t^3 + b_2t^2 + b_3t + b_4} \quad (8)$$

where $A=a_2-a_1b_1$, $B=(a_3-a_1b_2)-b_1A$, $C=(a_4-a_1b_3)-b_2A$, $D=a_5-b_3A$, $E=a_6-b_4A$.

When $|t|$ is large, the influence of a_1t+A to $f(t)$ becomes more dominant than that of $\frac{Bt^3+Ct^2+Dt+E}{t^4+b_1t^3+b_2t^2+b_3t+b_4}$. Thus, $f(t)$ approaches a linear function of the time.

When $|t|<1$, the influence of $\frac{Bt^3+Ct^2+Dt+E}{t^4+b_1t^3+b_2t^2+b_3t+b_4}$ to $f(t)$ becomes more dominant than that of a_1t+A. Then, $f(t)$ becomes more sensitive to the time.

For an actual AP, the duration is usually no more than 10 ms. In this case, we have only interest in the functional image and f(t) can break. When a smaller |a1| and other appropriate parameters are chosen, aimed to keep relative smooth of f(t) before a break occurs, the waveform of a typical AP can be recurred, as shown in Fig. 3.

B. Comparation between R-5/4 Model and Other Rational Models

Using a long division, it can be found that all the models whose order of numerator is larger than that of denominator and the difference between them is 1, such as R-4/3, R-3/2, R-6/5 and so on, have a similar characteristic with R-5/4.

Sometimes a better performance can be obtained when the order of function model is increased. However, the complexity of the model and the computing time will be increased. That is the reason why the models of a higher order, such as R-6/5, will not be accepted.

A lower-order model, such as R-3/2 model, can be used only for a large SD. Therefore, the R-5/4 model gives a good tradeoff between SD and the complexity of the model.

V. CONCLUSION

In this paper, a curve fitting arithmetic of the spikes in a high SNR spontaneous signal from the spinal cord of a rat has been made firstly based on least-squares. It is found that under the rule of minimization of SD, the optimal function model is the rational one whose numerator and denominator is a 5-order polynomial and a 4-order polynomial, respectively. Using this model, the patterns in a low SNR signal in the sciatic nerve of a rat can be recognized, even when the SNR is lower than 8 dB.

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