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Directed Information Measure for Quantifying the Information Flow in the Brain

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Abstract— **In neurophysiology, it is important to determine the causal relationships between neuronal sites. The major problem with existing methods for quantifying the causality in the brain, e.g. Granger causality, is that they assume an multivariate autoregressive signal model for the multi-channel EEG signals and do not take the nonlinear dependencies between neuronal oscillations into account. In this paper, we propose to quantify the causality of the interactions based on the directed information (DI) criterion, which measures the information flow between two signals over time. The proposed measure is applied to real EEG data from control and schizophrenic subject groups. The significance of the computed DI values are verified by Fourier bootstrapping technique.**

I. INTRODUCTION

Neuroimaging technology such as the electroencephalogram (EEG) makes it possible to record brain activity with high temporal resolution and accuracy. However, the current neuroimaging modalities reflect solely the local neural activities rather than the large-scale interactions between different parts of the brain. There has been evidence that large-scale functional integration of the brain is mediated by neuronal groups that oscillate in the gamma band range ($30 - 80$ Hz). It has been found that schizophrenic patients exhibit deficits in gamma band neural synchrony compared to normal subjects [1]. In order to get a better understanding of the human brain and to develop alternative treatments to brain diseases such as schizophrenia, measures of interdependence and causality between brain activity recorded at different neural sites are needed.

Different measures for quantifying the interaction and direction between two signals include cross-correlation, Granger causality, directed transfer function (DTF), partial directed coherence (PDC), and information-theoretic measures [2], [3]. The cross-correlation function gives the linear correlation between two variables X and Y as a function of the lag time, which reflects the causal relationship between the signals. Granger quantifies causality such that the time series X causes Y if the variance of the prediction error for Y at the present time is reduced by including past measurements from X in the linear regression model. DTF and PDC are similar to the Granger causality and quantify the linear interaction assuming a multivariate autoregressive (MVAR) model for the underlying signals. However, these approaches may be misleading when applied to EEG signals which are known to have nonlinear dependencies [4]. The nonlinear correlation coefficient, which describes the dependency of two signals in a more general way, was proposed to address this issue. However, inferring causality from the time delay is not always straightforward. Freiwald [5] proposed a Local Nonlinear Autoregressive model to test not only the causality but also the degree of nonlinearity in the interaction between neuronal sites. Nonetheless, the proposed model still requires a priori knowledge about the underlying signal models, which are often unknown and can introduce inaccuracies due to the difficulty of parameter estimation. Therefore, a model-free measure detecting both the linear and nonlinear relationships is desired.

Information-theoretic methods are used to measure the dynamic and directional information between two signals. These measures include transfer entropy, directed information and directed transinformation [6], [7], [8]. In this paper, we apply Directed Information (DI) to quantify the dynamic information flow across the different brain sites without assuming any underlying signal models. Moreover, we also evaluate whether there is increased information flow in the gamma band between the frontal and parietal lobes during target perception for control group compared to schizophrenic group.

In this paper, we first introduce the directed information measure in Section II. A statistical method to verify the significance of the the computed DI values is given in Section III. Finally, Section IV discusses the application of this measure to EEG data from a study of schizophrenia.

II. DIRECTED INFORMATION

A. Definition

Different information measures have been proposed to quantify the causal relationship between two random processes. Some common measures are the "transfer entropy", "directed transinformation" and "directed information". "Transfer entropy" is presented by [6], which is a conditional entropy defined by Kullback divergence, with the difficulty of appropriate conditioning of transition probabilities. "Directed transinformation" (T) introduced by Saito [7] measures the information flow from the current sample of one signal to the future samples of another signal given the past samples of both signals and has the problem of not discriminating between totally dependent and independent processes. The directed information measure introduced by Massey [8] is defined for two length N sequences $\mathbf{X} = X^N = X_1, \cdots, X_N$ and $\mathbf{Y} = Y^N = Y_1, \cdots, Y_N$ as

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follows:

$$DI(X^N \rightarrow Y^N) = \sum_{n=1}^{N} I(X^n; Y_n | Y^{n-1}), \quad (1)$$

where $X^n = (X_1, \ldots, X_n), Y^n = (Y_1, \ldots, Y_n)$ are the length n random sequences.. $I(X;Y)$ is the mutual information between two random variables X and Y.

Compared to other measures, the last measure is more discriminative for measuring the dependency between two time series and has been shown to be effective in different applications [9].

B. Directed Information Computation

Mutual information between two N dimensional random vectors can be written as $I(X^N;Y^N) = H(X^N) - H(X^N|Y^N)$, where $H(X^N)$ and $H(X^N|Y^N)$ are the Shannon entropy of X^N and the conditional entropy of X^N given Y^N, respectively. Using this definition, DI can be rewritten in terms of individual and joint entropies of X and Y.

$$\begin{aligned} DI(X^N \rightarrow Y^N) &= \sum_{n=1}^{N} [H(X^n|Y^{n-1}) - H(X^n|Y^n)] \\ &= \sum_{n=1}^{N} [I(X^n;Y^n) - I(X^n;Y^{n-1})] \end{aligned} \quad (2)$$

Therefore the computation of DI requires the estimation of joint probabilities of high dimensional random variables over time.

According to equation (2) one can observe that, as N increases, the estimation of high order densities becomes complicated. Therefore, in practice, the directed information measures are applied to every two samples of X and Y. The expression for every two samples at the k^{th} time sample can be written as [9]:

$$\begin{aligned} DI_k(X_k X_{k+1} \rightarrow Y_k Y_{k+1}) =& H(X_k) - H(X_k Y_k) \\ &+ H(X_k X_{k+1} Y_k) + H(Y_k Y_{k+1}) \\ &- H(X_k X_{k+1} Y_k Y_{k+1}). \end{aligned} \quad (3)$$

The estimation of the entropies in equation (3) requires the estimation of the underlying probability distributions. If we assume that X and Y are normally distributed, then equation (3) can be reduced to,

$$DI_k(X_k X_{k+1} \rightarrow Y_k Y_{k+1}) = \frac{1}{2} \log \frac{|X_k||X_k X_{k+1} Y_k||Y_k Y_{k+1}|}{|X_k Y_k||X_k X_{k+1} Y_k Y_{k+1}|}, \quad (4)$$

where $|Z_1, Z_2, \ldots, Z_n|$ is the determinant of the covariance matrix of n random variables $Z_1, Z_2, \ldots, Z_n$. In this way, we can obtain the DI between two successive time points over the whole time series.

III. Significance Testing

In order to test the significance of the directed information between two time series, we first normalize the DI coefficient [10].

From the definition of DI, we can observe that $0 < DI(X^N \rightarrow Y^N) < I(X^N;Y^N) < \infty$. Therefore, we use the following normalized version of DI, which maps DI to the $[0, 1]$ range [10]:

$$\rho_{DI} = \sqrt{1 - e^{-2 \sum_{i=1}^{N} I(X^i; Y_i | Y^{i-1})}}. \quad (5)$$

Once the normalized DI is obtained, its significance can be determined using Fourier bootstrapping technique. For this purpose, we generated 100 independent pairs of surrogate time series, as proposed by Theiler [11]. In this method, the surrogate data are constructed to have the same Fourier spectra as the raw data. First, each data set is independently transformed by a Fourier transform, then the phase is randomized by multiplying each complex amplitude with $e^{i\varphi}$, where φ is independently chosen for each frequency from the interval $[0, 2\pi]$ with $\varphi(f) = -\varphi(-f)$. Finally, the inverse Fourier transform is used to get the surrogate data. After obtaining the surrogate data, we compute the DI for each pair of data and determine a thresholds such that the probability of obtaining a particular DI value by chance is less than 0.1.

IV. Results

In this section, we test the effectiveness of directed information on real EEG data.

A. EEG Data

We examined the electrophysiological activity in 10 schizophrenia patients and 10 non-psychiatric control subjects who performed a continuous performance task (CPT). The directed information measure was computed over a window corresponding to the $P300$ response, $200 - 600$ ms after the stimulus, over all trials (94 trials) between 27 electrode pairs in the gamma frequency band ($30 - 55$Hz). The sampling frequency is 500Hz. In order to extract the signals in the gamma band, a second order Butterworth bandpass filter is applied and a window with 200 time samples, corresponding to the $P300$ range, is extracted.

B. Distribution of EEG data

The computation of DI in equation (4) is based on the assumption that the distribution of the signal is Gaussian. The real EEG data following a Gaussian distribution is rejected at significance level of 0.05 using Chi-square goodness-of-fit test. For each subject, we analyzed data at 27 electrodes over 200 time samples. At each time point, we tested whether the data from a particular electrode follows a normal distribution. The chi-square test shows that for more than 90% of the time points in the P300 window the hypothesis that the EEG amplitude follows a normal distribution cannot be rejected at the 5% significance level for all electrodes. Therefore, using equation (4) for estimating DI is a reasonable approximation.

C. Significant DI matrix

In order to reflect the information flow for control and schizophrenic subjects, we obtain the mean value of DI between all electrode pairs over the whole P300 window and a 27×27 normalized DI matrix is computed for each subject. Moreover, we obtain a 27×27 threshold matrix by generating 100 surrogate data sets using Fourier bootstrapping technique

to verify the significance of DI value for each subject and for each electrode pair. The threshold is determined such that the probability of obtaining a particular DI value by chance is less than 0.1. A new matrix consisting only of the significant DI values is constructed as:

$$DI_{sig}(i,j) = \begin{cases} DI(i,j) & \text{if } DI(i,j) > TH(i,j) \\ 0 & \text{if } DI(i,j) \leq TH(i,j) \end{cases} \quad (6)$$

where $DI(i,j)$ and $TH(i,j)$ are the DI value and the corresponding threshold at 90% significance level for electrode pair i and j, respectively. We then compute the average significant DI matrix for 10 subjects in each group. The results are shown in Fig. 1 and 2. The control group has slightly more significant electrode pairs than the schizophrenic group. For the control group, the average number of significant pairs is 555, whereas for the schizophrenic group it is 504 out of a total of 729 (27×27) electrode pairs.

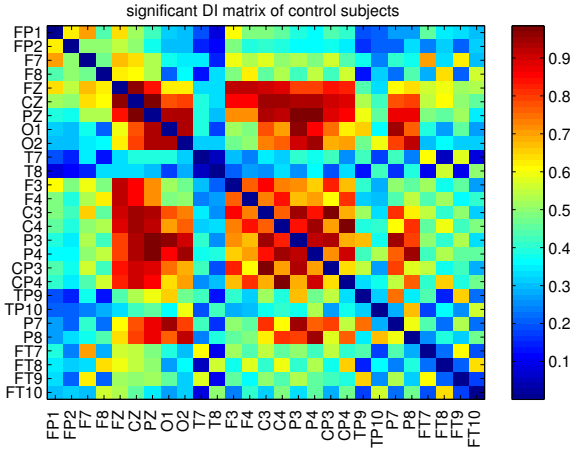

Fig. 1. Mean of the significant DI matrix for all control subjects

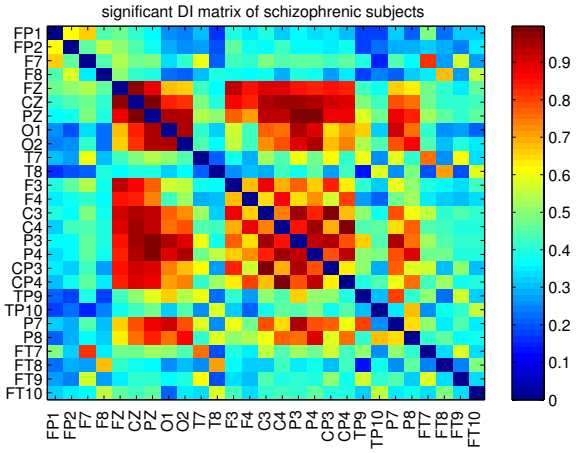

Fig. 2. Mean of the significant DI matrix for all schizophrenic subjects

Here, we compute the difference of DI value in two directions for each electrode pair i and j to determine the causality of the relationships. The directionally significant DI matrix (DDI) can be computed as:

$$DDI = DI_{sig} - DI_{sig}^{T}, \quad (7)$$

where the DI_{sig}^{T} is the transpose of the significant DI matrix. If the DI value from electrode i to j is the same with the DI value from j to i, then the entry of DDI matrix should be 0; if it is different, then it should be a non-zero value, indicating the causal relationship between the two electrodes. The average DDI matrices for control and schizophrenic groups are shown in Fig. 3 and 4. From these figures, it is observed that the information flow patterns in the brain are more symmetric for the control group compared to the schizophrenic group.

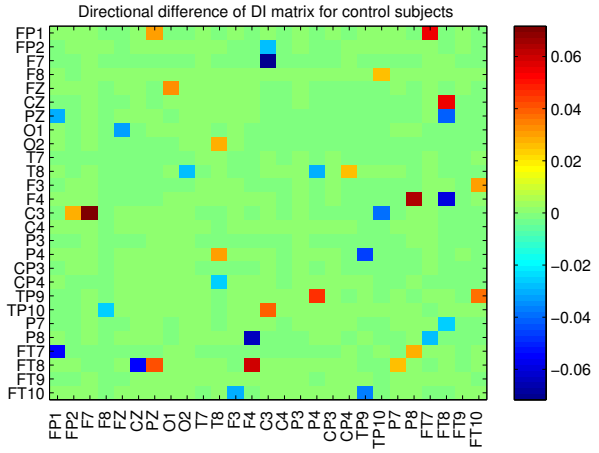

Fig. 3. Mean of the DDI matrix for all control subjects

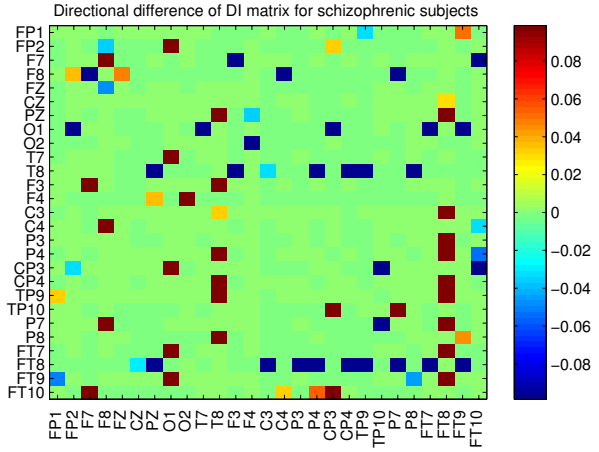

Fig. 4. Mean of the DDI matrix for all schizophrenic subjects

D. Comparison between the subject groups

For better comparison between the two subject groups, t-statistics is used to quantify the statistical power of the difference in information flow for each electrode pair between the control and schizophrenia subjects as seen in Fig. 5. The

t-statistics can be computed as:

$$T = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{SD_1 + SD_2}}, \tag{8}$$

where $\overline{X_1}$ and $\overline{X_2}$ are the average DI value for control and schizophrenia groups, respectively. SD_1 and SD_2 are the standard deviations for each group. The t-statistics for each electrode pair is shown in in Fig. 5. A positive value means that the control subjects have stronger information flow for a specific electrode pair than the schizophrenic subjects and the negative DI value denotes weaker information flow. Although the two groups have almost equal DI values for most electrode pairs, there still exists a number of significant differences in terms of information flow. Electrode pairs which have highest discrimination power are shown in Table I. As it can be seen the control group has stronger information flow from frontal to central (F8→C4) and parietal lobes (F8→CP4), whereas most of the significant information flow for the schizophrenic subjects occurs between electrodes that are close to each other. This is in alignment with previous research that states that effective connectivity within frontal-parietal neurocognitive networks is disrupted in schizophrenia [1].

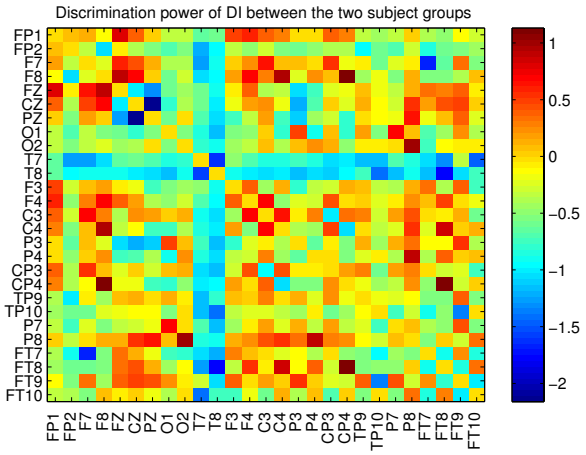

Fig. 5. The discrimination power between control and schizophrenic subjects

V. CONCLUSIONS AND FUTURE WORK

In this paper, we applied directed information measure to quantify the causality and dynamics of the interaction across the brain. The DI measure is implemented on EEG data collected from a group of control and schizophrenic subjects and is shown to discriminate between the two subject groups in terms of the information flow patterns.

Future work will include using non-parametric estimation techniques to compute the DI measure and extending the current work to quantify the information flow patterns for multiple time samples at a time, i.e. $N > 2$. Moreover, the proposed measure can be extended to discriminate between direct and indirect interactions using a network inference framework.

TABLE I

ELECTRODE PAIRS WITH HIGH DISCRIMINATION POWER

Control	Schizophrenia
FZ→FP1	FT7→F7
F8→FZ	FT8→F8
F8→C4	CZ→PZ
F8→CP4	T7→T8
P8→O2	
P8→P4	
FT8→CP4	
FT8→C4	

VI. ACKNOWLEDGMENTS

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