31st Annual International Conference of the IEEE EMBS
Minneapolis, Minnesota, USA, September 2-6, 2009

# Electromagnetic Forward and Inverse Problems of Non-rotating Magnetoacoustic Tomography with Magnetic Induction

Yang Zhang[1], Guoqiang Liu[1], *Member IEEE* , Chunjing Tao[2], Hao Wang[3], Wenjing He[1]
1 Research Department of Frontiers, Electrical Engineering Institute of Chinese Academy of Sciences, China ,100190
2 National Research Center for Rehabilitation Technical Aids, China,100190
3 Department of Biomedical Engineering, Boston University, Massachusetts, USA, MA02215

*Abstract*—**The analysis of electromagnetic forward and inverse problems is very important in the process of image reconstruction for magnetoacoustic tomography with magnetic induction (MAT-MI). A new analysis method was introduced in this paper. It breaks through some illogical supposes that the existing methods applied and can improve the spatial resolution of the image availably. Besides it can avoid rotating the static magnetic field which is very difficult to come true in application, therefore the development of MAT-MI technique can be promoted greatly. To test the validity of the new method, two test models were analyzed, and the availability of the method was demonstrated.**

## I. INTRODUCTION

MAGNETOACOUSTIC tomography with magnetic induction (MAT-MI) is a new imaging method which brought forward by B He etc. in 2003. It is the combination of Electromagnetic tomography (EMT) and ultrasonic tomography, so it combines the good contrast of EMT with the good spatial resolution of ultrasonic tomography[1]. Compared with other electromagnetic tomography techniques, MAT-MI has several unique features. Firstly, it has no shielding effect, that is it will not be affected by the low-conductivity layer of the tissue at/near the surface of the human body[2]. Secondly, for the eddy current which produces the Lorentz force distributes throughout the imaging body, so the ultrasonic waves which taking valid information of magnetic parameter character are abundant enough, and this is the main reason why the spatial resolution of MAT-MI is good. B He etc. have studied profoundly for MAT-MI, and both their simulation and phantom image results have a good spatial resolution in 2D and 3D[3].

Image reconstruction algorithm is the key problem of MAT-MI, and the analysis of electromagnetic forward and inverse problems is the main part of the image reconstruction process. The existing algorithms are not perfect. They either need to adopt some illogical approximate hypothesis which will decrease the quality of the reconstructed image greatly, such as looking the time varying impulse as a step one and supposing the distribution of object is piecewise, or need to rotate the1 static magnetic field which is very difficult to come true in application, and these have baffled the improvement of MAT-MI greatly. Therefore new image reconstruction algorithms of MAT-MI technique are needed especially.

Project is supported by the National Natural Science Foundation of China(No.60802086)and the National High Technology Research and Development Program(863)of China(No.2007AA06Z212)

Refer to the reconstruction method of magnetic resonance electrical impedance tomography (MREIT) which apply the single component of magnetic induction intensity to reconstruct the distribution of electromagnetic parameters[4-6], we brought forwards a new image reconstruction method of MAT-MI technique. Through substituting the single component of the divergence of Lorentz force density, the new method can not only break through the illogical supposes, but also can avoid rotating the static magnetic field which is the choke point of MAT-MI. Besides, the method applies the transient field analysis method to take into account the electromagnetic effect produced by the time-varying excitation. Therefore, it can improve the trueness of the image, and can also promote the development of MAT-MI effectively.

## II. Formulations

For MAT-MI, the imaging object is in a static magnetic field $B_0$ and a time-varying magnetic field $B_1$ . The time-varying magnetic field induces eddy current in the object. Consequently, the object will emit ultrasonic waves through the Lorentz force produced by the combination of the eddy current and the static magnetic field. The ultrasonic waves are then collected by the detectors located around the object. Then, through the image reconstruction the image which can reflect the tissue's electromagnetic parameters can be reconstructed.

The process of image reconstruction is very important, and this process can be divided into two steps. In the first step, the divergence of Lorentz force's density distribution will be reconstructed from sound pressure through the analysis of sound field reverse problem. In the second step, the distribution of electromagnetic parameters will be reconstructed from the divergence of Lorentz force density through the analysis of electromagnetic field reverse problems. The first step can be accomplished with the back-projection algorithm[7], and it is mature comparatively. The second step is more challenging, and is also the mainly part that we studied.

To avoid rotating the static magnetic field, in the new method we applied the method of substituting the single component of the divergence of Lorentz force density.

Besides, to improve the trueness of the image reconstruction, we applied the transient field analysis method to take into account the effect of the time-varying excitation accurately.

Because the reverse problem of electromagnetic field is based on the forward one, so at first the forward problem had been studied.

### *A . Forward problem of electromagnetic field*

The field equation system with $A-\phi, A$ formulations with the Coulomb gauge can be given as:

$$\begin{cases} \nabla\times\frac{1}{\mu}\nabla\times A=-\sigma\left(\frac{\partial A}{\partial t}+\nabla u\right) & (a) \\ \nabla\cdot\left[\sigma\left(\frac{\partial A}{\partial t}+\nabla u\right)\right]=0 & (b) \end{cases} \quad (1)$$

Where, $A$ is the magnetic vector potential, $u$ is the electric scalar potential, $\mu$ and $\sigma$ are the magnetic reluctivity and conductivity respectively.

In the forward problem of electromagnetic field, the known quality is the distribution of $\sigma$, and the calculating target is the divergence of Lorentz force density $\nabla\cdot\left(\vec{J}\times\vec{B}\right)$.

To ensure the symmetrical characteristics of equation coefficient matrix, variance $\nu$ was introduced, where $u=\frac{\partial \nu}{\partial t}$, and the matrix form of the transient field equation can be given as

$$[K]\{A_\nu\}+[D]\frac{\partial}{\partial t}\{A_\nu\}=\{F\} \quad (2)$$

Where, $[K]$,$[D]$ are coefficient matrices of the field equation, $\{F\}$ is the known quantity matrices of source, and $\{A_\nu\}$ is the unknown variances matrices of magnetic vector potential and electric scalar potential.

Discrete the time item by finite difference method, and formulation (2) can be written as the following one.

$$\left[\theta K^{n+1}+\frac{D}{\Delta t}\right]\left\{A_\nu{}^{n+1}\right\}=\theta\left\{F^{n+1}\right\}+(1-\theta)\left\{F^{n}\right\}+\left[\frac{D}{\Delta t}-(1-\theta)K^{n}\right]\left\{A_\nu{}^{n}\right\} \quad (3)$$

In formulation (3), $n$ is the number of time step, and $\Delta t$ is the length of time step.

Formulation (3) can be solved by Crank-Nicholson methods, and $\{A_\nu\}$ of every time step can be calculated. Based on the corresponding relation between $A$ - $B$ and $A-J$, magnetic induction intensity $B$ and eddy current $J$ can be calculated accordingly, and then the divergence of Lorentz force density $\nabla\cdot\left(\vec{J}\times\vec{B}\right)$ at each time step can be obtained.

When conductivity of the imaging object is small enough, the magnetic field produced by the eddy current is too small to be taken into account, and then $A$ is only correlate with $B_1$, so it can be looked as $A_0$ [3],

$$\vec{A}_0=\frac{1}{2}B_1\left(-(y-a)\vec{e}_x+x\vec{e}_y\right) \quad (4)$$

Where, $B_1$ is the time-varying excitation, $a$ is the center of the exciting coil, and $(x,y)$ is the position of the point. Then the four calculated unknown variances $\lfloor A_x \quad A_y \quad A_z \quad \nu\rfloor$ of every point in formulation (2) can be reduced to only one unknown variance $\nu$, and the computational effort of 3D electromagnetic field problem in the process of image reconstruction is reduced evidently. This is very important for the inverse problem.

### *B . Inverse problem of electromagnetic field*

In the inverse problem of electromagnetic field, the known quality is the divergence of Lorentz force density $\nabla\cdot\left(\vec{J}\times\vec{B}\right)$, and the calculating target is the distribution of $\sigma$.

Through calculating we find that comparing with the static magnetic field $B_0$, the magnetic field $B_2$ produced by the eddy current are small enough to be neglected, so the divergence of Lorentz force density can be regarded as $\nabla\cdot\left(\vec{J}\times\vec{B}'\right)$ .When $B_1$ is small enough and can be neglected ,then $\vec{B}'=\vec{B}_0$, and

$$\nabla\cdot\left(\vec{J}\times\vec{B}'\right)=\vec{B}_0\cdot\nabla\times\vec{J}\Big|_z \quad (5)$$

When $B_1$ is big and can not be ignored, then

$$\nabla\cdot\left(\vec{J}\times\vec{B}'\right)=\left(\vec{B}_0+\vec{B}_1\right)\cdot\nabla\times\vec{J}\Big|_z \quad (6)$$

For $\vec{B}_0$ is a constant we known, and $\vec{B}_1$ can be calculated, so $\nabla\times J\big|_z$ can be obtained.

Besides formulation (1), $\sigma$ and $A$ also satisfy the following equation,

$$\left(\nabla\sigma\times\frac{\partial\vec{A}}{\partial t}\right)\Bigg|_z+(\nabla\sigma\times\nabla\phi)\big|_z+\sigma\nabla\times\frac{\partial\vec{A}}{\partial t}\Bigg|_z=-\nabla\times\vec{J}\Big|_z \quad (7)$$

Replace $A$ by $A_0$, then formulation(1) and (7) can be written as formulation (8) and (9)

$$\frac{\partial}{\partial x}\left(\sigma\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\sigma\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial u}{\partial z}\right)+\frac{1}{2}\frac{\partial B_1}{\partial t}\left[\frac{\partial\sigma}{\partial x}(a-y)-\frac{\partial\sigma}{\partial y}x\right]=0 \quad (8)$$

$$\frac{1}{2}\frac{\partial\sigma}{\partial x}\frac{\partial B_1}{\partial t}x+\frac{1}{2}\frac{\partial\sigma}{\partial y}\frac{\partial B_1}{\partial t}(y-\alpha)+\frac{\partial\sigma}{\partial x}\frac{\partial u}{\partial y}-\frac{\partial\sigma}{\partial y}\frac{\partial u}{\partial x}+\sigma\frac{\partial B_1}{\partial t}=-\nabla\times J\big|_z \quad (9)$$

Through iterative calculating between formulation (8) and (9), the distribution of $\sigma$ can be reconstructed.

## I. APPLICATION

To verify the availability of the new method, test model 1 was analyzed. There are two coils and a conductive object which composed by 16 blocks with different conductivity in the model. The outer radius of the coils are 150mm, and the inside radius are 120mm. Distance of the two coils is 120mm. The conductive object is the imaging object. Center points of the conductive object and the two coils are coincident.

The effect of the coils is creating the time-varying magnetic field. When an impulse current is added to the coils, a transient changed magnetic field will be induced. For the structure of the coils approach to helmholtz coil, and the conductive object was placed in the uniform range of magnetic field, so in the conductive object area the magnetic field is homogeneous spatially.

The purpose of analyzing the test model is to test whether $A$ can be substituted by $A_0$ when the conductivity of imaging object is small enough. The structure of the imaging object and the subdivision of the whole model are shown in Fig.1 and Fig.2.

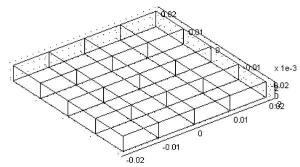

Fig.1 Structure of the imaging object in model 1

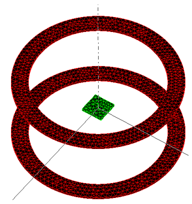

Fig.2 Subdivision of test model 1

Curve of the impulse current is shown in Fig.3. The maximal value of the impulse current is 98.71A, and the actuation duration is 5.2us. The value of the static magnetic field $B_0$ we added to the imaging object is 1T. If the total magnetic induction intensity is $B$, then

$$B = B_0 + B_1 + B_2 \qquad (10)$$

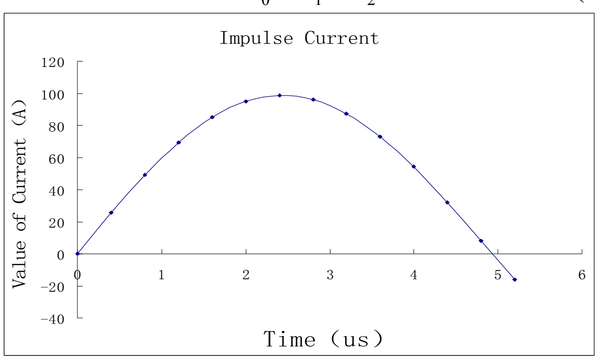


Fig.3 Curve of the impulse current

Based on the theoretical analysis, we have created a program and applied it to analyze the test model 1. $B$ together with the eddy current $J$ and Lorentz force density $J \times B$ are calculated. The curves of the calculated results at three sample points are shown in Fig.4-6.

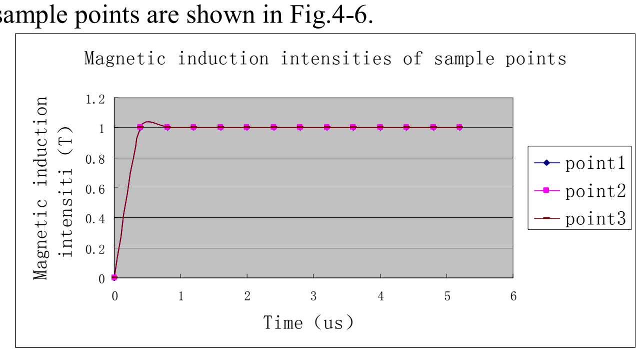


Fig.4 Curve of magnetic induction intensity results

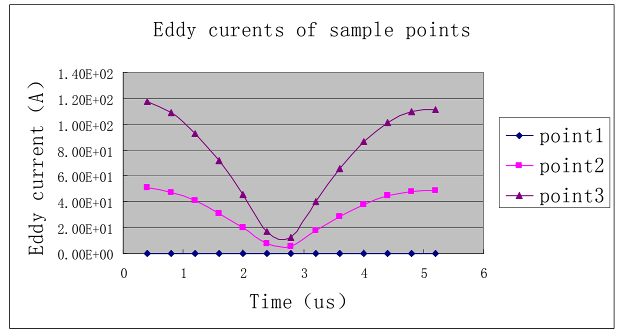


Fig.5 Curve of eddy current results

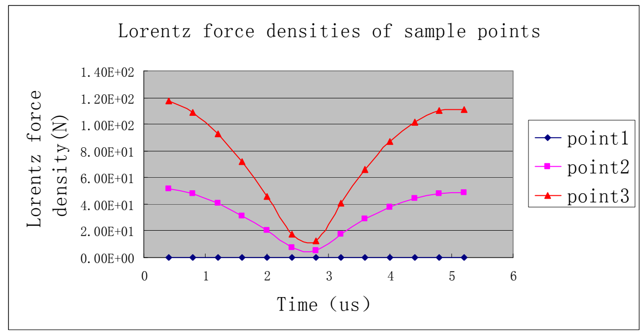


Fig.6 Curve of Lorentz force density results

Besides, $B_1$ and $B_2$ of the three sample points are calculated respectively, as shown in Fig.7,8.

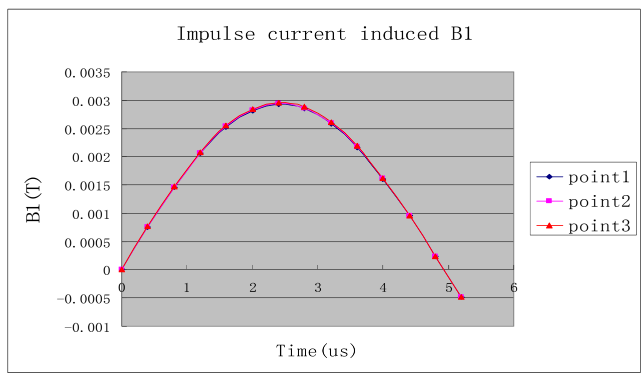


Fig.7 Curve of the time varying magnetic field $B_1$

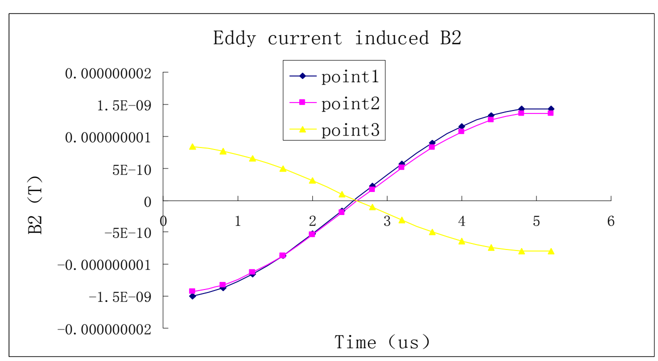


Fig.8 Curve of the eddy current induced magnetic field $B_2$

Through the curves we can see that comparison with $B_0$, $B_2$ are surely small enough to be neglected, so the suppose of formulations (4) is reasonable. Because $B_1$ is big comparatively and should not be neglected, $\nabla \times J|_z$ should be calculated through formulation (6). For $B_1$ is time varying, so the inverse problem of electromagnetic field is also a transient problem, and should be analyzed at every time step when the time item is discrete.

In the analysis of the test model 1, the number of the time step is 14, and the image reconstruction results of the distribution of $\sigma$ on 4 slices along z axis at time step 6 are shown in the following pictures.

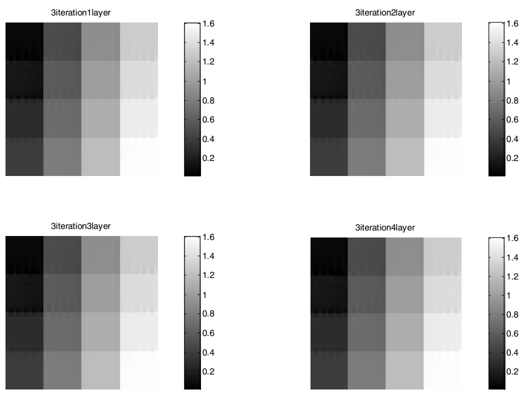


Fig.9 Image reconstruction results of model 1 after 3 iterations

From Fig.9, we find that after 3 iterations, the result seems to be very close to the origin conductivity distribution. To test its correctness quantitively, we introduced the relate error(RE).

$$RE = \frac{\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}(\sigma_{i,j} - \sigma_{R\,i,j})^2}}{\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{i,j}^{2}}} \qquad (11)$$

We calculate $RE$ of each slice at each time, and show them in table 1. We find that the result is convergent, and the error is small.

TABLE 1 $RE$ of Model 1

| | Slice1 | Slice2 | Slice3 | Slice4 |
|---|---|---|---|---|
| Iteration 1 | 7.8597 | 7.8597 | 7.8598 | 7.8597 |
| Iteration 2 | 0.14059 | 0.14055 | 0.14054 | 0.14058 |
| Iteration 3 | 0.0602 | 0.0602 | 0.0602 | 0.0602 |

Above is the reconstruction process of test model 1. The result is encouraging. However, there is no change of conductivity distribution in the z direction. Therefore, we set up another model(fig.10).

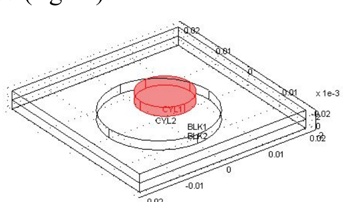


Fig.10 Structure of the imaging object in model 2

The background is divided into 4 areas: 2 cuboids and 2 cylinders. Each of them have different homogeneous conductivity. Similar to model 1, we obtain the result of 3 iterations( fig.11).

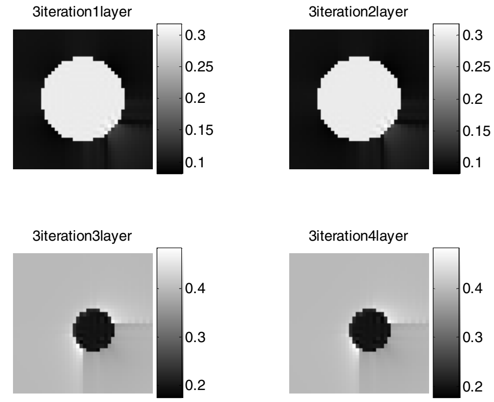


Fig.11 Image reconstruction results of model 2 after 3 iterations

From fig.11, we can see the result is also close to the given conductivity distribution.

## II. Conclusion

A new analysis method of electromagnetic forward and inverse problems in the process of image reconstruction for MAT-MI was presented in this paper. Through substituting the single component of the divergence of Lorentz force density, the problem of rotating static magnetic field can be solved availably, therefore the application of MAT-MI technique will be promoted. Besides, the transient magnetic field analysis was used to take into account the effect of the time-varying excitation, and accordingly the quality of image could be improved effectively. Two test models were analyzed to test the availability of the new method, and the availability of the new method was demonstrated.

## References


[1] X.Yuan, B. He. Magnetoacoustic Tomography with Magnetic Induction(MAT-MI) [J]. Phys. Med. Biol. 2005. Vol.50, pp.5175-5187.

[2] Yuan Xu, Bin He. Magnetoacoustic Tomography with Magnetic Stimulation (MAT-MS). International Journal of Bioelectromagnetism, 2005, (7): No. 2.

[3] Wang Hao, Liu Guoqiang. et.al. Inverse Problem of Magnetoacoustic Tomography with Magnetic Induction. Proceedings of NFSI & ICFBI 2007.290~292.

[4] Birgul O, Eyuboglu BM, Ider YZ. 2003. Experimental results for 2D magnetic resonance electrical impedance tomography (MR-EIT) using magnetic flux density in one direction, Phys. Med. Biol, 48:3485-3504.

[5] J. K. Seo, J. R. Yoon, E. J. Woo, and O. Kwon, “Reconstruction of conductivity and current density images using only one component of magnetic field measurements,” IEEE Trans. Biomed. Eng., vol. 50, no. 9, pp. 1121–1124, Sep. 2003.

[6] Ider Y. Z., Onart S. Algebraic Reconstruction for 3D Magnetic Resonance-electrical Impedance Tomography (MR-EIT) Using One Component of Magnetic Flux Density. Physiological Measurement, 2004, (25):281-294.

[7] Ma QY and He B. Magnetoacoustic Tomography With Magnetic Induction: A Rigorous Theory. IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 55, NO. 2, FEBRUARY 2008.