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Pseudo 2D Random Sampling for Compressed Sensing MRI

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Abstract—**The paper presents a novel approach of pseudo 2D random sampling scheme for application of compressed sensing in Cartesian magnetic resonance imaging (MRI). The proposed scheme is realized by a pulse sequence program which switches the directions of phase encoding and frequency encoding during data acquisition such that both k_x and k_y directions can be undersampled randomly. The resulting random sampling pattern approximates the ideal but impractical 2D patterns. Both the simulation and experiment results show the proposed method is superior to the existing 1D random sampling and similar to the ideal 2D random sampling in terms of the reconstruction quality. This method can potentially improve the MR imaging speed through the application of compressed sensing in conventional MRI.**

I. INTRODUCTION

CONSIDERABLE attention has been paid to compressed sensing (CS) in the MRI community recently [1-8]. Compressed sensing is a novel reconstruction theory which allows exact recovery of a sparse signal from a highly incomplete set of samples [9, 10], and thus has the potential for significant reduction in MRI scan time.

Most existing work in Cartesian sampling has focused on 1D random sampling scheme. Although the 2D random sampling scheme is known to perform better than the 1D scheme, it cannot be physically realized in an MRI scanner because of the hardware constraints [1]. To address this challenge, multi-excitation is a straightforward solution. However, the modification of hardware in conventional MRI scanners increases the cost.

In this paper, we propose a pseudo 2D random sampling scheme for compressed sensing MRI with Cartesian trajectories. The sampling scheme first randomly undersamples the k-space along one direction, and then changes to the other. The proposed scheme is realized by a pulse sequence program which alternatively switches the directions of phase encoding and frequency encoding during data acquisition. The resulting random sampling pattern approximates the 2D patterns. The experimental results show that the proposed scheme can be easily realized in conventional MRI scanners. In addition, the reconstructions with the proposed scheme is superior to that with 1D sampling scheme and approaches that with 2D scheme, when the same reduction factor is used. The proposed method should enhance application of compressed sensing in conventional MRI scanner.

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II. BACKGROUND

Compressed sensing (CS) is used to recover a signal x with size n from its linear measurements y with size m: $y = \mathbf{\Phi} x$, where m is assumed to be smaller than n. If a vector, x, has a sparse representation under some sparse transformation $\mathbf{\Psi}$, then x can be recover from a sample y, by solving a convex program: $\min_x \|\mathbf{\Psi} x\|_1$ s.t. $\mathbf{\Phi} x = y$. To achieve faithful recovery from very few measurements, some sufficient conditions need to be satisfied [11]: (a) the signal is sparse after a known sparsifying transform $\mathbf{\Psi}$, (b) the encoding matrix $\mathbf{\Phi}$ is incoherent with the sparsifying transform $\mathbf{\Psi}$, and (c) the measurements exceed the minimum requirement, which is usually 2 to 5 times the sparsity of x.

SpareseMRI [1] is one of the methods to apply CS to conventional MRI with Cartesian sampling. Although simulation shows that SparseMRI can achieve a much higher reduction factor with 2D random sampling, the practical constrains limit the random sampling to be along 1D only. The method fully samples along the frequency encoding direction and randomly undersamples along the phase encoding direction using a variable-density sampling scheme with denser sampling near the center of the k-space. The desired image x is reconstructed by solving the following equation:

$$\arg\min_x \left\{ \|y - \mathbf{F_u} x\|_2^2 + \lambda_1 \cdot \|\mathbf{\Psi} x\|_1 + \lambda_2 \cdot TV(x) \right\} \tag{1}$$

where y is the measured k-space data, $\mathbf{F_u}$ is the random subset of the rows of the Fourier encoding matrix, $\mathbf{\Psi}$ is the sparsifying transform matrix, and $TV(\cdot)$ is total variation.

III. METHOD AND IMPLEMENTATION

Our objective is to design a practical random sampling scheme for conventional Cartesian MRI that performs similarly to the 2D random sampling scheme. The 2D random sampling uses a 2D variable-density sampling scheme with denser sampling near the center of the k-space. The probability function for sampling at a location $\vec{r}$ in k-space is [1]:

$$p(\vec{r}) = \begin{cases} 1 & r \le R \\ \dfrac{1}{(1-r)^p} & r > R \end{cases} \tag{2}$$

where $r = \|\vec{r}\|_2$, $\vec{r} = [r_x \quad r_y]^T$, and R is a real number and $0 < R < 1$. Since the 2D random sampling cannot been physically implemented in conventional MRI scanners, 1D random sampling is adopted instead. The 1D random sampling uses a 1D variable-density sampling scheme with denser sampling near the center of the k-space. The corresponding probability function is:

$$p(\vec{r})=\begin{cases}1 & r\le R\\ \dfrac{1}{(1-r_x)^p} & r>R\end{cases} \tag{3}$$

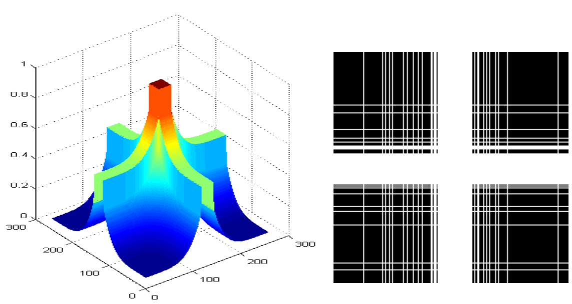
where the parameters are defined as same as Eq. (2).

Our proposed sampling scheme adopts a distribution function different from either 2D or 1D random sampling. It uses two 1D variable-density sampling that are orthogonal to each other in Cartesian k-space, also with denser sampling near the center of the k-space. The corresponding probability function is:

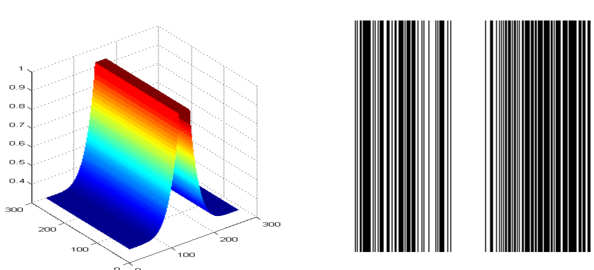
$$p(\vec{r})=\begin{cases}1 & r\le R\\ \dfrac{a}{(1-r_x)^p}+\dfrac{1-a}{(1-r_y)^p} & r>R\end{cases} \tag{4}$$

where the parameters are defined as same as Eq. (2). a is a constant that depend on the number of sampling lines along k_x direction of k-space.

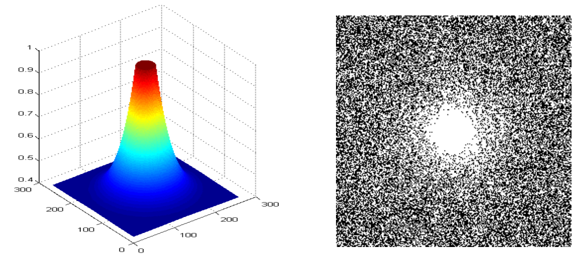
Figure 1 compares the probability functions of the three random sampling methods. It is obvious that the proposed pseudo 2D random sampling is closer to the 2D random sampling than the 1D random sampling is. As a result, the proposed pseudo 2D random sampling should outperform the 1D random sampling in CS reconstruction.



(a) Proposed pseudo 2D random sampling



(b) 1D Random Sampling



(c) 2D Random Sampling

Fig. 1 Probability function (left) and the corresponding sampling pattern (right) of the three random sampling schemes

In conventional Cartesian MRI, phase encoding gradient and frequency encoding gradient are applied on orthogonal directions in k-space [12]. The amplitude of phase encoding gradient is changed at each excitation to sample a different line in k-space. The 1D random sampling can be easily implemented by changing the amplitudes of phase encoding randomly. However, it is extremely difficult (if not impossible) to implement 2D random sampling which requires simultaneous change of the phase encoding and frequency encoding gradients randomly. In current systems, gradients are limited by maximum amplitude and maximum slew-rate. In addition, high gradient amplitudes and rapid switching can produce peripheral nerve stimulation.

The proposed pseudo 2D random sampling scheme can be implemented on a conventional MRI scanner as easily as the 1D scheme. To realize the proposed sampling scheme, both the amplitude and the direction of the phase encoding gradient have to change accordingly. For example, if a total of N lines are to be acquired, the first N_1 lines are acquired with phase encoding along k_x direction and frequency encoding along k_y direction, and the rest (N-N_1) lines with phase encoding along k_y direction and frequency encoding along k_x direction. In either case, these data are randomly undersampled along the phase encoding direction only, while keeping the frequency encoding direction fully sampled. The value of N depends on the undersampling factor to achieve, and the value of N_1 is equal to $a \cdot N$, where a is the constant in Eq. (4).

Figure 2 shows the timing diagram of the pulse sequences for the proposed pseudo 2D random sampling. The gradients shown in solid red represent the 1D random sampling sequence with phase encoding along y and frequency encoding along k_x direction. In the proposed pseudo 2D random sampling, we alternate between the gradients in solid red and those in dashed green. It corresponds to switching phase and frequency encoding directions alternatively.

As seen in Fig. 1 (a), at certain locations of the k-space, the data are acquired twice – one from the horizontal lines and the other from the vertical lines. The two values are averaged to represent the value of the data at this point. There are a total of $N_1 \times (N$-$N_1)$ such points. Although there is redundancy in such data acquisition, the averaging can improve the signal to noise ratio of the acquired data and thus the reconstructed image.

With the data acquired using the proposed sampling scheme, the image is reconstructed by the following nonlinear convex program [13, 14]:

$$\arg\min_{x}\left\{\|\mathbf{\Psi}x\|_1 + \alpha\cdot TV\left(x\right)\right\} \ \ s.t.\ \|\mathbf{F}_{\mathbf{u}}x - y\|_2 < \varepsilon \quad (5)$$

where x is the image to be reconstructed and is presented as a vector; $\mathbf{\Psi}$ denotes the linear operator that transforms the image from a pixel representation into a sparse representation, such as wavelet; y is the acquired k-space data; $\mathbf{F}_{\mathbf{u}}$ is Fourier Transform associated with the proposed undersampling pattern; $TV(\cdot)$ is the total-variation; and ε controls the fidelity of reconstruction to the measured data. The threshold parameter ε is usually set below the expected noise level.

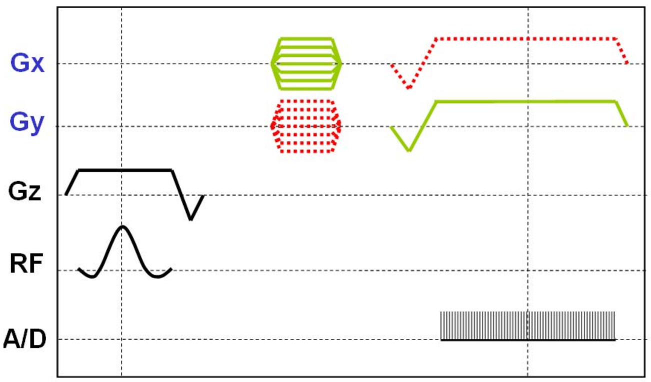


Fig. 2 Timing diagram of the pulse sequences for the proposed pseudo 2D random sampling

IV. EXPERIEMENTS

Both simulation and experiment were carried out to compare the reconstruction results of the proposed pseudo 2D random sampling with the existing 1D random sampling and the desired ideal 2D random sampling. For validating the proposed pulse sequences, Bloch simulation [15] was done to demonstrate the feasibility of the proposed sampling scheme. A ball with 5cm radius is used as the desired object. The image of size 128×128 is reconstructed from the simulated data using the CS algorithm. Figure 3 shows the reconstruction results with the proposed sampling and the existing 1D sampling. With the same reduction factor of 2, the image using the 1D sampling method shows undersampling artifacts (ripples horizontally in background) and the image using the proposed sampling method is seen to be free of such artifacts.

In the experiment, a phantom was scanned using a sequence designed in Figs. 4 and 5. Specifically, the phantom was scanned with phase encoding along y first and then along x afterwards. The data were acquired in full with size of 256×256 and then manually undersampled according to the three random sampling patterns to simulate the desired reduction factors. Identity transform was used as the sparse representation in Eq. (5). The reduction factors of 2 and 2.86 were used. The results show that the proposed pseudo 2D random sampling scheme performs similar to the 2D random sampling scheme, and is superior to the 1D random sampling scheme.

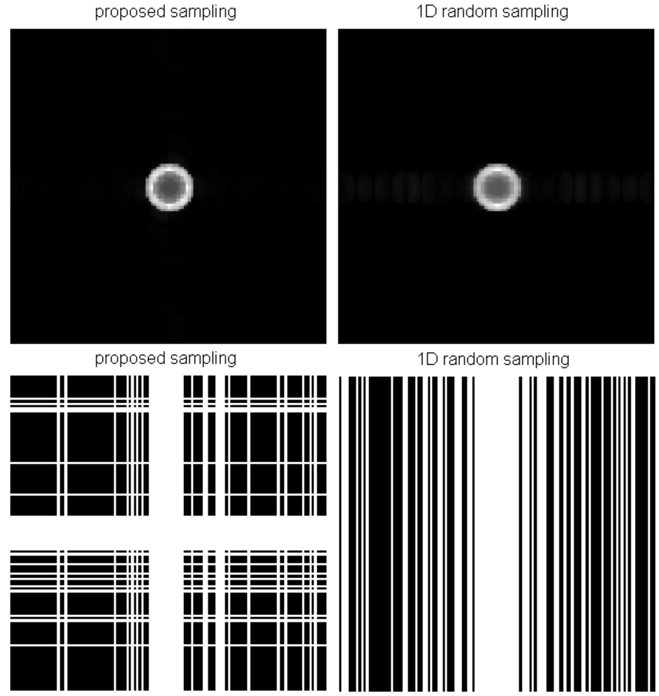


Fig. 3 Bloch simulation results of the proposed pseudo 2D random sampling scheme (left) and the existing 1D random sampling scheme (right). The image size is 128×128, and the reduction factor is 2. The top are the reconstruction results, and the bottom are the corresponding sampling patterns.

V. CONCLUSIONS

We have presented a novel random Cartesian sampling technique for applications of CS in conventional MRI. The simulation and experiment results have shown promising results to accelerate imaging speed with high reconstruction quality.

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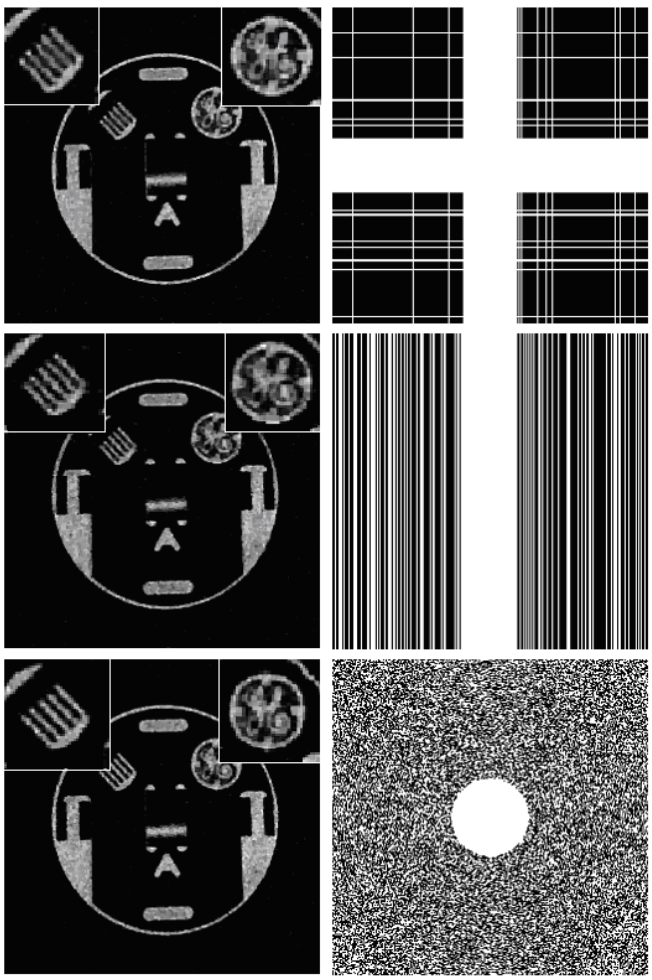

Fig. 4 Phantom reconstructions (left) and the corresponding sampling patterns (right) of the proposed, 1D random and 2D random sampling methods. A reduction factor of 2 is used.

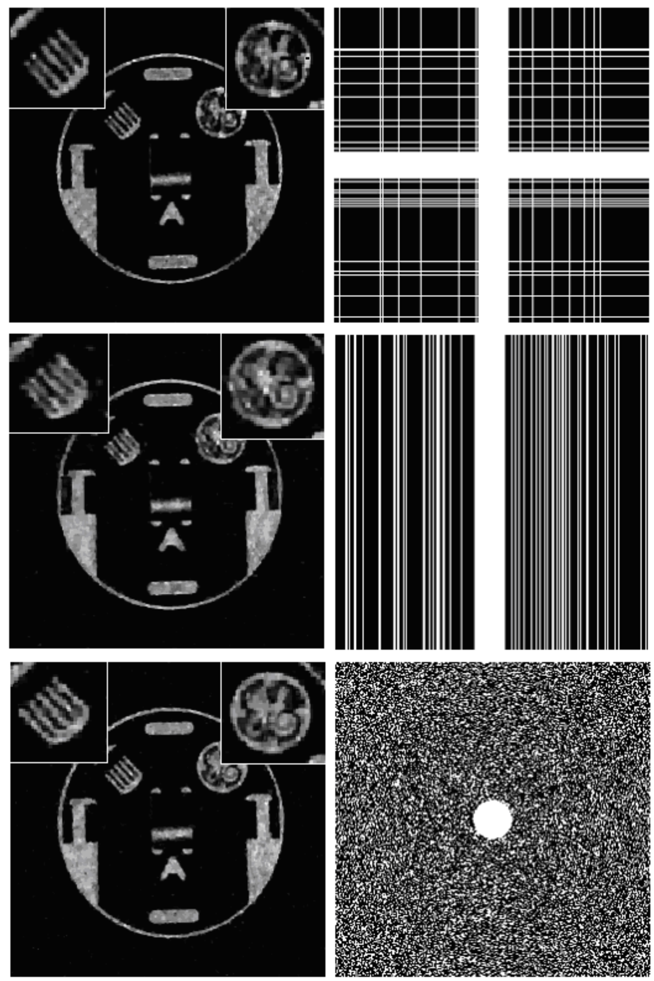

Fig. 5 Phantom reconstructions (left) and the corresponding sampling patterns (right) of the proposed, 1D random and 2D random sampling methods. A reduction factor of 2.86 is used

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