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Improved Reconstruction of Non-Cartesian Magnetic Resonance Imaging Data through Total Variation Minimization and POCS Optimization

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***Abstract*—In the article, an iterative reconstruction algorithm based on total variation minimization and POCS optimization for non-Cartesian K-space data is proposed. The proposed algorithm interpolates non-Cartesian data onto a 2D Cartesian grid using gridding method first, and then during the iterative process of total variation minimization, the frequency values on grid points near the measured data are replaced with the interpolated ones according to POCS. The experiments on simulated and real data show that the proposed method can reconstruct image more accurately and rapidly than constrained total variation minimization method.**

***Index Terms*- MRI; non-Cartesian sampling; Total Variation; POCS**

I. Introduction

NON-CARTESIAN scanning techniques, such as spirals, radial lines have benefits associated with fast data acquisition and insensitive to flow, so they are widely used in various applications such as contrast-enhanced angiography, functional brain imaging. However, the data from non-Cartesian sampling do not distribute on the equal spaced grid points, and can not be reconstructed by fast Fourier transform (FFT) directly. The usual way is to interpolate the non-Cartesian K-space data onto a 2D Cartesian grid firstly, and then perform FFT to get the image, such as gridding [1], Non-uniform Fast Fourier Transform (NUFFT [2]). However, the radial or spiral scanning usually samples high frequency data in K-space with low sampling density, which can result in large approximation error on the grid points which are far away from measured data in high frequency domain when interpolation methods are applied.

The feasible methods may be iterative reconstruction methods with prior constraints [6]. The regularization functional is the key to restricting noise and artifacts efficiently during iteration. As classical $L2$ norms tend to produce images with blurred edges, some authors choose $L1$ norm, total variation (TV, $L1$ norm of image's gradient), as the regularization functional. Because TV constraints can restrict artifacts and noise efficiently without smoothing edges, it has been widely used in other image processing areas such as restoration[4], reconstruction[5][6] since it was first introduced to image denoising [3].

The Constrained TV (CTV) minimization reconstruction algorithm was proposed by Xiao-Qun Zhang and Jacques Froment [6], it only regards TV functional as the objective function to be minimized, which reduces the computation, and the simulated experiment in [6] also shows that CTV algorithm is efficient. As the constraint used by CTV is defined in the frequency domain, it is of potential use for MR data reconstruction. However, the constraint used in it are frequency intervals defined using minimum and maximum of measured data lying in the neighbor of grid points, which weakens the data consistency. At the same time, the computation of frequency intervals is highly computation intensive. For example, the computation complexity of constraint is $O(N^2M)$ for spiral scanning data where M is the number of measured data and $N\times N$ is image's size.

In this work, an iterative reconstruction algorithm based on TV minimization and POCS optimization for non-Cartesian MR data is proposed. The proposed method can be considered as an improved version of CTV algorithm, it interpolates non-Cartesian data onto a 2D Cartesian grid first, and then in the iterative process of TV minimization, the Fourier values on grid points which are close to measured K-space data are replaced with the interpolated one according to POCS principle, which imposes the data consistency of constraint. The proposed algorithm is abbreviated as POCS-TV in following part of this paper.

II. Theory

A. Total Variation

The TV of an image f which is defined on the bounded, open and convex region $\Omega \in R^2$, can be formulated as follows [3] [6]:

$$\mathrm{TV}(f) = \int_{\Omega} |\nabla f| dx \tag{1}$$

where ∇f is the gradient vector of image f. The TV of a discrete image f ($N\times N$) can be expressed as:

$$\mathrm{TV}(f) = \sum_{i,j=1}^{N} \sqrt{\left|(\nabla_x f)_{i,j}\right|^2 + \left|(\nabla_y f)_{i,j}\right|^2} \tag{2}$$

where $\nabla_x f$ and $\nabla_y f$ are the elements of vector ∇f.

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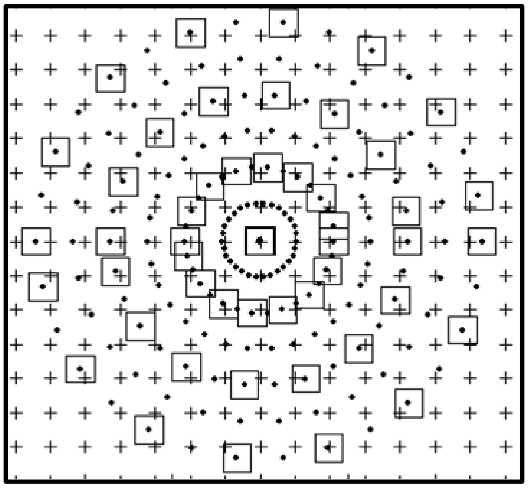

Fig.1 Illustration of calculating mask. Where '.' stand for measured data, '+' stand for the elements of mask, while small rectangle represents the neighbor of measured data.

B. Constrained Total Variation Minimization Algorithm

The algorithm is proposed for parallel CT reconstruction and it defines data consistent constraint in frequency domain [6], which makes it be of potential use for MR data reconstruction. The reconstruction problem is written as:

$$f^* = \arg\min_{f\in D} \mathrm{TV}(f) \quad (3)$$

$$D=\{f\in R^{N^2} : F(f)_{i,j} \in [F^-_{i,j}, F^+_{i,j}], \forall i,j=-N/2,\ldots,N/2-1\} \quad (4)$$

where f^* is the image to be reconstructed, D is the constraint, $F(f)_{i,j}$ denotes Fourier transform value of f on the grid point(i,j) while $F^-_{i,j}, F^+_{i,j}$ represents the minimum and maximum of frequency values lying in the circle neighbor of grid point(i,j) respectively. During the process of TV minimization, $F(f)_{i,j}$ is allowed to vary freely inside$[F^-_{i,j}, F^+_{i,j}]$.

III. METHOD

A. POCS in MRI Reconstruction

Assume H is Hilbert space of square-integrable functions, if partial K-space data $G(u,v)$, $(u,v)\in Z$ are given where Z is a closed region in the Fourier plane, then a convex constraint [11] $C=\{f\in R^{N^2} : F(f)_{u,v} = G(u,v), (u,v)\in Z\}$ and $C\subset H$ is determined. For any image $f(x,y)\in H$, assume that $F(u,v)$ are the Fourier transform of $f(x,y)$, the projection of $f(x,y)$ onto C can be realized as follows:

$$Tf = \begin{cases} G(u,v) & (u,v)\in Z \\ F(u,v) & (u,v)\notin Z \end{cases} \quad (5)$$

B. Definition of Constraint

Assume that the size of the image to be reconstructed is $N\times N$. The non-Cartesian MR data are interpolated onto a 2D Cartesian grid (size $N\times N$) using gridding algorithm, the data after interpolation are denoted by $V(i,j), i,j=-N/2,\ldots,N/2-1$. The principle of redefining the constraint can be described as: For grid point (i,j),if it lies in the neighbor around real data, the interpolated data $V(i,j)$ on (i,j) is preserved, otherwise the corresponding frequency value on grid point(i,j) produced in the process of TV minimization is preserved. The process of computing constraint is sketched as follows:

1) Calculating a mask (size $N\times N$) based on the coordinates (x,y), $x\in R$, $y\in R$ of non-Cartesian K-space data, mask(i,j). Assume that $S_{x,y}$ is the set of grid points lying in the square neighbor whose center is (x,y) and width is d.

$$S_{x,y} = \{(i,j) : |i-x| \le d, |j-y| \le d\} \quad (6)$$

Then mask can be defined as:

$$\mathrm{mask}(i,j) = \begin{cases} 1 & (i,j)\in S_{x,y} \\ 0 & (i,j)\notin S_{x,y} \end{cases} \quad (7)$$

Fig.1. is the illustration of calculating mask.

2) Set $L = \{(i,j) : \mathrm{mask}(i,j) = 1\}$, $F(f)$ is the Fourier transform of f, then data consistent constraint is defined as:

$$C=\{f\in R^{N^2} : F(f)_{i,j} = V(i,j),\ (i,j)\in L\} \quad (8)$$

Selection of d in (6) is the key to C, if d is too large or too small, the consistency of constraint will be weakened. The complexity of computing constraint is O(M) for both radial and spiral sampling data where M is the number of measured data and $N\times N$ is the size of image.

C. The Improved Reconstruction Algorithm

The improved algorithm can be written as:

$$f^* = \mathop{argmin}_{f\in C}(\mathrm{TV}(f)) \quad (9)$$

with $C=\{f\in R^{N^2} : F(f)_{i,j} = V(i,j), (i,j)\in L\}$

where f^* is the reconstructed image.

Generally, image f reconstructed by inverse Fourier transform is complex one, $f = f_{re} + i\cdot f_{im}$, so the model of TV used in our algorithm is[8]:

$$\mathrm{TV}(f) = \sum_{i,j}\sqrt{|\nabla_x f_{re}(i,j)|^2 + |\nabla_y f_{re}(i,j)|^2} + \sum_{i,j}\sqrt{|\nabla_x f_{im}(i,j)|^2 + |\nabla_y f_{im}(i,j)|^2} \quad (10)$$

The solution of (9) can be found by projection subgradient optimization method [6][9]:

$$f_{k+1} = P(f_k - t_k g(f_k)) \quad t_k > 0 \quad f_0 \in C \quad (11)$$

where $g(f)$ is a subgradient of TV(f) at f, P is the projection onto C. k stands for the k^{th} iteration. One condition can guarantee (11) convergent is [10]:

$$\sum_{k=0}^{+\infty} t_k = +\infty,\ and \sum_{k=0}^{+\infty} t_k^2 < +\infty \quad (12)$$

For example, $t_k = \frac{a}{k+1}$ where a is a positive constant.

The algorithm can be described as follows:

1) Interpolate non-Cartesian data onto a 2D Cartesian grid, get frequency values on grid point(i,j) $V(i,j), i,j=-N/2,\ldots,N/2-1$.

2) Calculate a mask(i,j) using coordinates of non-Cartesian

data.

3) Set $L=\{(i,j)\text{:mask}(i,j)=1\}$,then determine the constraint $C=\left\{f\in R^{N^2}:F(f)_{i,j}=V(i,j),\ (i,j)\in L\right\}$.

4) Get initial image f_0 and determine iterative number *Num*, step size t_k and set $k=0$.

5) Calculate $f_t=f_k-t_k g(f_k)$ and the projection of f_t onto constraint according POCS, assume $F(f_t)$ is the Fourier transform of f_t and *PF* is the frequency data after projection:

$$PF_{i,j}=\begin{cases}G(i,j) & (i,j)\in L\\ F(f_t)_{i,j} & (i,j)\notin L\end{cases} \tag{13}$$

Then take inverse Fourier transform of *PF* and set $Pf_t=IFFT2(PF)$ where *IFFT2* stands inverse FFT.

6) Set k=k+1, if k<*Num*, return to step 5, or stop iteration.

IV. RESULTS AND DISCUSSION

In order to validate POCS-TV algorithm, we have experimented it on both simulated data and real data reconstruction. The initial image used in all experiments is reconstructed by girdding algorithm and $t_k=\dfrac{a}{k+1}$.

A. Simulated Experiment

Simulated data (radial sampling, size 180 ×512) is computed from a 256×256 Shepp-Logan image with Gaussian noise (its variance is 0.02 while the pixel value of image falls in [0,1]) using discrete Radon transform and 1D FFT. The reconstruction grid is oversampled to 512×512. Total iterative number in CTV and POCS-TV are 15, parameters used in CTV are a=0.005, r=3 (parameter r is given in [6], though parameter a is not given in [6], we chose the one that can produce best reconstruction result) while in POCS-TV are a =0.005, d=0.1(parameters a and d are experience values).

The normalized mean square error (NMSE) of images is also calculated to compare reconstructed images in quantity:

$$\text{NMSE}(m_r)=\frac{\sum_{i,j=1}^{N}(m_r(i,j)-m_s(i,j))^2}{\sum_{i,j=1}^{N}m_s^2(i,j)} \tag{14}$$

where m_r is the reconstructed image while m_s is standard Shepp-Logan image.

Table I shows NMSE of images and runtime for simulated experiment. Fig.2 shows the curves of variation of NMSE. Because of the role of TV functional, NMSE decreases significantly for both CTV and POCS-TV after iteration, and NMSE of image reconstructed by POCS-TV starts to be smaller than that of image reconstructed by CTV. However, as data consistency is weakened in CTV, it can't prevent image from being excessively smoothed by TV functional, which results in that NMSE increases after 10 time iteration. For POCS-TV, thanks to high data consistency of constraint, the NMSE decreases all the time during iterative process.

TABLE I
NMSE AND RUNTIME FOR SIMULATED EXPERIMENT

	Gridding	POCS-TV	CTV
NMSE	0.0548	0.0444	0.0499
Runtime(s)	1.91	21.76	32.38

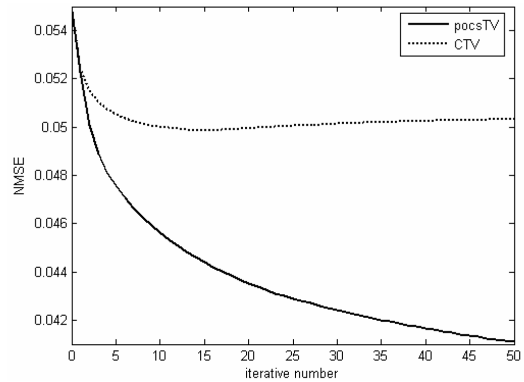

Fig.2 Varying curves of images' NMSE vs iterative number

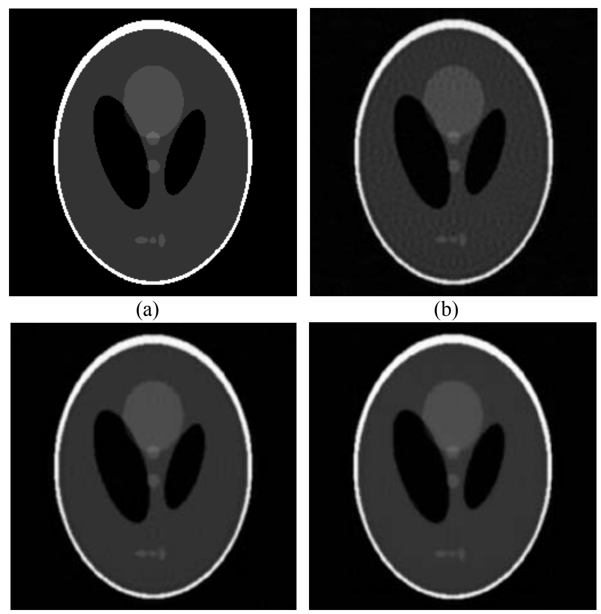
(a) (b) (c) (d)

Fig. 3 Reconstruction results of simulated experiment (image size 256×256). (a) Shepp- Logan image, (b) image reconstructed by gridding, (c) by CTV,(d) by POCS-TV

Fig.3 presents the reconstruction results of simulated experiment and Fig.4 illustrates the profiles of corresponding images. Visibly, image reconstructed by gridding (Fig. 3(b)) contains much noise and artifacts, which can be ascribed to low interpolation precision in high frequency domain. Thanks to TV functional, images reconstructed by CTV (Fig.3(c)) and POCS-TV (Fig.3 (d)) have little noise and artifacts though they are poorer than standard Shepp-Logan image.

In addition, NMSE of image in POCS-TV is smaller than that in both CTV and gridding (see TABLE I), which indicates that image reconstructed by POCS-TV is more similar to the standard one. It can also be derived from image profiles (Fig.4) where the pixel value oscillations of image in POCS-TV is smaller than that in both CTV and gridding, The speed of POCS-TV is faster than that of CTV but not

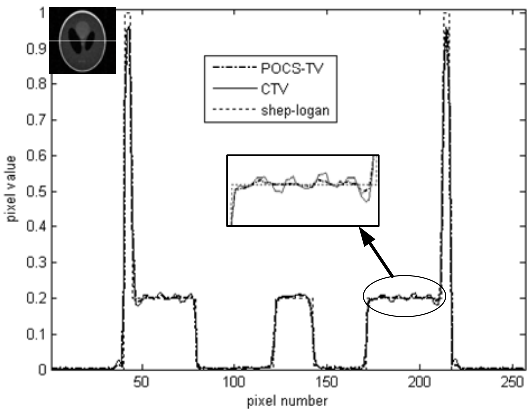

Fig. 4 Profiles of images reconstructed from simulated data along the centers of images in horizontal direction

significant (see TABLE I), because the complexity of computing constraint in POCS-TV is O(M) which is near to O(N^2) in CTV for radial sampling data, where M is the number of K-space data and N×N is the size of image.

As the limitation of pages, we did not discuss parameters' influence on reconstruction results.

B. Real Data Experiment

The real data is T2 weighted brain spiral sampling data (got from Workshop on Non-Cartesian MRI 2007, size 9×13624, we only used 1/3 of it, 9×4542). The reconstruction grid is oversampled to 512×512. Fig. 5 shows the reconstruction results of real data experiment. It is obvious that the image reconstructed by POCS-TV is the best in vision while that reconstructed by gridding is the worst, which also can be derived from image profiles presented in Fig. 6 where the pixel value oscillations of image reconstructed by POCS-TV is the smallest while that in gridding is the largest. In addition, the time cost by POCS-TV, CTV and gridding are 23.17s, 1343.13s and 4.12s respectively. Obviously, as an iterative method, POCS-TV is slower than gridding, but it is much faster than CTV for spiral sampling data, because the complexity of constraint computation of POCS-TV is O(M) while that of CTV is O(N^2M).

V. CONCLUTION

By combination of TV regulation and accurate data fidelity constraints imposed by POCS principle, the proposed POCS-TV algorithm in this paper can reconstruct images for non-Cartesian MR data with less noise and artifacts than gridding and CTV algorithms. In addition, POCS-TV algorithm is also much faster than CTV algorithm when used for spiral MR data. Finally, POCS-TV can be extended for the reconstruction of sparse MRI data.

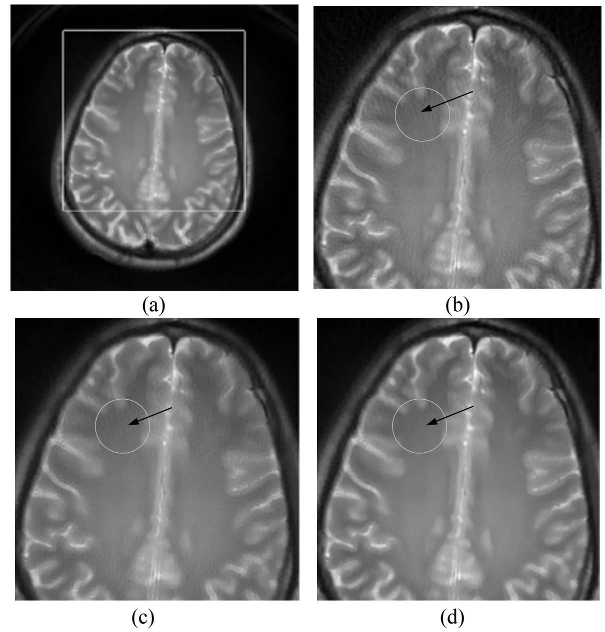

Fig. 5 Reconstruction results of T2 weighted brain data(image size 320×320). (a) Shepp-Logan image, (b) image reconstructed by gridding,(c) CTV, (d) POCS-TV. (b), (c), (d) are got from enlarging part of (a) 2 times.

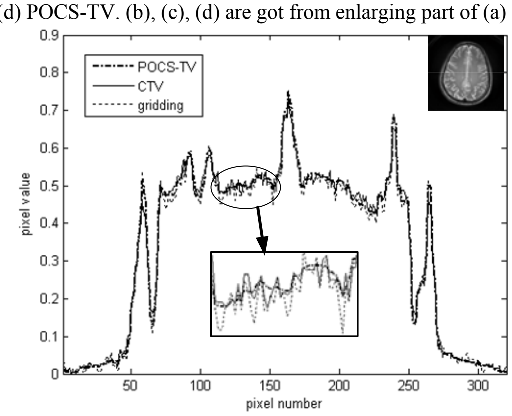

Fig. 6 Profiles of images reconstructed from T2 weighted brain data

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