

Evaluation of Biaxial Mechanical Properties of Soft Tubes and Arteries using Sonometry

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Abstract—Arterial elasticity has become a topic of importance in the past decades, as it has shown that it can be used to predict cardiovascular diseases and mortality. Several *in vivo* and *ex vivo* techniques have been developed to characterize the mechanical properties of vessels. *In vivo* techniques tend to ignore the anisotropy of the vessel wall components. While *ex vivo* techniques tend to be destructive and do not account for the geometry of the arteries. In this paper we present a technique using sonometry to study the elasticity of soft tubes and excised pig carotids in different directions. The method uses piezoelectric crystals to track the strain in the circumferential and longitudinal directions while the tubes or vessels are being pressurized. We compare the Young's moduli obtained from sonometry experiments performed in two different types of tubes with the mechanical testing done in the material used to make these tubes. We also present data obtained in the excised pig carotids and show the differences in the longitudinal versus the circumferential directions. The technique we propose has a potential for the non destructive study of soft material cylindrical shapes and can be used to study the mechanical properties of vessels.

I. INTRODUCTION

In the past few decades the study of the mechanical properties of arteries has gained importance as a predictor of cardiovascular events [1]. *Ex vivo* and *in vivo* testing of vessels is important, first to understand the intrinsic mechanical properties of the tissues, and second, to relate this to the physiology of the system to which they belong. Several *in vivo* approaches to study the properties of vessels had been developed such as pulse wave velocity [2, 3], imaging techniques [4, 5] and augmentation index analysis [6]. Even though these techniques have the potential to be applied in a clinical setting, the complexity of the measurements and analysis has not allowed them to become widely used. Another disadvantage to *in vivo* testing is that it tends to ignore that arteries consist of anisotropic materials; therefore the mechanical properties are not fully characterized.

Ex vivo testing techniques have concentrated their efforts on understanding the differences in mechanical properties in the circumferential and longitudinal directions. Fung *et al*, proposed a biaxial technique, in which the arteries are cut

and opened as thin films, then are placed on a machine that stretched them in two directions while recording the stresses [7, 8]. Other attempts to characterize the arteries, used an inflation method where they pressurized a thin film of vessel wall to create a bubble while optically tracking the shape in the directions corresponding to the axial and longitudinal axes [9, 10]. Kassab's group developed a testing technique that measured the circumferential, longitudinal and torsional properties independently from each other by pressurizing the vessels while keeping them at a fixed length, and later stretching them in the axial direction at a fixed pressure [11].

In this paper, we propose the use of piezoelectric crystals and sonometry to track the strain in the longitudinal and circumferential directions when pressurizing soft tubes. This is a nondestructive testing technique, as there is no need to modify the geometrical shape of thin tubes or to exceed the elastic region of the materials (no plastic deformations). To study the strain-stress relationships in the different directions we use the constitutive equations for isotropic materials. Later, we describe the experiments using sonometry on soft tubes using piezoelectric crystals and the mechanical testing done on the urethane used for making the tubes. Finally, we show the comparison from the simulation results, the sonometry experiments and the mechanical testing. We believe that this technique can be easily transferred to the study of arteries by using the proper constitutive equations for orthotropic materials.

II. METHODS

A. Theory

According to the theory of elasticity for plane stress, the constitutive equations for isotropic materials are described by the following system of equations [12]

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1+\nu \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{Bmatrix} \quad (1)$$

Solving for the elastic modulus for the different directions we obtain (2), (3) and (4)

This work was supported in part by the National Institute of Health under grant EB02640.

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$$E = \frac{\sigma_{11}(1-\nu^2)}{\epsilon_{11} + \nu\epsilon_{22}} \quad (2)$$

$$E = \frac{\sigma_{22}(1-\nu^2)}{\nu\epsilon_{11} + \epsilon_{22}} \quad (3)$$

$$E = \frac{\sigma_{12}(1-\nu^2)}{\epsilon_{12}(1+\nu)} \quad (4)$$

where σ_{11} , σ_{22} and σ_{12} are the stress in the circumferential (hoop stress), longitudinal (axial stress) and thickness directions respectively and ϵ_{11} , ϵ_{22} and ϵ_{12} are the strains in the respective directions; ν is the Poisson's ratio and E is the Young's modulus. During the analysis the stress in the thickness direction was assumed to not be significant, therefore, our analysis is focused on the longitudinal and circumferential directions.

The circumferential stress (hoop stress) was calculated as $\sigma_{11} = Pr/t$ and the longitudinal (axial stress) as $\sigma_{22} = Pr/2t$. The strains in the circumferential directions were calculated as $\epsilon_{11} = (r - r_0)/r_0$ and $\epsilon_{22} = (l - l_0)/l_0$ respectively, where P is the transmural pressure, r and l are the changing radius and length due to the pressure variation, r_0 and l_0 the initial radius and length and t is the thickness.

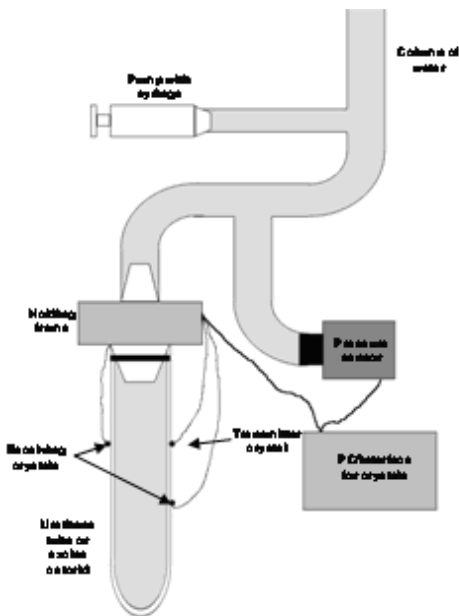


Fig. 1. Diagram of sonometry experimental set up. Three piezoelectric crystals were attached to the urethane tube or vessels as shown in the figure. The tube was pressurized using a syringe, an infusion pump and a column of water. The pressure was recorded using a pressure sensor that was connected to a PC.

B. Experiments

1) Urethane tubes

To produce customizable tubes for our experiments, we developed an injection process to create our own tubes out of urethane. The urethane that we used (Reoflex 20, Smooth-On Inc, Easton, PA) comes in two different components (A and B), which are mixed in equal amounts. To soften the urethane mixture there is a third component that can be added (So-Flex, Smooth-On Inc, Easton, PA). The percentages were determined by the weight of the

components used. We made two tubes, one had no So-Flex, which we defined as 0%, meaning 0% of So-Flex to the quantity of A or B and one tube with 15% So-Flex. The liquid urethane mixture was then injected into a glass tube with an internal diameter of 4.98 mm. Once the tube was filled with the urethane, a stainless steel rod of 3.2 mm diameter was introduced and centered from one end of the tube to the other. Centering of the rod was guaranteed by two machined nozzles that were at each end of the glass tube. The urethane was allowed to cure for at least 24 hours before taking the rod and the urethane tube out of the glass mold.

2) Sonometry experiments

The sonometry experiments were performed using a Sonometrics system (Sonometrics Corp. London, ON, Canada) with 1 mm diameter piezoelectric crystals. The sonometry method tracks the displacement in the circumferential and the longitudinal directions as the tubes are being pressurized by using three piezoelectric crystals attached to the tubes (Fig. 1). One of the crystals is set as a transmitter and the other two as receivers; the transmitter was placed on one side of the tube, and the crystal used to measure the diameter changes was placed at the same level on opposite side of the tube. The crystal used to track the longitudinal motion was placed on the same side as the transmitter somewhere along the length of the tube. The experimental set up is described in Fig. 1. The location for all the crystals was marked before the attachment using a ruler, to make sure that the crystals used to measure the longitudinal strain were in line with each other, and the crystal across the diameter was at the same height as the transmitter and on the exact opposite side of the tube.

For the urethane tubes, the crystals were attached using silicone glue (732 Multi-Purpose Sealant, Dow Corning Corp., Midland, MI). After attaching the crystals we let the glue cure for at least 10 minutes. For the excised arteries, a small point (less than 1 mm diameter) of super glue (Aron Alpha, Elmer's Products Inc., Columbus, OH) was put on each location until it dried. The crystals were then attached to the super glue using skin adhesive (Medical Adhesive 7730, Hollister Inc., Libertyville, IL) which was let to cure for 5 minutes. The tubes or the artery with the attached crystals were then immersed in a water or saline bath as the ultrasound pulse from the crystals needed a medium to propagate. We proceeded to maximize the sensitivity of each crystal which is an option available at the interface with the computer. The pressurization of the system was performed using an infusion pump (KDS210, KD Scientific, Holliston, MA) which was set to deliver water or saline at a specific rate to a column of water as described in Fig. 1. Before recording data 4 to 5 cycles of pressurization were performed for preconditioning of the tube and the artery. This was repeated every time the crystals were attached. The crystals data was acquired by the Sonometrics interface and software and a PC; the sampling rate was set to 483.7 Hz, the length of the transmitting pulse to 406.3 ns and an inhibit delay of 3 mm (as the crystals were at all times farther than this). The system also recorded the output from the pressure sensor (PX319-015G5V, Omegadyne Inc. Sunbury, OH) in

one of its analog inputs, The sensor was calibrated using a column of water before the series of experiments.

During the analysis of the piezoelectric crystal data, a linear regression on the radius changes and the length changes was done in order to extrapolate the values of r and l to a 0 mmHg pressure and to decrease the noise. To assess the goodness of the fit the R^2 values of the regressions were calculated. The data from the fitting was used in the equations described before to make the calculations of stress and strain and Young's modulus.

3) Mechanical testing

Controlled compression tests were performed on the urethane samples using a DMA machine (Q800 by TA Instruments, New Castle, DE). These tests were done on samples of about 5 x 5 x 5 mm from the same urethane composition as the tubes used for the sonometry experiments. The controlled force test was done to determine the Young's modulus in a linearly elastic portion of the material. In this test the loading rate was 1 N/min to a maximum of 0.75 N, which translates to about 5% strain. The same test was repeated in five different samples from the same batch to measure the reproducibility of the compression test.

III. RESULTS

The results of the piezoelectric crystal experiments on the two tubes are shown in Fig. 2. In this case we tested the 0% and 15% tubes, to evaluate the sensitivity of the technique to changes in the Young's modulus. In panels A and B are the changes of the radius and the length through the pressure loop. The pressure profile applied to the tubes is shown in panel C. The solid lines in panels A and B are the linear regression on the radius and length change. The R^2 values were 0.9876, and 0.9866 for the three radius changes, and

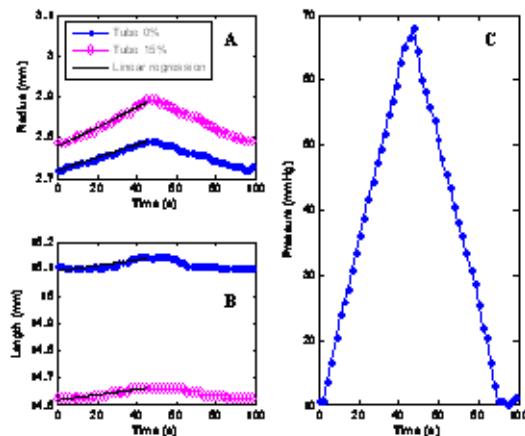


Fig. 2. Results of sonometry experiment using the two 0% tubes and the 15% tube. Panel A shows the change in the radius for each tube during the pressurization and depressurization. Panel B, similarly, shows the change in the length. The solid lines in both panel A and B represent the linear regression done on the experimental data. Panel C shows the pressure profile.

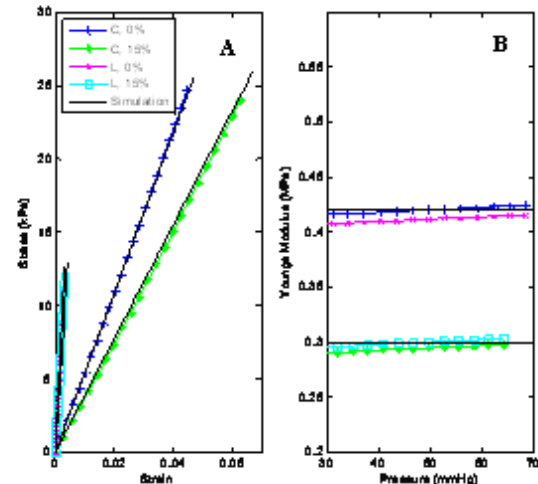


Fig. 3. Calculated strains, stresses and Young's moduli for the two urethane tubes. In panel A, stress and strain relationships for the urethane tubes in the circumferential (C) and in the longitudinal (L) directions. Panel B, shows the calculated Young's modulus in MPa. The legend from panel A applied to panel B

0.6728, and 0.9030 for the length changes. The linear regression was used to decrease the noise of the piezoelectric crystals signal and to extrapolate the values of r and l to 0 mmHg of pressure. In Fig. 3, panel A, are the circumferential and longitudinal stresses and strains calculated using the values from the linear regression and the equations described before. In panel B is the Young's modulus as calculated by (2) and (3). In solid lines are the results of a simulation using the same equations as for the experimental data and a linear pressure profile from 0 to 70 mmHg. The values for the Young's moduli for 0% and 15% used in the simulation to match the experimental data were 0.42 and 0.30 MPa respectively. The results of the compression mechanical test to the 0% and 15% softex urethane samples are summarized in Table I. The results of the sonometry experiments on the excised pig carotid are shown in Fig. 4. This figure shows the stress and strain relationship in the longitudinal and circumferential directions when pressurizing the artery from 10 to 100 mmHg.

IV. DISCUSSION

The results of the piezoelectric crystal experiments presented in Fig. 2 shows the linear profile of pressure that was applied to the tubes. The response of the tubes in both directions was also linear, corroborating our hypothesis that the testing took place in the linearly elastic portion of the material, and therefore no plastic or permanent deformations were caused. It also allowed us to perform a linear regression of the data to decrease noise and get radius and length values for pressures equal to 0 mmHg, which experimentally was not feasible but for the calculation of the Young's modulus was important because it gave an estimate of the radius and the length at a zero stress state. In Fig. 2, panel A, it is evident that the changes in the radius for the

0% and 15% tubes are different, and the slopes of the regression are also

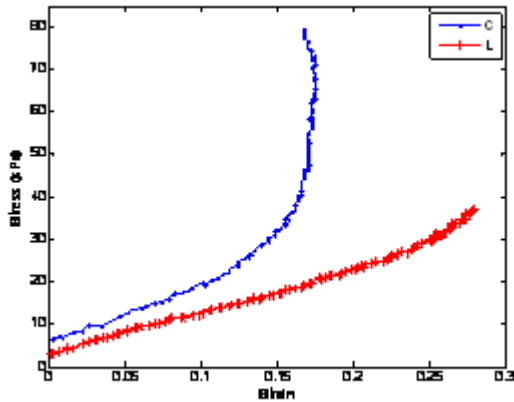


Fig. 4. Piezoelectric crystal experiment on excised pig carotid. Stress-strain relationship in the circumferential and longitudinal directions when pressurizing the artery to 100 mmHg.

TABLE I
MECHANICAL TESTING RESULTS

Sample # \ Material	Young Modulus (MPa)	
	0% Soflex	15% Soflex
1	0.4133	0.3364
2	0.4033	0.3523
3	0.4185	0.3468
4	0.4279	0.3457
5	0.4176	0.3427
Mean	0.4161	0.3448
Std	0.0089	0.0058

Table 1. Results of the compression tests done in the 0% and 15% sofex urethane used in the elaboration of the tubes used in the sonometry experiments.

different. Comparison of the length changes is difficult as there was small strain in this direction for the two tubes. In Fig. 3, the stress-strain relationships in the circumferential direction for the 0% and 15% tubes significantly differ. The differences are less noticeable in the longitudinal direction as the strains in this direction were very small. Using these values for the stress and strain, and a Poisson's ratio of 0.45, it was possible to calculate the elastic moduli as a function of the transmural pressure. The results for the two tubes showed that the method has the sensitivity to differentiate between the 0% and the 15% compounds. The results from the compression controlled force test for the different samples and materials presented in Table I are in good agreement with the results from the piezoelectric crystal technique, which makes us very confident that the technique can be used to investigate properties of soft materials in a cylindrical shape. Also, the correlation of the simulation to the results from the experiment presented in Fig. 3 (solid lines), gives us good confidence that the sets of equations that we are using are proper for this case. The changes in the radius and length are comparable between the two scenarios and the calculated stresses and strains are very close.

In comparison with the urethane tubes, the stress-strain relationships for the arteries showed a nonlinear behavior as it was expected, especially in the circumferential direction (Fig. 4). It also showed how the relationship in the two

directions differs significantly. This might be attributed to the fact that there was no prestress in the longitudinal direction which might delay the nonlinearity of the longitudinal strain. Another significant difference between the artery and the tube results were the maximum values of strain achieved at similar pressures. Notice that the maximum strain for the urethane tubes in the longitudinal directions was significantly lower than in the circumferential directions; the opposite happened in the arteries. The results in tubes and arteries give us good confidence that the technique can be used for studying the properties soft tubes and arteries in two different directions. We still need to work on the proper equations for anisotropic or transversely isotropic materials for arteries applications to be able to calculate Young's modulus from the stress-strain relationships.

V. CONCLUSION

This paper presents a technique to evaluate the biaxial mechanical properties of isotropic materials in cylindrical shapes and arteries using piezoelectric crystals and sonometry. The constitutive equations that govern the technique for isotropic materials are presented and are correlated with experimental data with very good agreement. Also, mechanical testing data on the same materials is presented to validate the method. Therefore we are confident that the piezoelectric crystal method can be used to investigate the material properties of soft tissues and can be applied to the study of vessels by developing the constitutive equations for anisotropic materials.

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