31st Annual International Conference of the IEEE EMBS
Minneapolis, Minnesota, USA, September 2-6, 2009

# A mechanical model of soft biological tissue - An application to lung parenchyma -

Nele De Geeter, Clara Ionescu, *Member, IEEE* and Robin De Keyser

***Abstract*— This paper presents a fractal mechanical model for branching systems, with application to the respiratory system. Assuming a dichotomously branching tree, each airway tube is modeled by a Kelvin-Voigt model (a spring in parallel with a dashpot) using morphological values. The model allows investigations on the viscoelastic properties within the context of inter-connections between levels of the respiratory tree. The results are in agreement with physiological expectancy. The model presented in this paper can also serve to derive a mechanical model for other branching systems, i.e. the circulatory system.**

## I. Introduction

FRACTIONAL order systems are dynamical systems whose model can be represented in a natural way by non-integer order parameters. They acknowledge some specific phenomena; fractal structure, diffusion and/or viscoelasticity. The respiratory system poses all tree enumerated properties; the airway distribution has a fractal structure, in the alveoli gas exchange by means of diffusion takes place and the lung parenchyma is viscoelastic. The clinicians prefer a simple, yet accurate model from whose parameter values they are able to detect whether a patient has a lung pathology or not. It is therefore interesting to characterize the lung function in terms of its mechanical properties as stress, strain and viscoelasticity, which can be directly related to changes in airway duct geometry.

Viscoelasticity of lung parenchyma determines the mechanical properties of the overall lung function. Lung parenchyma consists of tissue fibers interwoven in a network of collagen and elastin strings [2], [7]. Since the system acts as a whole, it is important to characterize the mechanical properties as they propagate within consequent levels. Several research groups investigate the viscoelasticity of the lung parenchyma in animal and human studies (ex-vivo) [7], [10]. Their investigations are based on excised lung tissue strips, neglecting the inter-connection to the rest of the system.

This study is a sequel from modeling the respiratory tree with an electrical equivalent [5]. By electro-mechanical analogy, a simple mechanical model can be derived. The mechanical model allows predictions upon the stress-strain relationship calculated at the entrance of a level in the respiratory tree.

The paper is organized as follows. The respiratory tree and its modeling by the electrical equivalent is briefly explained in the next section, along with the mechanical model derivation for obtaining the stress-strain relation. Simulation results and their interpretation are detailed in the third section, while a conclusion section summarizes the main outcome of this investigation.

N. De Geeter, C. Ionescu and R. De Keyser are with the Faculty of Engineering, Department of Electrical energy, Systems & Automation, Ghent University, Technologiepark 913, 9052 Ghent, Belgium Nele.DeGeeter@UGent.be; Clara@autoctrl.UGent.be; Rdk@autoctrl.UGent.be

## II. Models for the respiratory tree

THE respiratory tree is an asymmetric branching structure of airway ducts, in which a certain degree of symmetry can be recognized [11], [12]. For simplicity, in this paper we treat the *symmetric* case of morphological values for the airways, which assumes a dichotomously equivalent bifurcation of the airways in sub-sequent levels [8], [9], [11], [12]. Gas enters and leaves the lung through a bifurcating system of tubes that get successively smaller in diameter (fractal structure). The respiratory system consists of two zones: the *conductive zone*, from level 1 to 15, and the *respiratory zone*, from level 16 to 24, with level 1 denoting the trachea and 24 the alveoli [4]. For the purpose of this study, we investigate the airways within the respiratory zone, in which the air is involved in the process of gas exchange. The airway tube parameters are presented in Table I.

TABLE I
THE AIRWAY TUBE PARAMETERS [9], [12].

| Level $m$ | Length $\ell$ (cm) | Radius $r$ (cm) | Wall thickness $h$ (cm) | Cartilage fraction $\kappa$ |
|---|---|---|---|---|
| 16 | 0.810 | 0.125 | 0.0086 | 0.0329 |
| 17 | 0.770 | 0.120 | 0.0083 | 0.0308 |
| 18 | 0.640 | 0.109 | 0.0077 | 0.0262 |
| 19 | 0.630 | 0.100 | 0.0072 | 0.0224 |
| 20 | 0.517 | 0.090 | 0.0066 | 0.0000 |
| 21 | 0.480 | 0.080 | 0.0060 | 0.0000 |
| 22 | 0.420 | 0.070 | 0.0055 | 0.0000 |
| 23 | 0.360 | 0.055 | 0.0047 | 0.0000 |
| 24 | 0.310 | 0.048 | 0.0043 | 0.0000 |

### A. Electrical Equivalent

By analogy to electrical networks, one can consider voltage as equivalent for respiratory pressure $P$ and current as equivalent for air-flow $Q$. Electrical resistances $R$ represent respiratory resistance that occur as a result of airflow dissipation in the airways, electrical capacitors $C$ represent volume compliance of the airways which allows them to inflate/deflate.

From the geometrical and mechanical characteristics of the airway tube, and from the air properties, one can express the

parameters for one airway tube [5]:

$$R = \ell \frac{\mu \delta^2}{\pi r^4 \acute{M}_{10}} \sin(\acute{\varepsilon}_{10}) \quad (1)$$

$$C = \ell \frac{2\pi r^3 (1-\nu_P^2)}{Eh} \quad (2)$$

with $\ell$ the length, $r$ the radius, $h$ the thickness, $\nu_P = 0.45$ the Poisson coefficient, $\mu = 1.86 \cdot 10^{-5}$ kg/m-s the viscosity of air and $\delta = r\sqrt{\frac{\omega\rho}{\mu}}$ the Womersley parameter [13], where $\rho = 1.075\ kg/m^3$ is the density of air, $\omega = 2\pi f$ and $f$ is the frequency in Hz. $\acute{M}_{10}$ and $\acute{\varepsilon}_{10}$ are respectively the modulus and phase angle of Bessel functions of the first kind and order 0 and 1 [1], denoted by:

$$\acute{M}_{10} e^{j\acute{\varepsilon}_{10}} = 1 - \frac{2J_1(\delta j^{3/2})}{J_0(\delta j^{3/2}) \delta j^{3/2}} \quad (3)$$

in which $j = \sqrt{-1}$ is the complex number. The effective elastic modulus $E$ is considered in function of the airway tissue structure. We take into account the fraction amount $\kappa$ of corresponding cartilage tissue (index $c$) and soft tissue (index $s$) for each level (see Table I), with $E_c = 400\ kPa$ and $E_s = 60\ kPa$ [5].

$$E = \kappa E_c + (1-\kappa) E_s \quad (4)$$

The balance between the cartilage and soft tissue percent varies with each respiratory level and with disease. It is therefore important to include this information in our model. Using equations (1-2) and with $e$ the voltage and $i$ the current represented as in Fig. 2, the equations for the electrical model are given by:

$$e_0 = R_1 i_1 + e_1;\ e_1 = \frac{R_2}{2} i_2 + e_2 \quad (5)$$

$$i_1 = i_2 + C_1 \dot{e}_1;\ i_2 = 2C_2 \dot{e}_2 \quad (6)$$

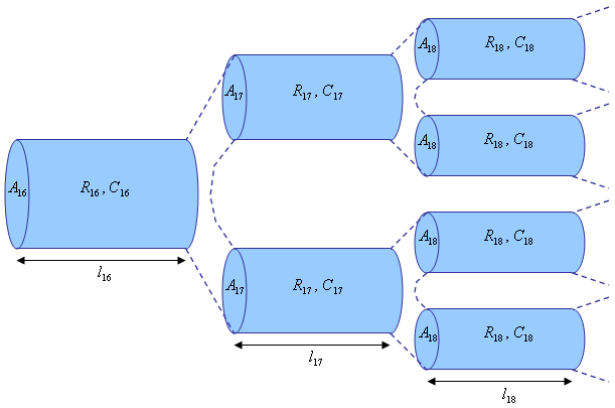


Fig. 1. A schematic representation of the electrical model for the lung parenchymal tissue (starting from level 16).

### B. Mechanical Equivalent

Using the electro-mechanical analogy from Table II, we can derive an equivalent mechanical model. This can be done starting from the electrical model equations (5-6). The electrical element (resistance in series with capacitor) corresponds to the mechanical Kelvin-Voigt element (dashpot in parallel with spring):

TABLE II

THE ELECTRO-MECHANICAL ANALOGY.

| **Electrical** | **Mechanical** |
|---|---|
| Voltage $e$ $[V]$ | Force $f$ $[N]$ |
| Current $i$ $[A]$ | Velocity $v$ $[m/s]$ |
| Resistance $R$ $[kPa-s/l]$ | Damping constant $B$ $[Ns/m]$ |
| Capacitance $C$ $[l/kPa]$ | Spring constant $1/K$ $[m/N]$ |
| Inductance $L$ $[kPa-s^2/l]$ | Mass $M$ $[Ns^2/m]$ |

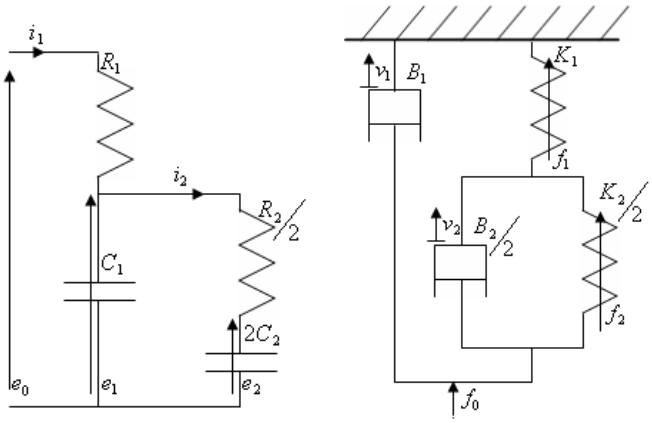


Fig. 2. An illustrating example of the first two levels in the electrical and the mechanical networks.

$$f_0 = B_1 v_1 + f_1;\ f_1 = \frac{B_2}{2} v_2 + f_2 \quad (7)$$

$$v_1 = v_2 + \frac{1}{K_1} \dot{f}_1;\ v_2 = \frac{2}{K_2} \dot{f}_2 \quad (8)$$

The values of resistors and capacitors are calculated with the model from Fig. 2 and relations (1-2): $R_1 = 0.2\ kPa-s/l$ and $C_1 = 0.25\ l/kPa$. The total parameter values for each level $m$ are then given by $R_m^* = R_m/2^{m-1}$ and $C_m^* = 2^{m-1} C_m$. From these values one can calculate the equivalent $B_m^*$ and $K_m^*$:

$$B_m^* = \frac{f_m}{v_m} = \frac{P_m}{Q_m} A_{Pm} A_{Qm} = R_m^* 4\pi^2 r_m^4 (1-\nu_P^2) \quad (9)$$

$$K_m^* = \frac{f_m}{x_m} = \frac{P_m}{V_m} A_{Pm} A_{Qm} = \frac{1}{C_m^*} 4\pi^2 r_m^4 (1-\nu_P^2) \quad (10)$$

with $P$ the pressure in $Pa$, $Q$ the flow in $m^3/s$, $V$ the volume in $m^3$, $A_{Pm}$ and $A_{Qm}$ areas, $r$ the radius of a tube, $x$ the axial displacement and $\nu_P = 0.45$ the Poisson coefficient.
Fig. 3 depicts the evolution of the parameters in the entire level $m$. The evolution in a single tube in consecutive levels is quasi-linear for both parameters. However, since the total parameter values (indicated by the superscript $*$) depend on the total number of tubes within each level, they change as an exponential decaying function. When represented on a logarithmic scale, one can observe a quasi-linear behavior.
In a similar manner as the electrical impedance is calculated, one may obtain $H(s)$, which defines the relation from velocity (input) to force (output) $f(s)/v(s)$, with $s$ the Laplace operator. For one damper and one spring in parallel, we have

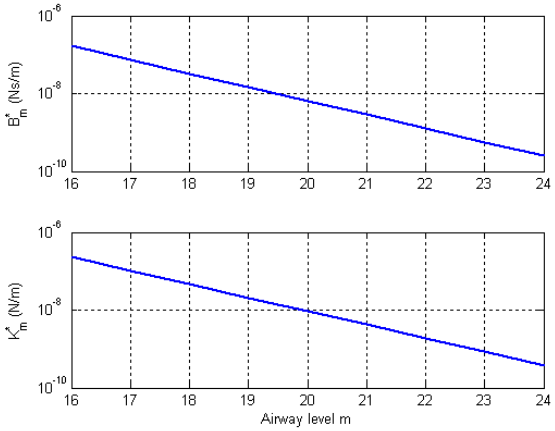


Fig. 3. Parameter evolution in the entire level, for levels 16–24.

that $H(s) = B + K/s$.

Due to the fact that the network is dichotomous and symmetric, we can obtain the *mechanical impedances* $H_{tot\ m}(s)$ using recurrent forms as in Fig. 2 and starting at level 24 with an impedance denoting the gas compression compartment. In Fig. 4 the Bode diagram of these transfer functions for the lung parenchyma are plotted; the fractional integrator with order 0.15 can be seen at $[10^{-2}, 10^{2}]\ rad/s$.

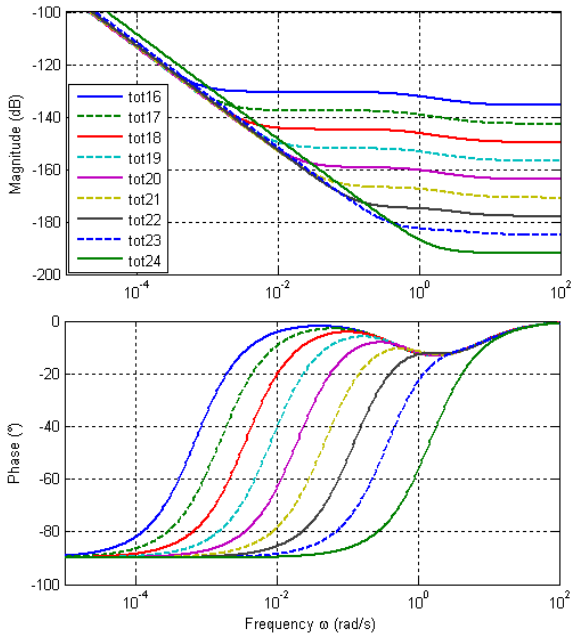


Fig. 4. The Bode diagram of the mechanical impedances.

The lung parenchyma consists of interwoven collagen (infinitely stiff) and elastin (elastic) fibers [7]. Each level in the respiratory tree has a specific balance between these two components. In our model we approximate this balance in function of the cartilage percent (4). Following this reasoning, a similar representation of the mechanical model is given in Fig. 5. Here, the cylinders represent the airway branches which are interconnected with inextensible unstressed strings. Once a string is taut, any further increases

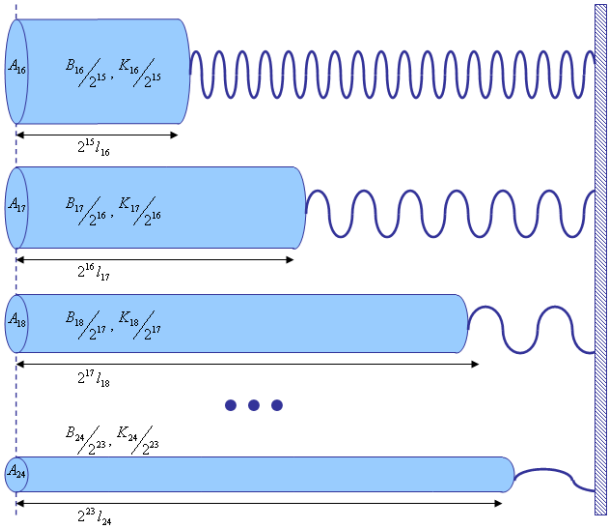

Fig. 5. A schematic representation of the mechanical model for the lung parenchymal tissue (levels 16–24).

in strain will cause its associated airway branch to become strained. Only those levels with taut strings bear stress. As the tissue is stressed progressively more of the strings become taut and the stiffness of the entire model increases accordingly. The lung elasticity is determined by elastin fibers, while collagen, which is virtually inextensible, limits the maximum lung dimensions.

This representation varies from that of Bates in that it represents the total collagen-elastin distribution in a level and not in a single tissue strip [2].

### C. Stress-strain derivation

The elastic modulus is defined as the ratio between stress and strain properties. The Kelvin-Voigt body is the simplest viscoelastic model that can store and dissipate energy, consisting of a perfectly elastic element (i.e. spring) arranged in parallel with a purely viscous element (i.e. dashpot). The corresponding differential equation is given by:

$$\sigma(t) = \frac{K\ell}{A_{cross}}\epsilon(t) + \frac{B\ell}{A_{cross}}\frac{d\epsilon(t)}{dt} \tag{11}$$

with $\sigma$ the stress, $\epsilon$ the strain, $\ell$ the length, $A_{cross} = 2\pi rh$ the cross section of the tube, with $r$ the radius and $h$ the thickness. $K$ and $B$ are the constants of respectively the spring and dashpot [3]. The stress can be defined as pressure, whereas the latter is given by force distribution over the area. The strain $\epsilon$ is defined as the ratio of the change in length over the initial length: $\Delta\ell/\ell$. Starting with an unstressed tissue, we apply a strain that increases in steps of 10% until it reaches 100%. The new length can be calculated as:

$$\ell_{new} = (1 + \epsilon)\ell_{old} \tag{12}$$

with the subscript *old* denoting the unstressed properties. Assuming a constant tissue volume $V_t$, the radius will decrease:

$$r_{new} = \frac{V_t}{2\pi\ell_{new}h} = \frac{r_{old}\ell_{old}}{\ell_{new}} \tag{13}$$

We neglect the changes in the thickness $h$ of the tube wall with changes in the strain. Applying an oscillatory flow $Q$ of constant amplitude $0.5\ l/s$ and a frequency of $0.25\ Hz$, the velocity $v$ can be calculated as:

$$v_{new} = \frac{5 \cdot 10^{-4}}{A_{Qnew}} \tag{14}$$

Since the $B$'s and $K$'s are time-invariant material properties, the transfer function $H$ will be independent of the strain. This mechanical impedance $H$ is defined as force over velocity. The new pressure is then given by:

$$P_{new} = \frac{f_{new}}{A_{Pnew}} = \frac{H \cdot v_{new} H}{A_{Pnew}} = \frac{H \cdot 5 \cdot 10^{-4}}{A_{Pnew} A_{Qnew}} \tag{15}$$

with the multiplication of the areas $A_{Pnew}A_{Qnew} = 4\pi^2 r_{new}^4 \left(1-\nu_P^2\right)$. The elongation of the tube can be expressed as [6]:

$$P + \frac{h}{r\left(1-\nu_p^2\right)}\left(\frac{K\ell}{A_{cross}}\epsilon + \frac{B\ell}{A_{cross}}\frac{d\epsilon}{dt}\right) = 0 \tag{16}$$

The stress $\sigma$ are then given by:

$$\sigma_{new} = -P_{new}\frac{r_{new}\left(1-\nu_p^2\right)}{h} \tag{17}$$

Now the stress and strain properties can be evaluated using equations (12-17).

## III. Results and Discussion

USING the formulas from section II-C, one obtains the stress-strain curves depicted in Fig. 6. The strain is increased in steps of 10% from 10 to 100%. Starting from level 24, one can then calculate the stress-strain curve at the input of each level. This then will give rheological information in the context of all levels interconnected.

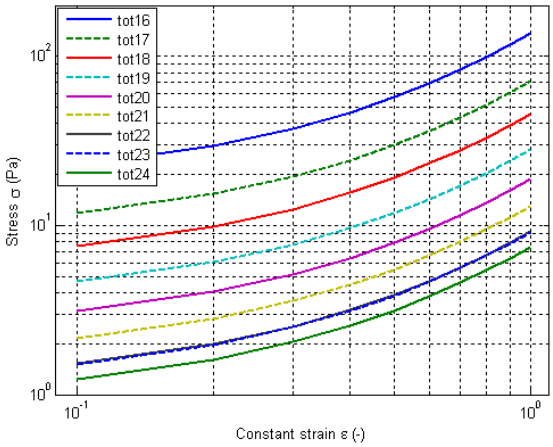


Fig. 6. The stress-strain curves.

As expected, the stress increases with the degree of elongation applied to the entire structure. The more levels we have in our structure, the higher the values of the stress-strain curve, due to higher amount of cartilage tissue (collagen). This is also illustrated in Fig. 5. The obtained results are qualitatively similar to those reported in literature [7], [10]. Quantitatively, it is not possible to make an evaluation of our model, since the values reported hitherto in the literature are based on excised tissue strips. One may expect that the mechanical conditions vary for an excised tissue and for a biological tissue analyzed in relation to the rest of the organ from which it belongs.

## IV. Conclusions

A MECHANICAL equivalent is derived in this paper, based on an electrical symmetrical model of the respiratory tree. The novel contributions are twofold: i) the elements are calculated with morphological values and preserve the geometry of the lungs, and ii) the stress-strain properties are evaluated at every level, but they are inter-related with the consequent levels within the network.

In a first instance, the model presented in this paper can serve to observe the evolution of the stress-strain relationship to changes in morphology. A second step is to verify how these results change for the case of an asymmetric tree.

The model can also serve to derive mechano-electrical models for other similarly branching systems, i.e. the circulatory system.

## References


[1] M. Abramowitz, I.A Stegun *Handbook of mathematical functions with formulas, graphs and mathematical tables*, New York: Dover Publications, ISBN 978-0-486-61272-0, (1972)

[2] J. Bates, "A recruitment model of quasi-linear power-law stress adaptation in lung tissue", *Annals of Biomedical Engineering*, **35**: 1165-1174, (2007)

[3] D. Craiem, R.L. Armentano, "A fractional derivative model to describe arterial viscoelasticity", *Biorheology*, **44**: 251-263, (2007)

[4] C. Hou, S. Gheorgiu, M.O. Coppens, V. Huxley, P. Pfeifer, "Gas diffusion through the fractal landscape of the lung: how deep does oxygen enter the alveolar system?", *Fractals in Biology and Medicine*; **4**: 17-30, (2005)

[5] C. Ionescu, P. Segers, R. De Keyser, "Mechanical properties of the respiratory system derived from morphologic insight", *IEEE Trans Biomed Eng*, **54(4)**: 949-959, (2009)

[6] C. Ionescu, W. Kosinski, R. De Keyser, "Viscoelasticity and fractal structure in a model of human lungs", *Archives of Mechanics*, Submitted, (2009)

[7] G. Maksym, J. Bates, "A distributed nonlinear model of lung tissue elasticity", *J Appl Physiol*, **82(1)**: 32-41, (1997)

[8] B. Mandelbrot, *The fractal geometry of nature*, New York: Freeman and Co, (1983)

[9] V. Sauret, P. Halson, I. Brown, J. Fleming, A. Bailey, "Study of the three-dimensional geometry of the central conducting airways in man using computed tomographic images", *J Anat*, **200**: 123-134, (2002)

[10] B. Suki, A.L. Barabasi, K. Lutchen, "Lung tissue viscoelasticity: a mathematical framework and its molecular basis", *J Appl Physiol*, **76(6)**: 2749-2759, (1994)

[11] E.R. Weibel, *Morphometry of the human lung*, Berlin: Springer, (1963)

[12] E.R. Weibel, "Mandelbrot's fractals and the geometry of life: a tribute to Benot Mandelbrot on his 80th birthday", *Fractals in Biology and Medicine*, **4**: 3-16, (2005)

[13] J. R Womersley, "An elastic tube theory of pulse transmission and oscillatory flow in mammalian arteries", Wright Air Development Center, Technical Report WADC-TR56-614, (1957)