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The influence of left-ventricular shape on end-diastolic fiber stress and strain.

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Abstract— **Passive filling is a major determinant for the pump performance of the left ventricle and is determined by the filling pressure and the ventricular compliance. We quantified the influence of left-ventricular shape on the overall compliance and the distribution of passive fiber stress and strain during the filling period in normal myocardium. Hereto, fiber stress and strain were calculated in a finite element analysis during the inflation of left ventricles of different shape, ranging from an elongated ellipsoid to a sphere, but keeping the initial cavity and wall volume constant. The passive myocardium was described by an incompressible hyperelastic material law with transverse isotropic symmetry along the muscle fiber directions. A realistic transmural gradient in fiber orientation was assumed. While compliance was not altered, the transmural distribution of both passive fiber stress and strain was highly dependent on ventricular shape, where more spherical ventricles exhibited a higher subendocardial gradient in both quantities.**

I. INTRODUCTION

The performance of the heart as a circulatory pump depends on the filling capacity of the left ventricle (LV), which is the muscular cardiac chamber that pumps oxygenated blood from the pulmonary system to the peripheral organs. This filling capacity will be determined by its passive mechanical behaviour, i.e. the compliance. This compliance in its turn depends strongly on the microscopic structure of the muscle wall, i.e. the myocardium, which is mainly composed of the muscle cells, called myocytes, and connective tissue. Experiments have shown that the myocytes are organized in a complex helical fiber network [1], indicating a mechanical behaviour that is locally anisotropic with a preferred direction along the fiber. The mechanics of the myocytes can therefore be described by estimation of local fiber stress and strain. However, a robust regional quantification of these quantities remains a challenge. Current methods to measure intramyocardial stresses are invasive and give only information about isotropic pressures [2], which leaves computational modeling as the only alternative to obtain regional fiber stress and strain values.

Several modeling studies have been performed with this aim, such as [3]-[9]. Different degrees of approximations in either geometry or fiber distribution have been adopted in these studies, for the sake of mathematical simplification or reduction of computational cost. However, the focus has been mainly on optimizing constitutive parameters or fiber orientations, starting from a certain chosen undeformed geometrical configuration of the left ventricle. Only limited attention has been paid to geometrical differences in the LV shape and it is not clear what influence this has on fiber stress and strain. In several patho-physiological conditions such as ischemia, changes in mechanical loading of the left ventricle is often accompagnied by a change in LV shape through a process called remodelling [10], in which the LV tends to become more spherical. From this point of view, it can be expected that changes in LV shape will be a factor of influence on regional mechanics. To test this hypothesis, a comparison by means of finite element (FE) modeling has been performed in the present study, in which the fiber stress and strain at the end of the passive filling phase, i.e. end diastole (ED), was examined for several LV shapes with different sphericity.

II. METHODS

A. Geometrical description

In this study, the geometry of the undeformed left ventricle was approximated as a truncated thick-walled ellipsoid. This simplification can be motivated by the strongly ellipsoidal character of the LV shape and is often adopted in mechanical studies of left-ventricular deformation ([3]-[6], [11]). The ellipsoidal geometry can most naturally be described using prolate spheroidal coordinates (λ, μ, θ), of which the transformations to Cartesian coordinates (x, y, z) are given by:

$$\begin{aligned} x &= f \sinh(\lambda) \sin(\mu) \cos(\theta) \\ y &= f \sinh(\lambda) \sin(\mu) \sin(\theta) \\ z &= f \cosh(\lambda) \cos(\mu), \end{aligned} \tag{1}$$

where f is the focal distance. As shown in Fig. 1, the λ-coordinate varies transmurally from the endocardial (inner) to the epicardial (outer) border, the μ-coordinate longitudinally from base to apex and the θ-coordinate in the circumferential direction. It is assumed that the undeformed configuration represents the LV in diastasis. In order to change the sphericity of the left ventricle, a reference shape was first defined based on the dimensions as given in [12]. The truncation angle at the basal side of this reference ventricle was chosen to be $\mu = \pi/3.3$. The equatorial cavity diameter and wall thickness was set to 3.8 and 1.3 cm respectively and the short-to-long-axis ratio of the cavity was taken to be 0.45. This choice of dimensions resulted in a cavity and wall volume of 56.6 ml and 125.4 ml respectively and a basal cavity diameter of 3.1 cm. The cavity short-to-long-axis ratio was subsequently changed to the values given in Table I to obtain ventricles with different sphericity while

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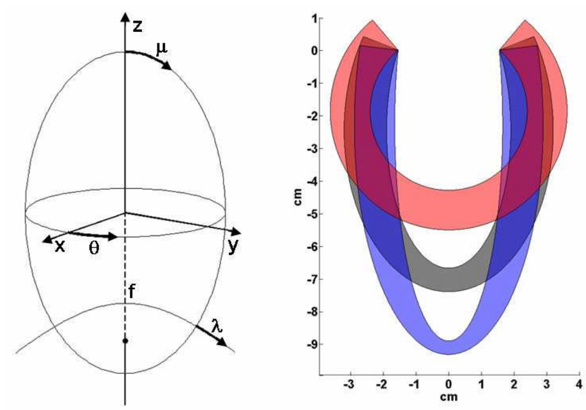

Fig. 1. Illustration of the ventricle geometry. The prolate spheroidal coordinates are shown on the left, while the different shapes of the ventricles with short-to-long-axis ratio of 0.25 (blue), 0.45 (grey) and 0.99 (red) are shown on the right.

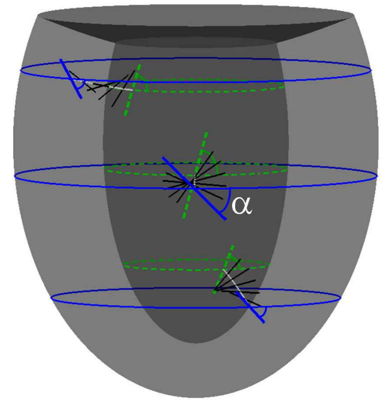

Fig. 2. Helical fiber orientations in the LV-wall, modeled via a transmural, linearly varying distribution of the fiber angle $\alpha(\lambda)$, shown at different longitudinal levels.

the cavity volume and basal diameter were kept constant at the reference values (see Fig. 1). By doing so, the equatorial cavity diameter changed to the values as given in Table I. The epicardial borders were determined by keeping the wall volume constant at the reference value, resulting in changes of the equatorial wall thickness as listed in Table I. To indicate the transmural position in the LV wall, a relative transmural position is defined that varies from 0 at the endocardial border to 1 at the epicardial border.

B. Finite element framework

During the cardiac cycle, the myocardium undergoes large deformations. As a consequence, the nonlinear theory of finite deformation [13] needs to be applied to describe its mechanical behaviour. As stated above, it is also often convenient to use curvilinear prolate spheroidal coordinates. Therefore, a FE framework as proposed by Costa et al. [11] was implemented to model finite elastic deformation of the left ventricle in a curvilinear coordinate system, allowing for a reduced number of elements in the FE mesh. This framework was integrated with the NOX and AztecOO solver-packages from the Trilinos project [14] to solve the resulting nonlinear equations using an iterative Newton based strategy. As validation of this FE implementation, the computed solution for a deformed cylindrical tube with the Mooney-Rivlin constitutive law was compared with the analytical solution derived by Rivlin [15]. In this study, only static equilibrium was considered without any body forces or couple stresses present. The solution was considered to be converged when the normalized two-norm of the residual vector becomes smaller than 10^{-6}.

TABLE I

SHORT-TO-LONG-AXIS RATIO AND EQUATORIAL CAVITY DIAMETER AND WALL THICKNESS FOR THE DIFFERENT VENTRICLE SHAPES.

axes ratio	0.250	0.375	0.450	0.675	0.990
eq. cavity diam. (cm)	3.302	3.620	3.800	4.272	4.810
eq. wall thickness (cm)	1.239	1.287	1.300	1.292	1.223

For each ventricle shape, a structured FE mesh was constructed in prolate spheroidal coordinates. Each mesh was composed of 10 transmural by 30 longitudinal elements, while only one element was taken in the circumferential direction because of the symmetry of the problem.

C. Boundary conditions

Because the undeformed configuration (zero-stress) of the LV was considered to be in the diastasic phase of the heart cycle, a cavity pressure of 8 mmHg (1.066 kPa) was imposed on the endocardial surface to simulate ED conditions. Following [3] and [11], the epicardial pressure was taken to be zero. The endocardial basal node was constraint in all coordinates to account for the relative stiff valve annuli and to prevent longitudinal translation.

D. Fiber distribution and constitutive law

For simplification, the anisotropic helical orientation of the myocytes in the LV wall was approximately modeled by a transmurally varying fiber angle distribution, $\alpha(\lambda)$, as illustrated in Fig. 2. This angle defines a local rotation around the transmural axis relative to the circumferential direction. Based on the study by Guccione et al [7], a linear variation from 75 degrees at the endocardium to -45 degrees at the epicardium was chosen. The LV myocardium was modeled as an incompressible hyperelastic material with transverse isotropic symmetry aligned with the local fiber direction. The strain energy function W^* as proposed by Guccione et al [7] was used:

$$W^* = W - p(J-1) \quad (2)$$

$$W = (C/2)(e^Q - 1) \quad (3)$$

$$Q = 2b_1(E_{RR} + E_{FF} + E_{CC}) + b_2 E_{FF}^2 + b_3(E_{CC}^2 + E_{RR}^2 + E_{CR}^2 + E_{RC}^2) + b_4(E_{RF}^2 + E_{FR}^2 + E_{FC}^2 + E_{CF}^2). \quad (4)$$

In (2), J is the determinant of the deformation gradient tensor. To obtain incompressibility, J must be equal to one,

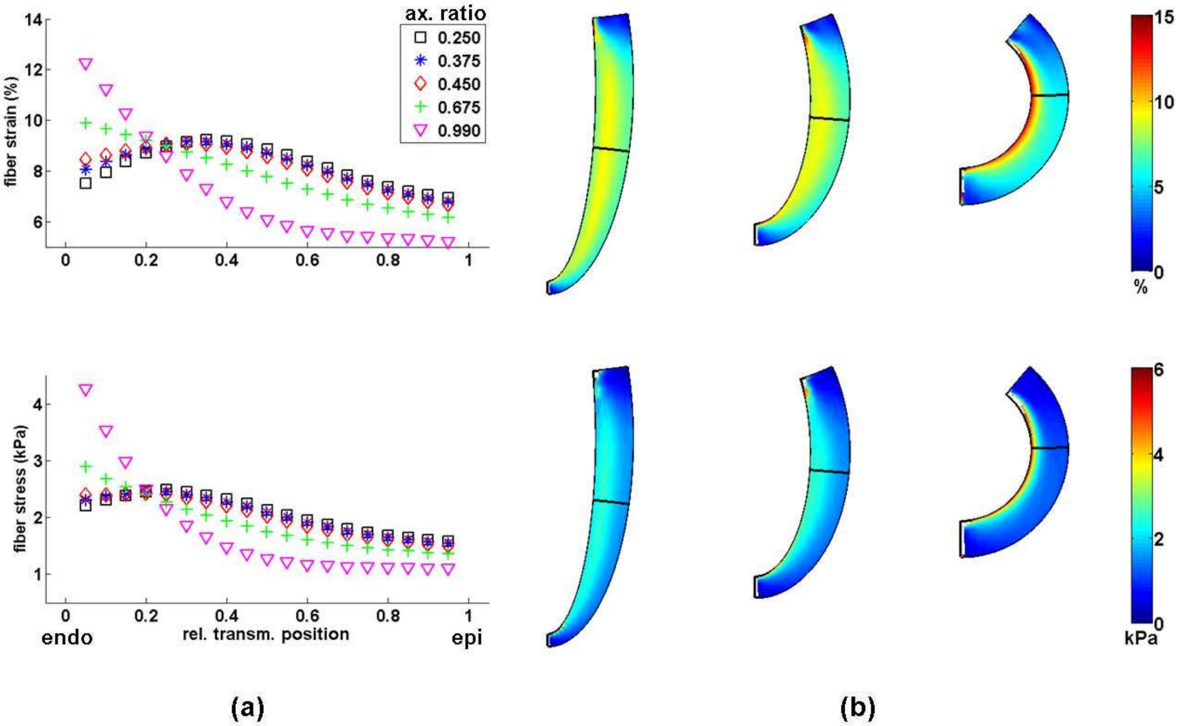

Fig. 3. ED fiber stress and strain values obtained for the different left-ventricular shapes, shown in the top and bottom row respectively. The transmural distribution at approximately mid-level, indicated by the black lines in (b), are plotted in (a) for all LV shapes. The distributions in a longitudinal-transmural cross-section are shown in (b) for the LV shapes with short-to-long-axis ratios of 0.25 (left), 0.45 (middle) and 0.99 (right).

which is imposed here via a Lagrange multiplier p. The isochoric elastic response is determined by the function W. In (4), F,C and R indicate the fiber, cross-fiber in-plane and transmural direction respectively. The Green-Lagrange strain tensor components referred to these fiber coordinate axes are given by E_{ij}. The values of the function parameters were chosen as suggested in [7]: $C = 0.644, b_1 = 2.547, b_2 = 15.09, b_3 = 0$ and $b_4 = 10.48$, which makes the passive myocardium stiffer in the fiber direction.

III. RESULTS

Because of the circumferential symmetry, the calculated fiber stress and strain distributions are only considered for a longitudinal-transmural cross-section, which are shown in Fig. 3. Both the fiber strain and stress are transmurally more heterogeneously distributed when the ventricle shape becomes more spherical. In the subendocardial region, both fiber stress and strain values increase strongly while the transmural gradients become steeper. In the mid-wall and subepicardium, on the contrary, the values decrease and the transmural gradients flatten.

The resulting ED volume of the LV cavity does not vary significantly between the different ventricles. An ED volume of 82.29 ml was obtained for the most elongated ventricle (axes ratio = 0.25) which only dropped slightly to 80.29 ml for the most spherical one (axes ratio = 0.99). The ventricle compliances, defined as Δvolume/Δpressure, therefore only changed from 24.10 ml/kPa to 22.22 ml/kPa respectively.

The deformed LV wall was characterized by a circumferential rotation which increased from the base towards the apex in all cases as shown in Fig. 4. It can be seen that when the sphericity of the LV wall increases, the amount of rotation decreases on every relative height and the torsion, i.e. rotation/relative height, becomes less linear.

IV. DISCUSSION AND CONCLUSIONS

In this study, the influence of left-ventricular shape on passive fiber stress and strain has been examined by changing the sphericity in a computational finite element model for a chosen transmural fiber angle distribution. The calculated values obtained for the reference LV model, which was considered to have a normal geometry, showed good agreement with the results reported in previous studies such as [7], [8] and [11]. Values in the apex and basal-endocardial region were considered unphysiological because of boundary artefacts, which were also obtained in [5]. From Fig. 3, it follows that changes in regional values and gradients of passive fiber stress and strain occur when the sphericity and hence the curvature of the LV wall is increased. The main influence seems to be in the transmural gradient, which becomes very steep in the subendocardium and flattens in the midwall and subepicardium, while the values themselves increase in the subendocardium but decrease in the midwall and subepicardium. As a consequence, the myocytes in an increasingly larger fraction of the midwall and subepicardium are pre-stretched with less than 5% when the ventricle becomes

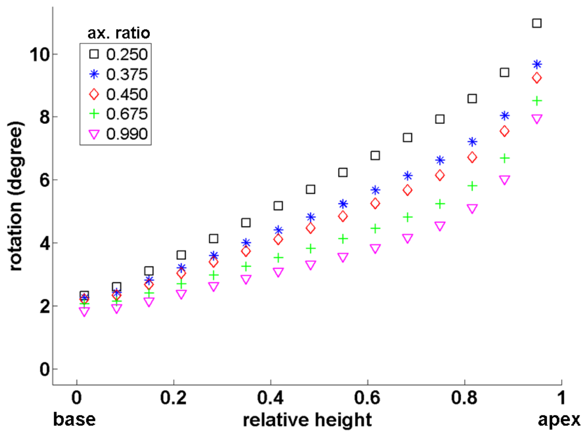

Fig. 4. The rotation in the midwall (relative transmural position = 0.5) of the deformed LV wall for all LV shapes. The relative height is defined as the relative distance along the z-axis from base to apex.

more spherical. Assuming that myocyte characteristics are homogeneous in the LV wall, this would lead to a severe imbalance in regional work load of the myocytes in the ejection phase, since active contraction of the myocytes depends on the amount of pre-stretch [16]. This may suggest that a more spherical ventricle would contract less efficiently. However, it is not certain that passive fiber stress and strain distributions are really homogeneous in the beating left ventricle. This has been hypothesized in [3]-[6] as a criterion to optimize the fiber angle distribution, while the results obtained by Guccione et al. [8] seem to indicate some level of heterogeneity. Surprisingly, the substantial change in shape does not seem to effect the filling capacity of the left ventricle, since the ED volumes and associated compliances obtained in this study hardly show any variability. The results also show that the helical fiber anisotropy causes a torsion of the deformed LV wall around the longitudinal z-axis, which has been observed in the beating heart [17]. However, compared to the values given in [17], the rotation of the LV wall seems to be overestimated in the present study, especially in the apical region, which may be caused by the simplification of the fiber angle distribution in the LV model. Nevertheless, Fig. 4 shows that the LV torsion decreases with increasing sphericity of the LV wall. The increase in the subendocardial stress gradient may be related to this decrease of LV torsion, since Guccione et al. [7] have shown that the presence of torsion can help in the reduction of subendocardial stress gradients in the passive myocardium.

For simplicity, no residual stress was considered in the undeformed configuration of the LV wall. The presence of residual stress has been observed experimentally in the unloaded ex-vivo heart [9] and has been shown to reduce subendocardial stress gradients [7]. The subendocardial fiber stress may therefore be overestimated in this study.

In conclusion, it has been shown in this computational finite element study that geometry can indeed be a factor of influence for the mechanics of the passive LV myocardium. Since only a simplified geometrical model and fiber distribution have been considered, further examination is required to determine whether this still applies for a more realistic model of the left ventricle which incorporates geometry and fiber distribution validated by measurements. However, the results presented here indicate that the choice of left-ventricular geometry must also be taken into account in the interpretation of calculated passive fiber stress and strain distribution, especially in optimization studies such as [3]-[6].

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