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# Estimation of Joint Stiffness with a Compliant Load

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***Abstract*—Joint stiffness defines the dynamic relationship between the position of the joint and the torque acting about it. It consists of two components: intrinsic and reflex stiffness. Many previous studies have investigated joint stiffness in an open-loop environment, because the current algorithm in use is an open-loop algorithm. This paper explores issues related to the estimation of joint stiffness when subjects interact with compliant loads. First, we show analytically how the bias in closed-loop estimates of joint stiffness depends on the properties of the load, the noise power, and length of the estimated impulse response functions (IRF). We then demonstrate with simulations that the open-loop analysis will fail completely for an elastic load but may succeed for an inertial load. We further show that the open-loop analysis can yield unbiased results with an inertial load and document IRF length, signal-to-noise ratio needed, and minimum inertia needed for the analysis to succeed. Thus, by using a load with a properly selected inertia, open-loop analysis can be used under closed-loop conditions.**

## I. Introduction

Joint stiffness defines the dynamic relationship between the position of the joint and torque acting about it. Past studies [1] have modeled joint stiffness as consisting of two components:

1. Intrinsic stiffness due to the viscoelastic properties of the joint, muscles, and connective tissues and the inertia of the limb.
2. Reflex stiffness due to torque generated by the stretch reflex in response to the stretch of the muscle.

In addition to these components, subjects instructed to maintain a fixed position may produce voluntary torques based on visual, proprioceptive or vestibular feedback.

It has been postulated that people can modulate the two stiffness components independently in a task-dependent manner. A popular paradigm used to test this hypothesis was the "maintain position"-"maintain torque" task [2-4] in which subjects were instructed either to maintain a constant torque while subjected to position perturbations or to maintain a constant position while subjected to torque perturbations. However in these studies the experimental environment was different for the two tasks, so that the changes in observed stiffness could not be ascribed to task-dependent reflex changes with certainty.

To obtain a clear answer, the experimental environment must be consistent throughout the experiment, with only the instructions to the subject changed. Since one condition requires the subject to control joint position, the experiment must be run using a compliant load. This results in a closed-loop system because a change in joint position will cause a change in torque due to joint stiffness, and this torque will, in turn, cause a change in the position. This may cause problems for system identification; it is known that when there is strong feedback, applying open-loop analysis to data acquired in closed-loop will result in biased estimates. Indeed, previous work has shown that using open-loop algorithms to estimate joint stiffness under closed-loop conditions does produce biased estimates [5]. Algorithms are being developed to estimate joint stiffness from closed-loop data [6], though their usefulness remains to be demonstrated.

In this paper, we investigate the factors that will result in biased estimates when open-loop analysis is used to measure the joint stiffness of subjects controlling compliant loads. First we demonstrate analytically that, the bias in stiffness estimates obtained using open-loop, non-parametric methods depends on the properties of the load, the power of the noise, and the length of the estimated impulse response functions (IRF). We then show that the open-loop analysis fails when an elastic load is used, but succeeds when an inertial load is used. We then estimate the maximum IRF length and the minimum signal-to-noise ratio needed to limit the effects of the feedback on the analysis. Finally, given the dynamics of the joint stiffness and the typical signal-to-noise ratio, we estimate the minimum inertia needed to produce unbiased estimates of ankle stiffness is 1 $kgm^2$.

## II. Analytical Studies

Figure 1 shows the system of interest; $S$ is the joint stiffness frequency response, $L$ is the frequency response of the load, $u$ is the perturbation, $x$ is the position, $y$ is the torque, and $v$ is additive noise. This analysis is being done in

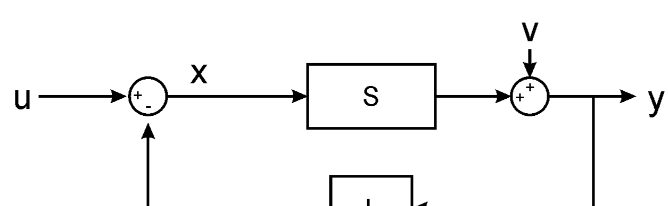


Figure 1. Schematic diagram of the closed-loop system. S is the joint stiffness, L is the inertial load, x is the position, y is the torque, u is the perturbation signal and v is the noise.

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the frequency-domain ($j\omega$), thus the signals $u$, $x$, $y$ and $v$ are all Fourier transforms of the time-domain signals. The objective is to estimate $S$ from measurements of the position and torque. This is considered to be a closed-loop identification problem because subjects produce torque in response to a change in position of their joint, and this torque is then applied to a compliant load, which alters the position of the load and the joint. If the load is very stiff, the torque will not alter the position of the joint and the problem can be considered as open-loop.

For the closed loop system [7],

$$\begin{aligned} y(j\omega) &= \frac{S(j\omega)u(j\omega) + v(j\omega)}{1 + S(j\omega)L(j\omega)} \\ x(j\omega) &= \frac{u(j\omega) - L(j\omega)v(j\omega)}{1 + S(j\omega)L(j\omega)} \end{aligned} \tag{1}$$

Under the assumption that the noise is not correlated with the perturbation, the cross-spectrum between $x$ and $y$ and auto-spectrum of $x$ are

$$\begin{aligned} \Phi_{xy}(j\omega) &= \frac{S(j\omega)\Phi_{uu}(j\omega) - L(j\omega)\Phi_{vv}(j\omega)}{\left(1 + S(j\omega)L(j\omega)\right)^2} \\ \Phi_{xx}(j\omega) &= \frac{\Phi_{uu}(j\omega) + L^2(j\omega)\Phi_{vv}(j\omega)}{\left(1 + S(j\omega)L(j\omega)\right)^2} \end{aligned} \tag{2}$$

And the frequency response estimate of the joint stiffness is

$$H(j\omega) = \frac{\Phi_{xy}(j\omega)}{\Phi_{xx}(j\omega)} = \frac{S(j\omega)\Phi_{uu}(j\omega) - L(j\omega)\Phi_{vv}(j\omega)}{\Phi_{uu}(j\omega) + L^2(j\omega)\Phi_{vv}(j\omega)} \tag{3}$$

Therefore if $S\Phi_{uu} \gg L\Phi_{vv}$ and $\Phi_{uu} \gg L^2\Phi_{vv}$ over the relevant range of frequencies, then the estimated frequency response will be unbiased. Thus, the bias in the frequency response estimate will depend on the input and noise spectra, the frequency responses of joint stiffness and the load, and the range of frequencies over which the spectra are estimated. (Note that the range of frequencies over which the spectra are estimated translates to the sample rate and the length of impulse response estimates).

## III. Simulation Studies

### A. Simulation Methods

To examine the effects of using open-loop analysis under closed loop conditions, we ran a series of simulations to determine whether the estimated systems matched the simulated ones.

We simulated both a closed loop and an open loop system (Fig. 1) using Simulink (The Mathworks Inc.) for 60 s with a sampling rate of 1 kHz. A pseudo-random binary sequence (PRBS) with a switching rate of 125 ms was used as the perturbation signal, and white noise filtered at 3 Hz was used as noise. The amplitude of the noise was set to generate a resultant signal-to-noise ratio (SNR) between 0 and 30 dB. Different loads were examined; for closed-loop experiments the load was modeled as either an elastic load, which consisted of proportional gain of 0.01, or as an inertial load ($\frac{1}{Is^2}$), where the inertia ($I$) varied amongst simulations. For open-loop experiments the load was set to 0, corresponding to an infinite inertia. Joint stiffness was modeled as a parallel-cascade system as described in [8]. The intrinsic stiffness was modeled with the transfer function

$$0.013s^2 + .81s + 80 \tag{4}$$

where $s$ is the Laplace variable. The reflex pathway was simulated as a differentiator, a delay of 60 ms, a half-wave rectifier, followed by the low-pass transfer function

$$\frac{3200}{s^2 + 49.2s + 400} \tag{5}$$

The parameter values for the intrinsic and reflex stiffness were taken from recent experiments in our lab.

Intrinsic and reflex stiffness were estimated using the parallel-cascade identification algorithm described in [8]. This yielded impulse response functions (IRFs) describing intrinsic and reflex dynamics that were compared to those of the simulation model. The quality of the estimated IRF (QF) was measured in terms of the percentage of the ideal IRF variance accounted for by the estimated IRF.

### B. Simulation Results

#### 1) Load Type

Figure 2 compares the modeled intrinsic compliance and reflex stiffness IRFs with those estimated from a closed-loop simulation with an elastic load and 10 dB SNR. The intrinsic compliance IRFs was slightly biased (Fig. 2a) with a QF of 85%. However, the reflex stiffness IRF estimate (Fig. 2b) was very poor with a QF of 0%. Clearly, the open-loop algorithm fails with an elastic load and an average SNR.

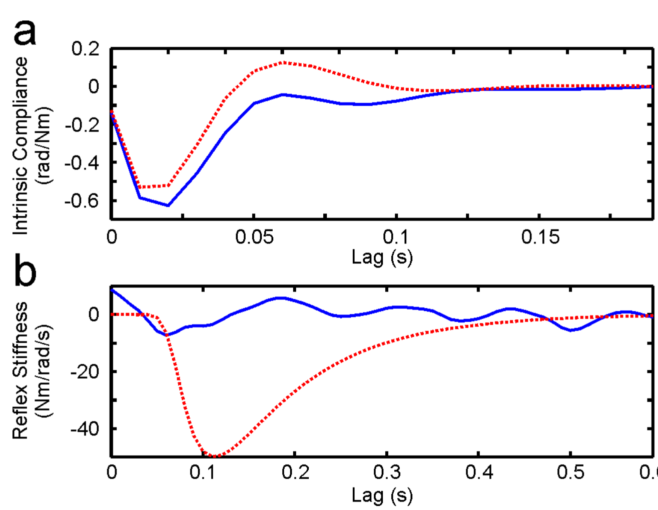


Figure 2. a) Intrinsic compliance and b) reflex stiffness estimated in closed-loop (blue-solid) simulations with an elastic load, compared to the theoretical IRFs (red-dotted).

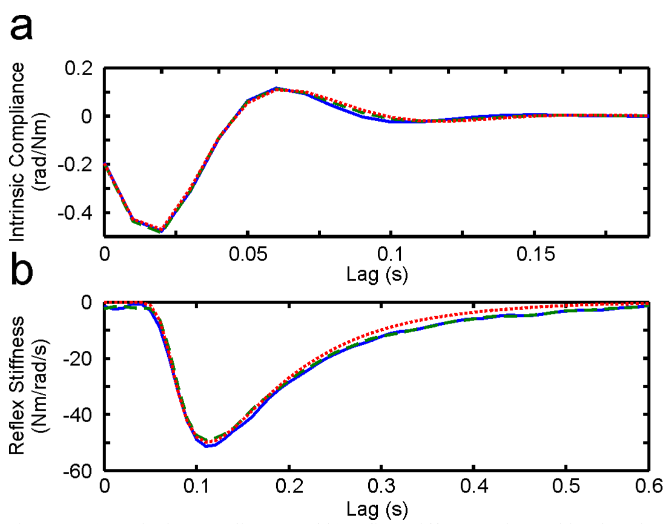


Figure 3. a) Intrinsic compliance and b) reflex stiffness estimated in closed-loop (blue-solid) with an inertial load and open-loop (green-dashed) simulations, compared to the theoretical IRFs (red-dotted). The IRFs were estimated from simulations where the SNR was 10 dB.

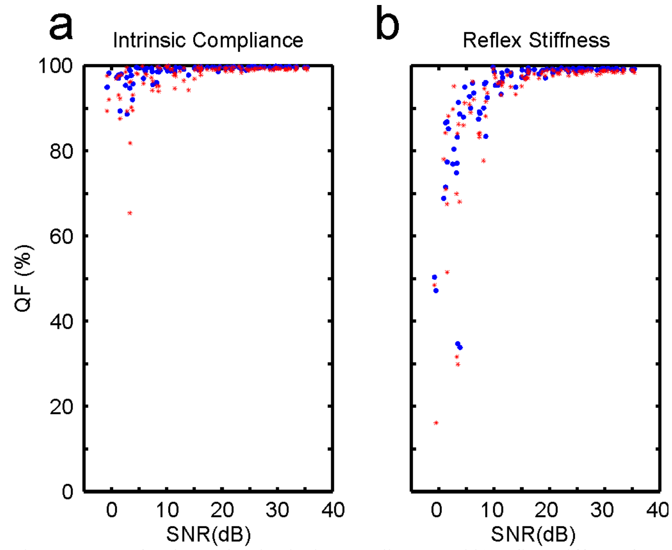


Figure 5. QF of estimated a) intrinsic compliance and b) reflex stiffness for both closed-loop (●) and open-loop (*) simulations as a function of SNR. The inertia was maintained at 100 kgm$^2$ for all simulations.

Figure 3a compares the modeled intrinsic compliance IRF with those estimated from the open-loop and closed loop simulations with an inertial load (I = 100 kgm$^2$). The closed-loop IRF matched the ideal IRF as well as the open-loop IRF (closed-loop QF = 99.5%, open-loop QF = 99.9%.) Figure 3b compares the reflex stiffness IRFs; the closed-loop and open-loop IRFs both matched the ideal closely (closed-loop QF = 99.8%, open-loop QF = 99.4%).

To determine why the parallel-cascade identification algorithm works with the inertial load and not the elastic load, we examined the power spectra from the previous simulation. Figure 4a shows that $S\Phi_{uu}$ was at least 100 times greater than $L\Phi_{vv}$ at all frequencies and at least 1000 times greater at frequencies above 0.3 Hz. Similarly, Figure 4b shows that $\Phi_{uu}$ was at least 100 times greater than $L^2\Phi_{vv}$ at frequencies above 0.3 Hz and at least 1000 times greater at frequencies above 0.7 Hz. Thus, limiting the analysis to frequencies above 0.3 Hz leads to less than 1 % bias, and 0.7 leads to 0.1% bias. This translates to IRFs that should be unbiased if their length is less than 3 seconds, or preferably 1.5 s. The IRFs estimated in the parallel-cascade algorithm are no more than 0.6 s long, and so should be unbiased.

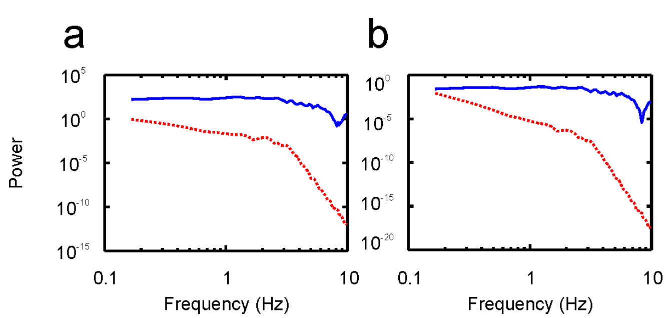


Figure 4. Power spectra of a) $S\Phi_{uu}$ (blue-solid) and $L\Phi_{vv}$ (red-dotted), and b) $\Phi_{uu}$ (blue-solid) and $L\Phi_{vv}$ (red-dotted).

*2) Signal-to-Noise Ratio*

Figure 5a shows the QF for the intrinsic compliance estimated in both open and closed-loop cases for a range of SNRs. The closed-loop IRFs fit the ideal IRF for all SNRs. Similarly, Figure 5b shows that the reflex stiffness IRFs estimated under closed-loop conditions matched the ideal IRF for SNRs above 10 dB SNR. Furthermore the QF of the closed-loop IRF was similar to the QF of the open-loop case at all SNR levels. This suggests the poor QF at low SNRs was due to the effect of the noise on the identification and not due to the bias introduced by the closed-loop.

*3) Inertia*

Figure 6 shows the QF for the intrinsic compliance and reflex stiffness when the inertia was varied. The parallel-cascade produced unbiased estimates of the joint stiffness component provided the inertia was greater than 1 kgm$^2$.

Thus we conclude that parallel-cascade algorithm estimates intrinsic and reflex stiffness in a closed-loop model as well as it does in an open-loop model, provided that the load is an inertia greater than 1 kgm$^2$.

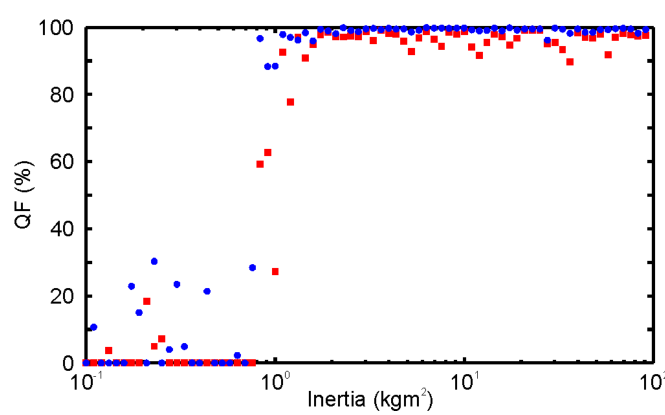


Figure 6. QF of estimated intrinsic compliance (●) and reflex stiffness (■) as a function of the inertia of the load. The SNR was 10 db for all simulations.

## IV. Discussion

Numerous studies have investigated joint stiffness in different contexts using the parallel-cascade algorithm to estimate intrinsic and reflex stiffness [8-11]. This algorithm was designed to estimate joint stiffness under open-loop conditions, i.e. when the position is controlled independently of the torque. As a result, the algorithm cannot be used when performing experiments with compliant loads. This rules out using the algorithm to estimate joint stiffness in some functional tasks such as a "maintain-position"-"maintain torque" task.

This paper explored the effects of using open-loop algorithms with closed-loop data. First, we showed analytically that the accuracy of the results of the open-loop analysis will depend on the properties of the load, the power of the noise, and the length of the estimated IRFs. We then demonstrated that using an elastic load, as was done in [12], resulted in an estimate of intrinsic compliance that was slightly biased, but an estimate of reflex stiffness that was very badly biased. However, when the load was a large inertia, the parallel-cascade algorithm estimated both joint stiffness components accurately.

We believe the algorithm can successfully estimate joint stiffness with an inertial load because the inertia suppresses the high frequency components. Thus, at high frequency the feedback is very weak whereas at lower frequencies it is very strong. By using short IRFs, we are focusing only on the higher frequency components and therefore eliminating the effects of the feedback. If this is the case, then not only will an inertia work as the load, but any load that suppresses high-frequency components adequately.

Since the quality of the estimates produced by the parallel-cascade algorithm depends on three factors, testing the limits of all three factors at once was not feasible. Instead, we fixed two of the factors and tested the third. Based on the dynamics of the joint stiffness and the SNR found in previous studies [8-11] we determined that: the IRFs must be limited to a maximum length of 3 s and preferably 1.5 s; that intrinsic compliance was properly estimated at all noise levels; and that reflex stiffness was estimated properly for SNRs above 10 dB. Furthermore, we found that reflex stiffness was estimated as accurately in the closed-loop case as in the open-loop case for all SNRs. At low SNRs both the open-loop and closed-loop estimates were in error. However, the QFs were similar in both open and closed-loop cases suggesting that the problem is due to the noise not the closed-loop effects.

The quality of the estimates depends on both the length of the estimated IRFs and the SNR, neither of which is under the experimenter's control. However the inertia of the load is adjustable. We found that algorithm successfully estimated joint stiffness when the inertia was greater than 1 $kgm^2$. This is much greater than that of the foot (~ 0.01 $kgm^2$) [9] but much less than that of the body (~ 80 $kgm^2$) [13]. Consequently, the open-loop, parallel-cascade algorithm could be appropriate for measuring joint stiffness in posture studies where subjects control body position, but is not appropriate for studies of free movement where subjects control foot position.

Despite the clear benefit of using an inertial load from an identification viewpoint there is also a drawback. Large inertial loads are very sluggish, so subjects cannot perform rapid movements.

Upright stance is commonly modeled as an inverted pendulum which includes an inertial load and a destabilizing force due to gravity. As a result the inverted pendulum is inherently unstable, and so may require changes in joint stiffness to achieve stability. The inverted pendulum model is clearly of great interest and the analysis of this more complex case is currently on-going.

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