31st Annual International Conference of the IEEE EMBS
Minneapolis, Minnesota, USA, September 2-6, 2009

A Comparison of Linear and Chaotic Measures for Rat Hippocampal EEG during Different Vigilance States

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Abstract—**The correlation dimension was used in this paper as a quantifier to describe the chaotic behavior of sleep EEG recorded from the hippocampus of adult rats during vigilance states of quiet-waking, slow-wave sleep, and REM sleep. A modified Grassberger-Procaccia method was implemented to compute the correlation integral using a Euclidean distance normalized by the embedding dimension. The performance of the correlation dimension as a measure to characterize the sleep EEG was compared to the quantitative measures derived from linear autoregressive models. Even though linear and chaotic measures are based on completely different theories and concepts, our experimental results have indicated them both effective in capturing the characteristic differences of sleep EEG during various states. The preliminary results have also shown the correlation dimension being particularly effective in emphasizing the differences in regard to the chaotic behavior between the EEG activity in SWS and QW and REM sleep.**

I. Introduction

Nonlinear and chaotic signal analysis techniques [1]-[3] have gained much popularity recently in many applications because of its ability to gain valuable information, otherwise impossible with traditional linear analysis approaches [4]-[5]. Nonlinear and chaotic behavior commonly appears in a wide variety of physical signals, including biological events [6]-[11]. Nonlinear dynamics studies open a door to new theoretical and conceptual interpretations and more understanding of many complex systems of interest [1]-[3]. Application examples of nonlinear dynamics and chaos are abundant in the literature [6]-[11]. Although nonlinear and chaotic analysis tools are equipped with unique signal processing advantages not shared by traditional linear analysis approaches, they are still relatively new in concept and demand a much steeper learning curve and heavy computation.

In EEG analysis [12], power spectral analysis [13]-[15] has been frequently utilized to provide valuable quantitative measures such as the peak frequency and the bandwidth. Sleep consists of different vigilance states; each of these states is distinct in regard to the correlation between EEG activity and the underlying neuro-anatomical and neurochemical mechanism [14]. Quantitative measures derived from spectral analysis are available in the literature [4]-[5], [15]-[17]. While the answer to the question, "Whether there exists hidden and crucial information of the EEG activity that cannot be retrieved, or ignored, by the linear spectral analysis?" is a straightforward "yes," a remaining, not yet fully answered, question that also stirs up more research endeavors is "How?"

Linear model based signal processing approaches have proven computationally efficient and are able to extract valuable information hidden in data measurements. However, the fundamental assumption of all linear models excludes the possibility of observing any nonlinear behavior hidden within. Nonlinear model based signal processing approaches have added new understanding with regard to the composition of the EEG [5], [16]-[17]. On the other hand, a common premise is that certain complex nonlinear behaviors such as the strange attractor can be effectively delineated in the phase space [10]. The correlation dimension is one such quantifier measure of chaos that examines the geometric aspect of strange attractors and provides an estimate of the fractal microstructure dimension and the Grassberger and Procaccia (GP) method was suggested [18].

In this paper, the correlation dimension was used to characterize the hippocampal EEG measured from adult rats during the vigilance states of quiet-waking (QW), slow-wave sleep (SWS) and REM sleep. In particular, a modified GP method was implemented to compute the correlation integrals $C(\tau)$ for varying embedding dimensions. By definition, the correlation integral function $C(\tau)$ is not a function of the embedding dimension. It was shown in [19] that correlation integrals estimated by mGP were less sensitive to the varying embedding dimension.

To assess the performance of the chaotic measure of correlation dimension in EEG analysis, results obtained from a linear model analysis were compared. The power spectra of QW, SWS, and REM sleep EEG were also computed through autoregressive (AR) modeling; the measures of the peak frequency and associated bandwidth were estimated using a second order AR model [5]. Thirty two hippocampal EEG epochs were used for each vigilance state. The statistics of both linear and nonlinear measures were calculated for comparison. The results show that both linear and chaotic measures are effective in characterizing the behavioral difference from two conceptually different angles of the EEG activities during different vigilance states.

Manuscript received April 23, 2009.

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II. METHODS

A. Correlation Dimension Estimation

In chaotic studies, the phase space trajectory is a graphical display of the evolution of a dynamic system over time. Some properties of the dynamical system can be accurately derived from the trajectory. One frequently adopted measure from the trajectory of a strange attractor is the fractal dimension. Grassberger and Procaccia [18] recognized that the computation of the fractal dimension is non-trivial and proposed an alternative that only requires the Euclidean distances from a point being computed. Their approach led to the formula of the *correlation integral* described below,

$$C(\tau) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \Theta(\tau - \|\vec{v}(i) - \vec{v}(j)\|), \qquad (1)$$

where ||.|| represents the Euclidean distance between two different state vectors reconstructed in phase space using Takens' time-delay embedding method shown in (2) and Θ is the Heaviside function.

$$\vec{v}(i) = [x(i), x(i+L), \ldots x(i+(m-1)L)]. \qquad (2)$$

L is the delay time (lag) and m is the embedding dimension. The value of L is often chosen to produce a state vector with independent or uncorrelated entries. Computation of $C(\tau)$ in (1) is to be repeated for an increasing radius τ by small steps (Δτ) until $C(\tau)$ converges. The computational load for $C(\tau)$ is very heavy and will increase if smaller step sizes are used or a larger range of τ is to be covered.

Because of the time-delay embedding in (2), the Euclidean distance value between two state vectors also increases with an increasing embedding dimension (m). To compensate for this undesired situation caused by a varying embedding dimension, a modified G-P method (mGP) as proposed to normalize the calculated Euclidean distance by the embedding dimension as follows,

$$\|\vec{v}(i) - \vec{v}(j)\| = \sqrt{\frac{\sum_{k=0}^{m-1}[x(i+kL) - x(j+kL)]^2}{m}}. \qquad (3)$$

Through the normalization, correlation integral $C(\tau)$ is expected to be less sensitive to the varying embedding dimension. It should be noted that there are *norms* other than the Euclidean norm, such as the maximum norm (or uniform norm) that can be used in (2) to measure the distance between two different state vectors.

The correlation dimension (D_2) is typically estimated as the slope of an identified linear scaling region in the log $C(\tau)$ versus log(τ) plot [1], [5], [12], i.e.,

$$D_2 = \lim_{d\tau \to \infty} \frac{\log C(d\tau)}{\log(d\tau)}. \qquad (4)$$

We report in this paper the results of using this normalized Euclidean norm in (3) for the computation of $C(\tau)$ and the estimation of D_2.

B. Power Spectrum Using AR modeling

The Burg AR modeling algorithm [4] was used in this paper to calculate AR model coefficients and power spectra for all vigilance states. The Burg method calculates the AR coefficients by minimizing the forward and backward prediction errors. In this paper, we used a second order AR model to extract two quantitative features of a power spectrum, i.e., the peak frequency and the bandwidth of an underlying EEG epoch. The two AR model coefficients $\{a_2, a_1\}$ were used to determine a pair of complex conjugate poles and the corresponding frequency. The peak frequency (Hertz) was determined by the sampling frequency (f_s) as follows [5]

$$peak\_freq = \frac{f_s}{2\pi} * \tan^{-1}\left(\sqrt{4a_2 - a_1^2}/a_1\right). \qquad (5)$$

The bandwidth at the peak frequency of the AR model based power spectrum is determined by the magnitude of the pole location. Burg's algorithm generates stable filters and the reflection coefficient (a_2) is less than one. The AR power spectrum has a very narrow bandwidth when the AR model poles are near the unit circle. One can use the magnitude of pole location to gauge the bandwidth of the peak frequency.

III. RESULTS AND DISCUSSION

The EEG epochs examined here were recorded from rats of 45 days of age from the hippocampus. The EEG activity was recorded for several hours. Thirty two 8-second epochs

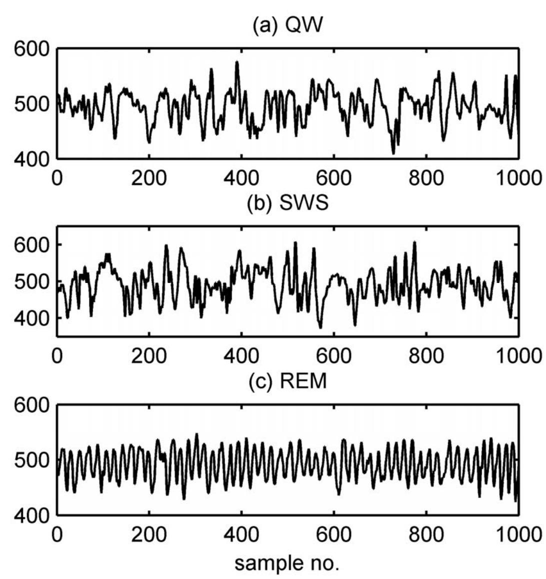

Fig.1 Hippocampal EEG during three vigilance states: (a) quiet-waking, (b) slow-wave sleep, (c) REM sleep.

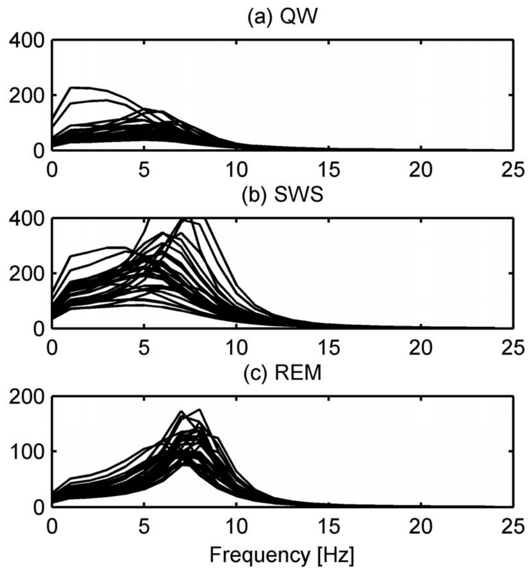

Fig.2 AR power spectra of hippocampal EEG during (a) quiet-waking, (b) slow-wave sleep, (c) REM sleep.

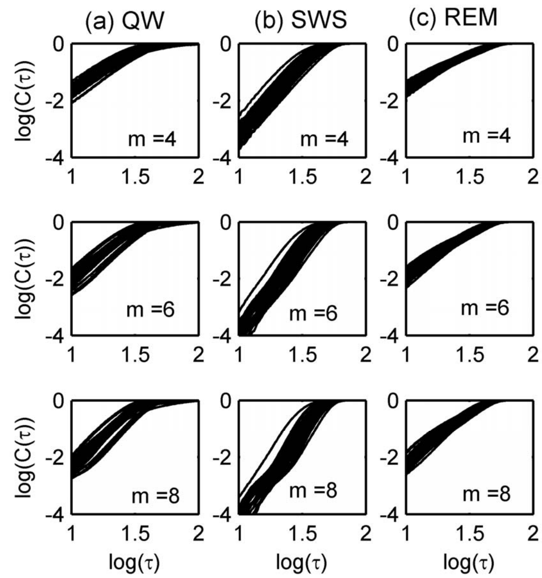

Fig.3 Correlation integrals of hippocampal EEG during (a) quiet-waking, (b) slow-wave sleep, (c) REM sleep.

were visually selected for QW, SWS, and REM sleep. An example of an 8-second EEG epoch of each vigilance state is shown in Fig.1. A second order AR model was used for each 8-sec epoch. The resultant AR model coefficients were used to estimate the AR power spectrum and extract the measures of *peak frequency* and associated bandwidth. Power spectra of 32 8-sec epochs of QW, SWS, and REM sleep are displayed in Fig.2.

The overlapped power spectra of the hippocampal EEG during REM sleep (Fig.2c) show a narrower bandwidth centered at a relatively higher peak frequency (near 7.5 Hz) than QW and SWS. The peak frequencies of QW and SWS are lower (5.8 and 6.2 Hz, respectively) than REM sleep, and the spread of the energy at peak frequencies are also broader than REM sleep. The linear measures derived from AR modeling are summarized in Table I. The differences between means, using a two-tailed student's t-test, lead to *p*-values of 0.0412 for QW vs. SWS, and <0.0001 for QW vs. REM and SWS vs. REM, respectively.

The hippocampal EEG epochs of the three vigilance states were used to compute the correlation integrals $C(\tau)$ in (1) using the modified Euclidean distance adjusted by the embedding dimension in (3). Without loss of generality, correlation integrals computed using three different embedding dimensions (*m=4, 6*, and *8*) were calculated and are shown in Fig.3. To assist comparison, correlation integrals of different embedding dimensions in all three vigilance states are displayed using the same axis range. The differences of $C(\tau)$ between SWS and REM are apparent—correlation integrals of REM converge to zero with significantly lower slopes than SWS.

TABLE I
LINEAR AND NONLINEAR MEASURES OF HIPPOCAMPAL EEG DURING THREE VIGILANCE STATES

Vigilance State	Measures	Mean ± STD
QW	Freq	5.873±0.754
	Pole	0.856±0.025
	D2 (m=6)	2.901±0.430
	D2 (m=8)	3.495±0.597
SWS	Freq	6.267±0.701
	Pole	0.866±0.028
	D2 (m=6)	4.106±0.312
	D2 (m=8)	4.684±0.453
REM	Freq	7.517±0.561
	Pole	0.913±0.012
	D2 (m=6)	2.524±0.337
	D2 (m=8)	3.009±0.432

[1]Freq is the dominant frequency in the EEG epoch. [2]Pole represents the pole magnitude of AR model. [3]D2 is the correlation dimension estimate.

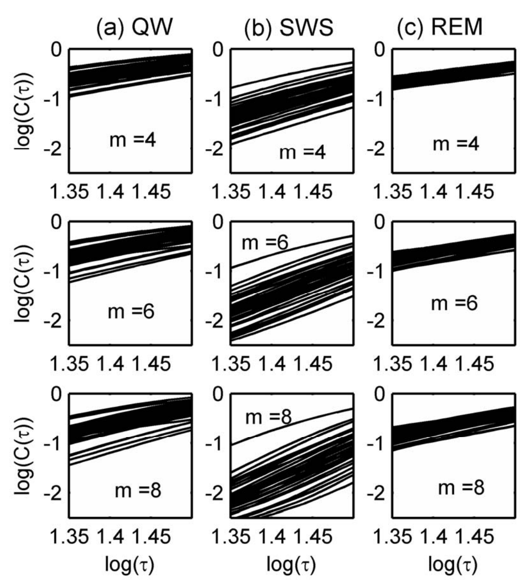

Fig.4 Linear scaling regions of correlation integrals: (a) quiet-waking, (b) slow-wave sleep, (c) REM sleep.

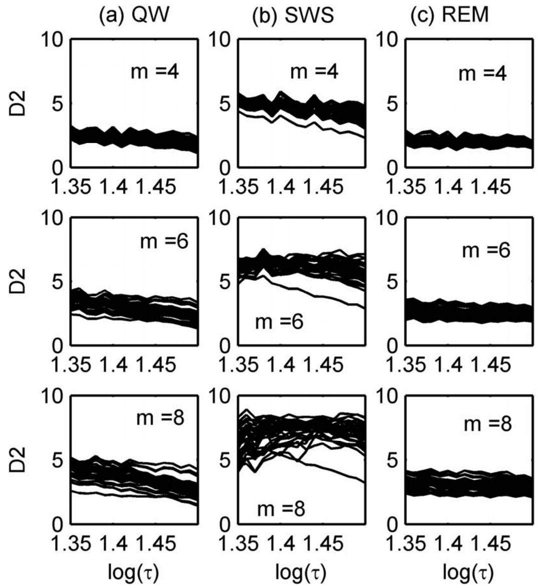

Fig.5 Correlation dimension (D2) estimates: (a) quiet-waking, (b) slow-wave sleep, (c) REM sleep.

Figure 4 depicts a chosen linear scaling region, *log(τ)* in [1.35, 1.5], for all vigilance states. The differences of $C(\tau)$ between SWS and REM (or QW) sleep are more prominent. The slopes for *m=4, 6*, and *8* in SWS (Fig.4b) are steeper than REM (Fig.4c) and QW (Fig.4a). In addition, $C(\tau)$ estimates during REM sleep are clustered tighter than QW and SWS. Slopes of *log C(τ)* versus *log τ* were computed for the chosen linear scaling region. Correlation dimension estimates of QW, SWS, and REM sleep EEG are shown in Fig.5.

Correlation dimension (D_2) estimates in QW (Fig.5a) and REM (Fig.5c) sleep are much smaller than those of SWS (Fig.5b). The statistics of nonlinear measures are given in Table I. It is clear from the numbers in D_2 (*m=6* and *m=8*) that SWS is statistically different from QW and REM sleep. The differences between means of D_2 in SWS and QW (or REM) lead to *p*-values <0.0001 for both *m=6* and *m=8* in a two-tailed student's t-test. Correlation dimension estimates are slightly increased when the embedding dimension was changed from *m=6* to *m=8* in all vigilance states. We found that, if different scaling regions had been selected for different embedding dimensions, correlation dimension estimates became closer.

IV. CONCLUSION

Linear measures derived from AR modeling and power spectral analysis were compared to the chaotic measure of correlation dimension in characterizing the EEG activity of adult rats during the vigilance states of QW, SWS and REM sleep. The results have suggested that the correlation dimension measure is a promising quantifier in EEG analysis and can better reveal the differences between SWS and REM sleep EEG from a different perspective than linear approaches. The following observations were noted through our study:

1. Linear measures derived from AR modeling can effectively delineate the peak frequency of each vigilance state.
2. The bandwidth of the peak frequency is characterized by AR model pole's closeness to the unit circle.
3. The correlation dimension derived from the trajectory of state vectors in the phase space provides a different perspective and effective means to characterize the differences between SWS and QW or REM sleep.

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