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Robust Complete Synchronization of Electrical Coupling Neurons under Uncertain Heterogeneous Disturbances Using Adaptive Internal Model

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Abstract—**An adaptive internal model control strategy is introduced into the robust complete synchronization of two gap-junction coupled FizHugh-Nagumo (FHN) neurons under uncertain heterogeneous disturbances which satisfies some general immersion condition. The synchronization problem can be converted into a robust stabilization problem of an augmented system consisting of the original given plants and an internal model. An adaptive law is employed against uncertain disturbances to make the estimate of internal model to converge to the ideal one. Following a proper state-feedback stabilizer is designed to guarantee the asymptotic stability of the resulting closed-loop system achieved for some appointed initial condition in the state space and for all possible values of the uncertain parameter vector. Finally, the simulation results demonstrate the validity of the proposed method.**

I. Introduction

A number of experimental studies have revealed the presence of electrical coupling via gap junctions in the mammalian brain [1-3]. Indeed, in many cases, electrical interactions via gap junctions have been related to observed synchrony in the dynamics of the underlying network. Recently, many theoretical studies have focused on the problem of to what extent the degree of synchronization can be controlled through the strength of the inter-oscillator coupling [4-5]. However, it has been argued that the transitions from the desynchronized state to the synchronized state are mediated not only by varying the coupling strength but also by changing the external stimulus such as man-made control applied to neurons. It is well known that the complete synchronization (CS) can be observed only in coupled systems with identical elements. In contrast, the exertion of proper external control is helpful to realize CS of the coupled neurons in present of uncertain disturbances. Theoretically, many nonlinear control methods, such as backstepping control [6], adaptive control [7], feedback control [8], sliding mode control [9] and internal model control [10] have been developed to achieve CS of the coupled living neurons.

Specially in [10], we have firstly introduced the internal model idea into the synchronization of two uncoupled homogeneous FHN neurons under periodical disturbances with uncertain amplitudes. However, the precise oscillating frequencies of disturbances must be known to be used for constructing the exact immersion system. In this note, this restriction is weakened by admitting periodical disturbance with not only uncertain amplitudes but also uncertain oscillating frequencies in the presence of two electrical coupling FHN neurons. Firstly, we employ an estimate of internal model to substitute the ideal one. And a state feedback stabilizer is proposed for the augmented system composed by the original coupled systems and the estimative internal model. Then, based on Lyapunov stability analysis, the adaptive tuning law is designed to guarantee estimative internal model to converge well to the ideal one. Finally, the simulation results are given to demonstrate the validity of the proposed method.

II. The Problem Statement

A. Description of a master-slave system

We define a master-slave system with two electrical coupling homogeneous FHN neurons as

$$\begin{aligned}
\frac{dX_M}{dt} &= X_M\left(X_M-1\right)\left(1-rX_M\right)-Y_M-g\left(X_M-X_S\right)+d_M\left(t\right)\\
\frac{dY_M}{dt} &= bX_M\\
\frac{dX_S}{dt} &= X_S\left(X_S-1\right)\left(1-rX_S\right)-Y_S-g\left(X_S-X_M\right)+d_S\left(t\right)+u\\
\frac{dY_S}{dt} &= bX_S
\end{aligned} \tag{1}$$

where (X_i,Y_i) are the state variables and d_i(t) are external uncertain disturbances of the master and slave system ($i=M$, S). $g\geq 0$ is the unknown inter-coupling strength; The master-slaver system is homogeneous with same unknown parameters b and r; u is the exerting control force which makes the dynamic behavior of slave one perfectly tracking that of master one. Here, we consider d_i(t) are periodic perturbations with both unknown amplitude and frequencies.

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B. Error dynamic system

Let e_X=X_S-X_M and e_Y=Y_S-Y_M, then the error dynamical system of (1) can be described as

$$\begin{aligned}\dot{e}_X &= f\left(e_X, e_Y, X_M, r\right) - d\left(t\right) + u \\ \dot{e}_Y &= b e_X\end{aligned} \tag{2}$$

where

$$f\left(e_X, e_Y, X_M, r\right) = X_S\left(X_S - 1\right)\left(1 - rX_S\right) - X_M\left(X_M - 1\right)\left(1 - rX_M\right) - 2ge_X - e_Y$$

and d(t)= d_M(t)-d_S(t). So our purpose is to solve the problem of robust CS of (1) by exerting u(t) when $d_M(t) \neq d_S(t)$.

III. Design of Robust Feedback Stabilizer with Estimative Internal Model

Utilizing the new state variable z_1=e_Y and $z_2 = \dot{e}_Y = be_X$, the error dynamic system (2) can be feedback linearized as the following system

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= bf_z\left(z_1, z_2, X_M, r\right) - bd\left(t\right) + bu\end{aligned} \tag{3}$$

where $f_z\left(z_1, z_2, X_M, r\right) = f\left(e_X, e_Y, X_M, r\right)$. Rewrite (4) as

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\left(bf_z\left(z_1, z_2, X_M, r\right) - bd\left(t\right) + bu\right) \tag{4}$$

with $\mathbf{z}=[z_1, z_2]^T$, $\mathbf{A}$=[0,1; 0,0] and $\mathbf{B}=[0, 1]^T$.

A. Ideal linear internal model of coupled FHN neurons

From (2), it can be verified that $f_z(0,0,X_M,r)=f(0,0,X_M,r)=0$ so that steady controller of (3) is $\bar{u}\left(t\right) = d\left(t\right)$. To construct a steady generator for (4), we introduce an immersion Assumption with regard to d(t).

***Assumption 1*:** There exists a number $q \in \mathbf{N}$ and a set of real numbers c_0, c_1,…, c_{q-1}, independent of the unknown system parameters of (1), such that $d(t)$ satisfies

$$d^{(q)} + c_{q-1}d^{(q-1)} + c_2 d^{(2)} + \cdots + c_1 d^{(1)} + c_0 d \equiv 0 \tag{5}$$

where $d^{(1)} = \dfrac{dd\left(t\right)}{dt}$ and $d^{(i)} = \dfrac{dd^{(i-1)}}{dt}$, ($i$=2, …,$q$), and the characteristic polynomial $\lambda^q + c_{q-1}\lambda^{q-1} + \cdots + c_1\lambda + c_0$ has distinct roots on the imaginary axis.

***Remark 1*:** In [10], we have solve the exactly known case for c_0, c_1,…, c_{q-1} by construct an ideal internal model. Here we suppose that these numbers may be unknown and an adaptive internal model is constructed in the following. By defining

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -c_0 & -c_1 & -c_2 & \cdots & -c_{q-1} \end{bmatrix} \in \mathbf{R}^{q\times q}, \quad \boldsymbol{\sigma} = \begin{bmatrix} d\left(t\right) \\ d^{(1)} \\ \vdots \\ d^{(q-1)} \\ d^{(q)} \end{bmatrix} \in \mathbf{R}^q$$

and $\boldsymbol{\Gamma} = [1, 0, \cdots, 0] \in \mathbf{R}^{1\times q}$, Assumption 1 implies existence of *a linear steady generator* of (4) with output d(t) as

$$\begin{aligned}\dot{\boldsymbol{\sigma}} &= \mathbf{S}\boldsymbol{\sigma} \\ d\left(t\right) &= \boldsymbol{\Gamma}\boldsymbol{\sigma}\end{aligned} \tag{6}$$

***Lemma 1*:** Given any known *Hurwitz matrix* $\mathbf{A}_\sigma \in \mathbf{R}^{q\times q}$ and any vector $\mathbf{J} \in \mathbf{R}^q$ such that the pair $\left(\mathbf{A}_\sigma, \mathbf{J}\right)$ is controllable, the Sylvester equation $\mathbf{M}_\sigma\mathbf{S} - \mathbf{A}_\sigma\mathbf{M}_\sigma = \mathbf{J}\boldsymbol{\Gamma}$ has a unique solution $\mathbf{M}_\sigma \in \mathbf{R}^{q\times q}$, which is non singular.

It can be seen that $\mathbf{M}_\sigma$ is unknown if $\mathbf{S}$ is unknown. Indeed $\mathbf{M}_\sigma\mathbf{S}\mathbf{M}_\sigma^{-1} = \mathbf{A}_\sigma + \mathbf{J}\mathbf{K}_\sigma$, where $\mathbf{K}_\sigma = \boldsymbol{\Gamma}\mathbf{M}_\sigma^{-1}$. Therefore, $\mathbf{S}$ is similar to $\mathbf{A}_\sigma + \mathbf{J}\mathbf{K}_\sigma$. Note that, since $\left(\mathbf{A}_\sigma, \mathbf{J}\right)$ is controllable, and $\mathbf{J}$ has just one column, the row vector $\mathbf{K}_\sigma$ is precisely the unique solution to the problem of assigning the poles of $\mathbf{S}$ to $\mathbf{A}_\sigma + \mathbf{J}\mathbf{K}_\sigma$. Thus, (6) can be immersed into the following linear autonomous system

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= \left(\mathbf{A}_\sigma + \mathbf{J}\mathbf{K}_\sigma\right)\boldsymbol{\eta} \\ y &= \mathbf{K}_\sigma\boldsymbol{\eta}\end{aligned} \tag{7}$$

where $\boldsymbol{\eta} \in \mathbf{R}^q$ and $\mathbf{K}_\sigma$ may be unkown.

***Remark 2*:** Specifically, the immersion map from (6) to (7) is given by $\boldsymbol{\eta} = \mathbf{M}_\sigma\boldsymbol{\sigma}$, which satisfies $d\left(t\right) = \mathbf{K}_\sigma\boldsymbol{\eta}$. This relation plays a crucial role in the sequel for adaptive internal model, as relation $d\left(t\right) = \boldsymbol{\Gamma}\boldsymbol{\sigma}$ does in the non-adaptive case.

According to the canonical internal model formulated in (9), we choose a linear internal model of coupled FHN neurons system (1) as the form

$$\dot{\boldsymbol{\xi}} = \left(\mathbf{A}_\sigma + \mathbf{J}\mathbf{K}_\sigma\right)\boldsymbol{\xi} + \mathbf{J}v\left(z, t\right) \tag{8}$$

where $\boldsymbol{\xi} \in \mathbf{R}^q$, v(z,t) is called added disturbance of control and chosen to satisfy that v(z,t)=0, as we will soon show.

B. Estimative internal model with state-feedback stabilizer

Owing to $\mathbf{K}_\sigma$ may unknown, the ideal internal model (8) for the coupled system (1) may not be used directly to construct the controller. Thus, we need to employ a method to asymptotically estimate the ideal internal model (8) as well. We replace $\mathbf{K}_\sigma$ in (8) with an estimates $\hat{\mathbf{K}}_\sigma$, governed by an adaptive tuning law of the kind $\dot{\hat{\mathbf{K}}}_\sigma = \boldsymbol{\Psi}\left(\boldsymbol{\xi}, z_a\right)$ which will be shown next. Thus, the estimate (8) is obtained as

$$\dot{\boldsymbol{\xi}} = \left(\mathbf{A}_\sigma + \mathbf{J}\hat{\mathbf{K}}_\sigma\right)\boldsymbol{\xi} + \mathbf{J}v\left(z, t\right) \tag{9}$$

Here we stress that $\boldsymbol{\xi}$ is available for feedback. And we choose the augmented error of the form $z_a = z_2 + k_a z_1$, where k_a>0 is a number yet to be determined. Under this new state variable, (4) can be translated as

$$\begin{aligned}\dot{z}_1 &= -k_a z_1 + z_a \\ \dot{z}_a &= f_{za}\left(z_1, z_a, X_M, r, k_a\right) + b\left(u - \bar{u}\right)\end{aligned} \tag{10}$$

where $f_{za}\left(z_1, z_a, X_M, r, k_a\right) = bf_z\left(z_1, z_2, X_M, r\right) + k_a z_2$. It is easy to verify that $f_{za}\left(0, 0, X_M, r, k_a\right) = 0$. Then, we design the steady feedback stabilizer for the augmented system composed by (9) and (10) as the form

$$u = \hat{\mathbf{K}}_\sigma\boldsymbol{\xi} + v\left(z, t\right) \tag{11}$$

with $v(z,t) = -k_v z_a$, where k_v>0 is adjustable parameter.

C. Asymptotic stability analysis of the closed-loop system and adaptive regulation

Defining the new error variable $\mathbf{\Phi} = \mathbf{\xi} - \mathbf{\eta} - \frac{1}{b}\mathbf{J} z_a$ and $\tilde{\mathbf{K}}_{\sigma} = \hat{\mathbf{K}}_{\sigma} - \mathbf{K}_{\sigma}$, the resulting closed-loop can be viewed as

$$
\begin{aligned}
&\dot{\mathbf{\Phi}} = \mathbf{A}_{\sigma}\mathbf{\Phi} + \frac{1}{b}\left(\mathbf{A}_{\sigma}\mathbf{J} z_a - \mathbf{J} f_{za}\left(z_1, z_a, X_M, r, k_a\right)\right) \\
&\dot{z}_1 = -k_a z_1 + z_a \\
&\dot{z}_a = f_{za}\left(z_1, z_a, X_M, r, k_a\right) + b\mathbf{K}_{\sigma}\mathbf{\Phi} \\
&\qquad + \left(\mathbf{K}_{\sigma}\mathbf{J} - b k_v\right) z_a + b\tilde{\mathbf{K}}_{\sigma}\mathbf{\xi} \\
&\dot{\tilde{\mathbf{K}}}_{\sigma} = \mathbf{\Psi}\left(\mathbf{\xi}, z_a\right)
\end{aligned}
\tag{12}
$$

which has a zero dynamics given by

$$
\begin{aligned}
&\dot{\mathbf{\Phi}} = \mathbf{A}_{\sigma}\mathbf{\Phi} - \frac{1}{b}\mathbf{J} f_{za}\left(z_1, 0, X_M, r, k_a\right) \\
&\dot{z}_1 = -k_a z_1
\end{aligned}
\tag{13}
$$

As $f_{za}\left(0,0,X_M,r,k_a\right) = 0$ and $\mathbf{A}_{\sigma}$ is Hurwitz, the result holds that the cascade system (13) is uniformly semi-global asymptotically stable in parameter k_a. We define a Lyapunov function $V_1\left(\mathbf{\Phi}, z_1\right) \overset{def}{=} \mathbf{\Phi}^T \mathbf{P}_{\mathbf{\Phi}} \mathbf{\Phi} + \frac{1}{2} z_1^2$ where the symmetric positive definite matrices $\mathbf{P}_{\dot{\mathbf{\Phi}}}$ is the solution of $\mathbf{P}_{\mathbf{\Phi}}\mathbf{A}_{\mathbf{\Phi}} + \mathbf{A}_{\mathbf{\Phi}}^T \mathbf{P}_{\mathbf{\Phi}} = -\mathbf{I}$. And define the compact set $\mathbf{\Omega}_{c_1} = \left\{\left(\mathbf{\Phi}, z_1\right) : V_1\left(\mathbf{\Phi}, z_1\right) \le c_1\right\}$ where c_1 is some positive constant. Thus, we have

$$
\dot{V}_1\left(\mathbf{\Phi}, z_1\right) \le -\left\|\mathbf{\Phi}\right\|^2 + \frac{2}{b}\left\|\mathbf{J}\right\|\left\|\mathbf{\Phi}\right\|\left|f_{za}\left(z_1, 0, X_M, r, k_a\right)\right| - k_a z_1^2 \tag{14}
$$

Owing to $f_{za}\left(z_1, 0, X_M, r, k_a\right)$ is locally Lipschitz, According to (14), for any c_1>0, there exist some positive design parameter k_a^* such that $\dot{V}_1\left(\mathbf{\Phi}, z_1\right) \le 0$, which guarantees that the state feedback $v(z,t) = -k_v z_a$ uniformly asymptotically stabilizes the equilibrium ($\mathbf{\Phi}$,z_1,z_a)=(0,0,0) of (12) in the case of $\tilde{\mathbf{K}}_{\sigma} = 0$. The positive define, locally quadratic function $V_2\left(\mathbf{\Phi}, z_1, z_a\right) = V_1\left(\mathbf{\Phi}, z_1\right) + \frac{1}{2} z_a^2$ is such that a number c_2>0 can always be found in such a way that the level set $\mathbf{\Omega}_{c_2} = \left\{\left(\mathbf{\Phi}, z_1, z_a\right) : V_2\left(\mathbf{\Phi}, z_1, z_a\right) \le c_2\right\}$ is compact and contains an arbitrary closed ball of $\mathbf{\Omega}_{c_1}$ in its interior. There exists a positive constant k_v^* and some positive function $\lambda(\cdot)$, locally quadratic around the origin, such that, if k_v is chosen greater than k_v^*, the derivative of $V_2\left(\mathbf{\Phi}, z_1, z_a\right)$ satisfies $\dot{V}_2\left(\mathbf{\Phi}, z_1, z_a\right) \le -\lambda\left(\left\|\left(\mathbf{\Phi}, z_1, z_a\right)\right\|\right)$ for all $\left(\mathbf{\Phi}, z_1, z_a\right) \in \mathbf{\Omega}_{c_2}$. Consider the Lyapunov function candidate $\bar{V}\left(\mathbf{\Phi}, z_1, z_a, \tilde{\mathbf{K}}_{\sigma}\right) = V_2\left(\mathbf{\Phi}, z_1, z_a\right) + \frac{1}{2\delta} b\tilde{\mathbf{K}}_{\sigma}\tilde{\mathbf{K}}_{\sigma}^{\mathbf{T}}$, where δ >0 is a design parameter. Select the level set

$$
\mathbf{\Omega}_{c_3} = \left\{\left(\mathbf{\Phi}, z_1, z_a, \tilde{\mathbf{K}}_{\sigma}\right) : \bar{V}\left(\mathbf{\Phi}, z_1, z_a, \tilde{\mathbf{K}}_{\sigma}\right) \le c_3\right\}
$$

is compact and contains an arbitrary closed ball of $\mathbf{\Omega}_{c_2}$ in its interior. Then, as shown above, values of the value of k_a and k_v can be determined such that the derivative of $\bar{V}$ satisfies

$$
\dot{\bar{V}}\left(\mathbf{\Phi}, z_1, z_a, \tilde{\mathbf{K}}_{\sigma}\right) \le -\lambda\left(\left\|\left(\mathbf{\Phi}, z_1, z_a\right)\right\|\right) + b\tilde{\mathbf{K}}_{\sigma}\left(z_a\mathbf{\xi} + \frac{1}{\delta}\mathbf{\Psi}^{\mathbf{T}}\left(\mathbf{\xi}, z_a\right)\right)
$$

for all $\left(\mathbf{\Phi}, z_1, z_a, \tilde{\mathbf{K}}_{\sigma}\right)$ in $\mathbf{\Omega}_{c_3}$. Obviously choose

$$
\dot{\hat{\mathbf{K}}}_{\sigma} = \mathbf{\Psi}\left(\mathbf{\xi}, z_a\right) = -\delta z_a \mathbf{\xi}^{\mathbf{T}}
$$

such that $\dot{\bar{V}}\left(\mathbf{\Phi}, z_1, z_a, \tilde{\mathbf{K}}_{\sigma}\right) \le -\lambda\left(\left\|\left(\mathbf{\Phi}, z_1, z_a\right)\right\|\right)$, which implies that asymptotic regulation is achieved.

IV. Simulation Results

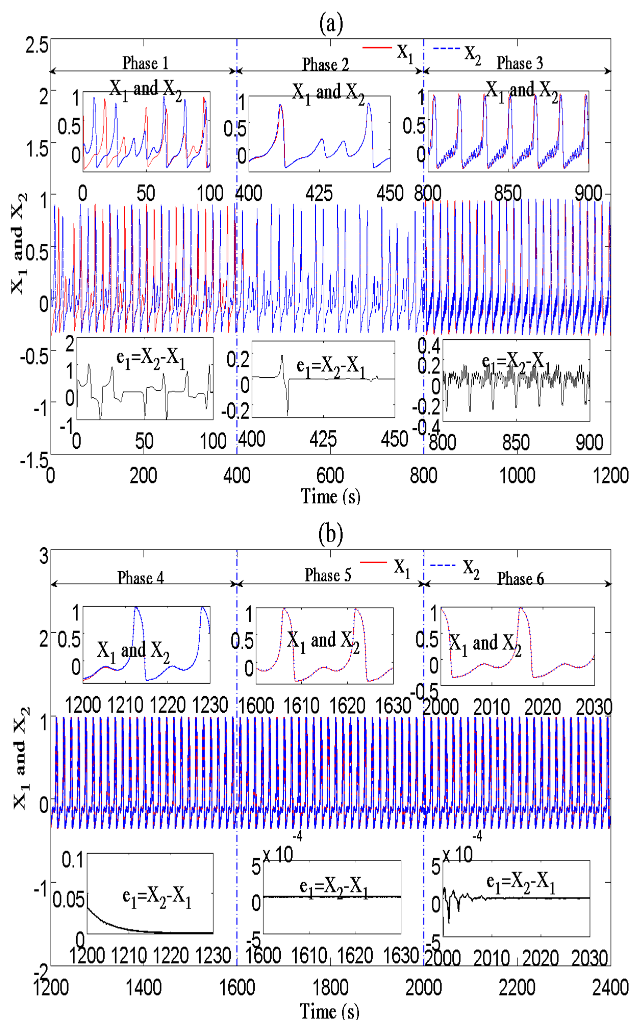

Fig. 1. State trajectories of Master and slave FHN neurons: (a) Phase 1 to 3 without external control; (b) Phase 4 to 6 with exerting the proposed controller

In this section, we consider the external disturbances d_i as the form $d_i=(A_i/\omega_i)\cos(\omega_i t)$ with the uncertain amplitudes A_i and uncertain oscillating frequency $\omega_i=2\pi f_i$. It can be easy to demonstrate that $d(t)$ satisfies Assumption 1 i.e. there exist following constants: $q=4$, $c_0=\omega_M^2\omega_S^2$, $c_1=0$, $c_2=\omega_M^2+\omega_S^2$ and $c_3=0$. Thus a linear steady generator can be written as (6) with $\mathbf{S}$=[0,1,0,0;0,0,1,0;0,0,0,1;-c_0,0,-c_2,0] and $\mathbf{\Gamma}$=[1,0,0,0]. In whole simulations, the fixed control action is implemented with the following parameters: $\mathbf{A}_\sigma$ is Hurwitz with the poles at (-1,-2±j,-3); $\mathbf{J}=[0,0,0,1]^T$; choosing proper control gains as k_a=0.2 and k_v=200; setting adaptive scale parameter $\delta=5\times10^6$. The initial simulation conditions are set as: ($X_M(0)$, $Y_M(0)$)=(1,2); ($X_S(0)$, $Y_S(0)$)=(0,0); $\xi(0)=[0,0,0,0]^T$; $\hat{\mathbf{K}}_\sigma(0)=[0,0,0,0]$; r=10 and b=1. The whole simulation is divided into the six phases: (1) Phase 1(0 to 400s): setting g=0.03, $f_M=f_S$=127.1Hz and $A_M=A_S$=0.1, without exerting external control; (2) Phase 2(400 to 800s): increasing g from 0.03 to 0.1; (3) Phase 3(800 to 1200s): changing external perturbations as f_M=127Hz, f_S=700Hz, A_M=0.3, A_S=2 and increasing g again from 0.1 to 1; (4) Phase 4(1200 to 1600s): introducing the proposed internal model control and turning the adaptation on; (5)Phase 5(1600 to 2000s): turning the adaptation off; (6)Phase 6(2000 to 2400s): changing external perturbation of the slave system again as f_S=530Hz and A_S=0.8, and turning the adaptation on.

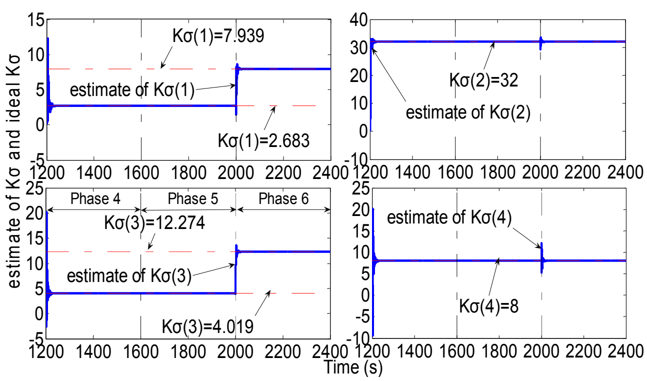

Fig. 2. Evolution of the estimates of $\mathbf{K}_\sigma$ in Phase 4 to 6 (**Note**: to verify the convergence of the estimates of $\mathbf{K}_\sigma$, here we manually compute $\mathbf{K}_\sigma$=[2.683,32,4.019,8] for Phase 4 and 5, and $\mathbf{K}_\sigma$=[7.939,32,12.274,8] for Phase 6 respectively.)

Simulation results are shown in Fig. 1, where (a) and (b) represent the case without external control from phase 1 to 3 and the case with control from phase 4 to 6 respectively. At first, the proposed controller is disconnected, where in Phase 1 different initial states result in the asynchronous behaviors with the weaker inter-coupling strength g=0.03 under homogeneous external disturbances, and the synchronization states are obtained by increasing g from 0.03 to 0.1 in Phase 2, and when there exist heterogeneous perturbations in Phase 3, even though the value of g is increased to ten times of that in Phase 2, the synchronization regulation can not be held on. We noticed a remarkable steady error until the adaptive internal model controller is connected in Phase 4. And Fig. 2 shows the evolution of the estimates $\hat{\mathbf{K}}_\sigma$ from Phase 4 to 6. At the beginning of Phase 5, the adaptation is removed. Remarkably, no error arises, which shows that the adaptive internal model has been tuned to the "ideal" one. Following, at the beginning of Phase 6, when we change the external perturbation of the slaver system again, it can be observed that zero steady state error can be maintained well by turning the adaptation on at the same time, as the internal model is tuned again to the another "ideal" one shown in Fig. 2.

V. Conclusion

Our previous studies in [10] need the complete knowledge of the frequencies of external disturbances. Here we solved the problem of robust CS for two electrically inter-coupling homogeneous FHN neurons under external heterogeneous disturbances with both unknown amplitudes and frequencies using the adaptive internal model control strategy. Under some proper Assumption about the external perturbations, the linear ideal internal model can be constructed for the original system. Then, a state feedback stabilizer is designed for the augmented system composed of the original system and an estimative internal model whose convergence to the ideal one is guaranteed by an adaptive tuning law.

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