31st Annual International Conference of the IEEE EMBS
Minneapolis, Minnesota, USA, September 2-6, 2009

Performance Evaluation of an Algorithm for the Identification of Time-Varying Joint Stiffness

Tanya Starret Visser, *Student Member, IEEE*, Daniel Ludvig, *Student Member, IEEE,* and Robert E. Kearney, *Fellow, IEEE*

Abstract—**Previously, we described a time-varying, parallel-cascade system identification algorithm that estimates intrinsic and reflex stiffness dynamics. It uses an iterative technique, in conjunction with established, time-varying, identification methods, to estimate the two pathways from ensembles of input and output realizations having the same time-varying behavior. This paper presents the results of a study that systematically evaluated the performance of the algorithm. Simulations were used to determine the algorithm's ability to track rapid changes in dynamic stiffness, and quantify its performance limits. There was close agreement between the simulated and estimated joint stiffness demonstrating that the algorithm estimates stiffness correctly even when it changes rapidly. However, the algorithm's ability to identify the reflex pathway was shown to depend on the relative contributions of the intrinsic and reflex pathways to the overall torque. As the intrinsic contribution to the output grew it became increasingly difficult to identify the reflex pathway accurately. The quality of the reflex identification greatly improved as the number of realizations in the data ensembles increased. More realizations were needed as the signal-to-noise ratio decreased and the relative contribution of the reflex pathway decreased. For good results, under typical time-varying experimental conditions, between 500 and 800 realizations are required.**

I. Introduction

Dynamic joint stiffness is defined as the relation between angular joint position and the torque acting about it. Joint stiffness can be separated into two components: an intrinsic component that arises from the mechanical properties of the joint, active muscle, and passive tissue; and a reflex component due to changes in muscle activation in response to the stretch reflex. Joint stiffness plays an important role in the control of movement and posture; it determines the amount of force needed to achieve the desired final position of the limb and the amount of movement that will result from an external perturbation. However, the exact role of the two stiffness components in the control of movement is still uncertain.

Manuscript received April 22, 2009. This work was supported in part by grants from the Canadian Institutes of Health Research (CIHR), and the National Sciences and Engineering Research Council of Canada (NSERC).

T. Starret Visser is with the Department of Biomedical Engineering, McGill University, Montreal QC H3A 2B4 Canada (phone: 514-398-6736; fax: 514-398-7461; e-mail: tanya.starret@mail.mcgill.ca).

D. Ludvig is with the Department of Biomedical Engineering, McGill University, Montreal QC H3A 2B4 Canada (phone: 514-398-6736; fax: 514-398-7461; e-mail: daniel.ludvig@mcgill.ca).

R. E. Kearney is with the Department of Biomedical Engineering, McGill University, Montreal QC H3A 2B4 Canada (phone: 514-398-6736; fax: 514-398-7461; e-mail: robert.kearney@mcgill.ca).

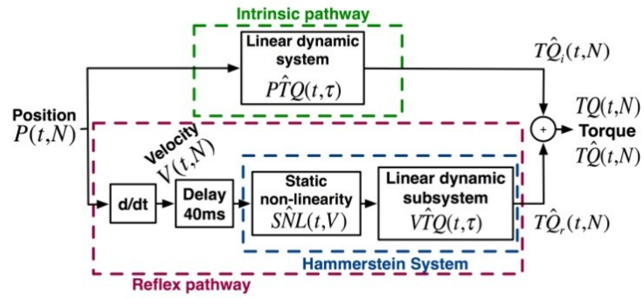

Fig. 1. Block diagram of the time-varying parallel-cascade model of ankle stiffness. The linear systems depend on both time, t, and lag, τ. The static non-linearity is a function of time and velocity. *N* represents the realization number.

Stationary, time-invariant studies of dynamic joint stiffness at the ankle have shown that both intrinsic and reflex stiffness vary greatly with ankle position and the level of voluntary torque [1]. Consequently, joint stiffness will change rapidly during movement and so cannot be measured using the time-invariant identification methods used under stationary conditions. To address this, we have developed a time-varying, parallel-cascade (TVPC) identification algorithm that uses ensemble methods to separate the intrinsic and reflex components of stiffness during time-varying conditions [2-4].

This paper presents the results of a systematic simulation study carried out to evaluate the accuracy with which the algorithm estimates rapid changes in intrinsic and reflex stiffness, and to explore the factors influencing its performance in the presence of noise.

II. Identification Algorithm

Previous studies have shown that dynamic ankle stiffness is described well by the model shown in Figure 1 [5]. The intrinsic pathway relates position to torque by a linear dynamic system with the transfer function $Is^2 + Bs + K$, where I, B, and K are inertial, viscous, and elastic parameters, respectively. The reflex pathway relates velocity to torque through a delay followed by a Hammerstein system, composed of a static non-linearity, similar to a half-wave rectifier, and a linear subsystem, modeled by a 2nd or 3rd order low-pass filter.

The time-varying (TV) identification algorithm uses an ensemble method that estimates the dynamics from multiple realizations of input/output pairs having the same TV behavior. Figure 2 shows a number of input and output realizations from such an ensemble. Since the system may be different at each time, the identification is achieved using

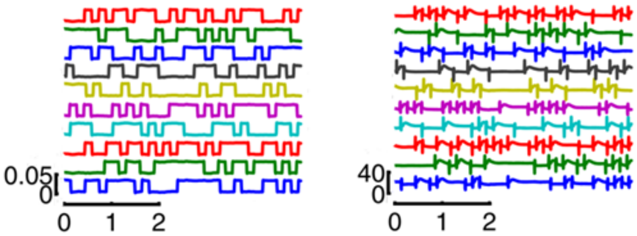

Fig. 2. Example input position (left) and output torque (right) ensembles.

estimates computed across the ensemble, rather than along time. This generates a different IRF for every time point, which is dependent on time, *t,* and lags, τ. The TVPC identification algorithm is described in detail in [4], but is reviewed briefly. It uses an iterative procedure to estimate the two parallel pathways as follows:

1. A first estimate of the TV intrinsic stiffness, $P\hat{T}Q$, is obtained at each time point by estimating the linear impulse response function (IRF) between position, P, and total output torque, TQ, using an algorithm that relates ensembles of input-output cross-correlations and input autocorrelations. Fixing the length of the intrinsic IRFs to less than the reflex delay eliminates any correlation between reflex torque and the input. The intrinsic torque, $T\hat{Q}_i$, is predicted by convolving the input position with the TV $P\hat{T}Q$.
2. The ensemble of intrinsic residuals, $T\hat{Q}_{iR}$, is calculated as $T\hat{Q}_{iR} = TQ - T\hat{Q}_i$ and is used as the output for reflex stiffness identification.
3. The non-linearity, $S\hat{N}L$, and the linear subsystem, $V\hat{T}Q$, of the Hammerstein system are estimated from the velocity and $T\hat{Q}_{iR}$ using an iterative process. $V\hat{T}Q$ is estimated first using a TV correlation method. $V\hat{T}Q$ is then fixed and a polynomial is fit using regression analysis to estimate $S\hat{N}L$. Then, the polynomial is fixed and a new estimate of $V\hat{T}Q$ is generated. This process repeats until the sum of squared errors fails to decrease. Hammerstein system estimates are generated for each sample time.
4. The estimated TV Hammerstein system is used to predict the reflex torque, $T\hat{Q}_r$.
5. $T\hat{Q}_i$ and $T\hat{Q}_r$ are summed to estimate the total output torque, $T\hat{Q}$.
6. The percent variance accounted for (%VAF) is calculated between TQ and $T\hat{Q}$ to quantify the quality of the identification. The %VAF between an observed signal, X, and its estimate $\hat{X}$ is defined as: $\%VAF(X,\hat{X}) = 100\left(1 - \frac{var(X-\hat{X})}{var(X)}\right)$
7. The procedure continues from step 1 by calculating a new estimate of intrinsic stiffness using reflex residuals as the output.

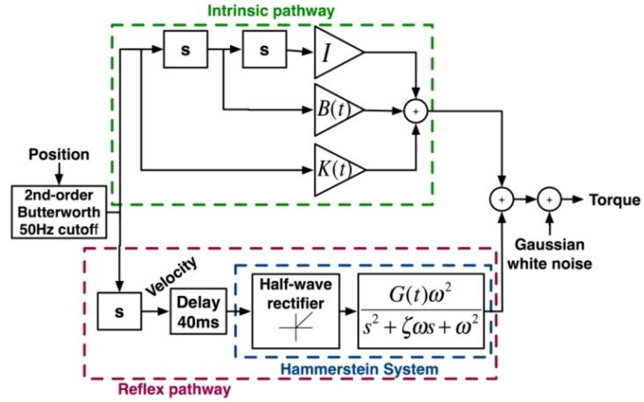

Fig. 3. Simulation model for joint stiffness. The values of *B, K,* and *G* varied with time. All other parameters were held constant.

8. The iteration continues until the %VAF fails to increase.

Following the identification, intrinsic stiffness is converted to compliance, $T\hat{Q}P$, since this is more readily interpreted.

III. Simulation Study

A. Methods

Time-varying dynamic ankle stiffness was simulated with Simulink (The Mathworks inc.) using the model shown in Figure 3. Intrinsic stiffness was represented by inertial, *I*, viscous, *B*, and elastic, *K*, parameters. Reflex stiffness was modeled as a Hammerstein system consisting of a half-wave rectifier followed by a 2nd-order low-pass filter with parameters gain, *G*, damping, ζ, and natural frequency, ω_n. The input was a 0.03 rad pseudo-random binary sequence (PRBS) with a 150ms switching rate, filtered with a 2nd order, low-pass Butterworth filter with a 50Hz cutoff.

B. Identification of rapid, time-varying changes

The goal of the first simulation study was to confirm that the TVPC algorithm could identify rapid, time-varying

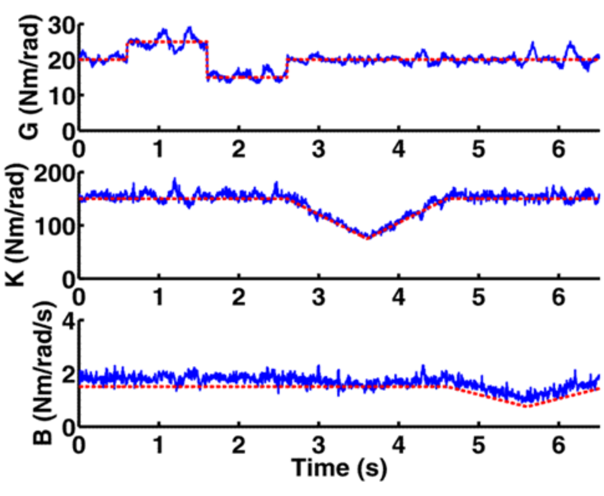

Fig. 4. Parametric fit results. The values of parameters used in the simulations are in red (dotted), and the values obtained by the parametric fits are in blue (solid).

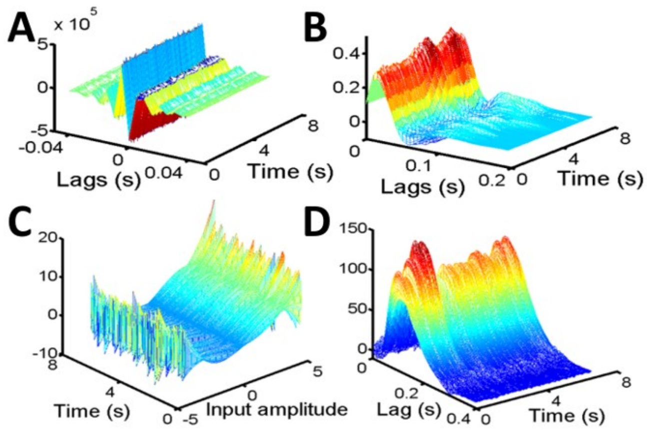

Fig. 5. Results of TVPC algorithm. A: $P\hat{T}Q$. B: $T\hat{Q}P$. C: $S\hat{N}L$. D: $V\hat{T}Q$

changes. Therefore, the intrinsic and reflex parameters were changed with time as shown in Figure 4. G changed stepwise at 3 times, while, K and B changed according to a ramp waveform. There was no additive noise. The simulation was run 800 times using a 1 kHz sampling rate, to generate the input and output ensembles. Note that we will use the term simulation to refer to the set of simulations used to generate the input and output ensembles. Individual input-output pairs will be referred to as realizations.

The time-varying simulation data was decimated to 200 Hz prior to applying the TVPC algorithm. Figure 5 shows the resulting stiffness and compliance estimates. The time-varying changes in stiffness dynamics are evident in $V\hat{T}Q$, and $T\hat{Q}P$. The %VAF between simulated torques and the estimates was high; the time average of the total, intrinsic, and reflex %VAFs, was 98.2%, 98.3%, and 94.1% respectively.

To determine how well the estimated system dynamics corresponded to the models simulated, parametric models were fit to the estimated TV IRFs. These fit the IRF estimates very well; the average %VAF between the parametric fits and the IRF estimates was 96.1% for the intrinsic and 99.7% for the reflex pathway. Moreover, as Figure 4 shows, the values of the parameters estimated for G, K, and B closely followed the simulated values. This demonstrates that the TVPC identification algorithm accurately tracks rapid TV behavior with no *a priori* information about its behavior.

C. Signal-to-Noise Ratio

The goal of the second simulation study was to define the performance limits of the algorithm. Specifically, we wanted to understand the effect the signal-to-noise ratio (SNR) had on the quality of the identification, and what conditions must be met for the identification to succeed. Initial investigation of the TVPC algorithm showed that the quality of the identification was not the same at each point of the TV behavior. For this reason, time-invariant simulations were used to quantify the precise system conditions required for successful identification.

Three parameters were varied between simulations: the noise power, the reflex gain, and the number of realizations in the data ensemble. Three values of reflex gain were used to explore how the relative size of the reflex influenced the quality of identification. For each value of reflex gain, the power of Gaussian white noise, added at the output, was varied to evaluate SNR effects.

The data ensembles, composed of 500 realizations, were decimated to 200 Hz prior to analysis. Each simulation was characterized by its SNR, and the relative contributions of intrinsic and reflex stiffness to the total output torque, quantified by the relation:

$$intrinsic/reflex\ ratio = \frac{\%VAF(TQ, TQ_i)}{\%VAF(TQ, TQ_r)}$$

The quality of the identification was evaluated by computing the %VAF between the noise-free, simulated torques and the predicted torques. Figure 6 shows the %VAF between the simulated and predicted total torque (A), intrinsic torque (B), and reflex torque (C) for different SNRs and intrinsic/reflex ratios: 1, 2, and 5. Each point represents the results of a single simulation. If the %VAF fell below zero, the identification was considered to have failed at those points, and they were set to zero.

Figure 6 shows that the total and intrinsic torques are well estimated for all intrinsic/reflex ratios, with %VAF above 80%; the quality of the identification drops only slightly as the SNR decreases. However, it is clear that the intrinsic/reflex ratio had a significant impact on how much noise the reflex identification could tolerate. For the intrinsic/reflex ratio of 5, the reflex identification failed for SNRs less than 10 dB, while it did not for the smaller ratios. The reflex estimation degraded at higher SNRs for larger ratios.

In light of the previous result, a new metric was a calculated: the effective reflex SNR, defined as the ratio of the reflex torque power and the noise power. Figure 7, shows the %VAF between the simulated and predicted reflex torque as a function of effective reflex SNR, for three intrinsic/reflex ratios. When plotted against the effective reflex SNR, the quality of the reflex estimates exhibited the same degradation behavior; beginning to fall off rapidly at an effective reflex SNR of 0dB.

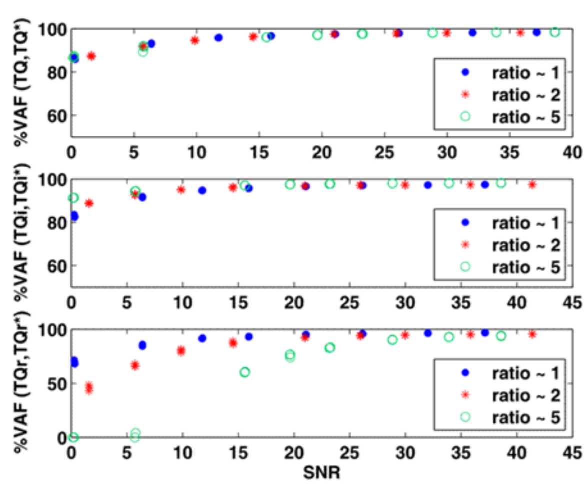

Fig. 6. Percent VAF between simulated and predicted (A) total, (B) intrinsic, and (C) reflex torque as a function of SNR for three intrinsic/ reflex ratios. 500 realizations were used in the identification.

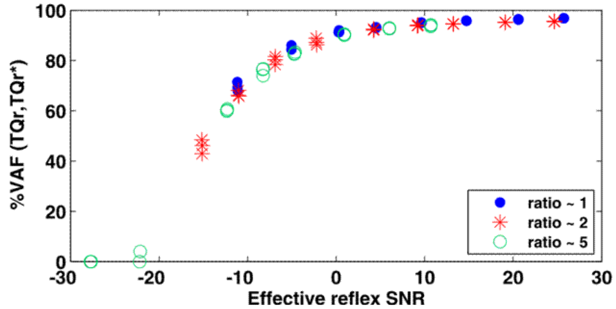

Fig. 7. Percent VAF between simulated and predicted reflex torque as a function of effective reflex SNR for three intrinsic/ reflex ratios.

D. Number of Realizations

As more noise was added to system, we expected that more realizations would be needed to identify the system. Similarly, as more realizations were used in the data ensembles, we expected the estimates to improve. To investigate these hypotheses, we first examined how the quality of the identification varied with the number of realization. A fixed SNR of 10dB and an intrinsic/reflex ratio of 2 were used because this is similar to what is expected experimentally.

Figure 8A shows the results of this simulation. The total and intrinsic torques were consistently well modeled; improving only slightly with the number of realizations. The reflex estimates were more sensitive, the %VAF increased from about 60% with 400 realizations, to more than 90% with 900 realizations.

In a second simulation study the number of realizations and the SNR were varied, to determine the minimum number of realizations required to estimate stiffness dynamics reliably under various noise conditions. Reliability was determined by repeating the simulations 5 times at the same SNR, with the same sized ensembles. If all identifications were successful the identification was considered reliable. However, if the identification failed for one or more of the simulations, the identification was deemed unreliable. The intrinsic/reflex ratio was set to 2.

Figure 8B shows the results. The minimum number of realizations needed for reliable identification decreased as the SNR increased. Note that although the algorithm requires relatively few realizations to perform the identification; adding realizations will improve the estimates.

IV. Discussion and Conclusion

These simulation studies show that the time-varying, parallel-cascade algorithm can identify rapid changes in system dynamics. Indeed, even step changes were tracked accurately. This means that there are few limitations on the type of movement that can be studied.

However, one limitation is the size of the intrinsic/reflex ratio, which may change during movement. That the reflex identification cannot tolerate as much noise when the intrinsic component dominates the output is not surprising because of the iterative nature of the TVPC algorithm. The intrinsic component is identified first, and its contribution to

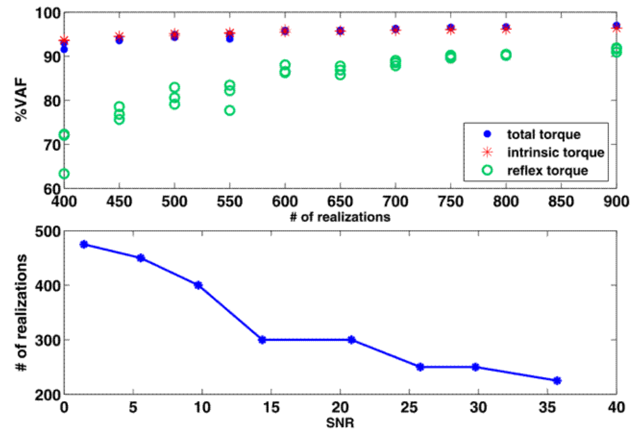

Fig 8. (A) %VAF between the simulated and estimated total, intrinsic and reflex torques vs. the number of realizations used in the data ensembles. (B)The minimum number of realizations required for reliable identification vs. SNR.

the output torque is removed prior to the reflex identification. However, all the noise is still present during reflex identification. Furthermore, since the reflex is smaller with larger intrinsic/reflex ratios, the effective SNR for the reflex identification is much lower than for the total torque.

More realizations will be required to obtain reliable identification as the SNR or effective reflex SNR decreases and the intrinsic/reflex ratio increases. Under typical experimental conditions (total SNR of approximately 10dB and intrinsic/reflex ratios between 2 and 4) between 500 and 800 realizations of the time-varying behavior will be necessary. Realizations are typically 3 seconds long, and it would, therefore, take 40 minutes to acquire 800 realizations, which is a reasonable and realistic amount of time. With a carefully developed experimental procedure, the time-varying, parallel-cascade algorithm will be a useful experimental tool.

This algorithm will be used to study the changes in intrinsic and reflex stiffness as the ankle moves or the level of voluntary torque is varied. This will provide further insight into the role of reflexes in motor control.

V. References

[1] Mirbagheri, M.M., H. Barbeau, and R.E. Kearney, "Intrinsic and reflex contributions to human ankle stiffness: variation with activation level and position," *Experimental Brain Research*, 2000. **135**(4): p. 423-436.

[2] Lortie, M. and R.E. Kearney, "Identification of physiological systems: estimation of linear time-varying dynamics with non-white inputs and noisy outputs," *Medical & Biological Engineering & Computing*, 2001. **39**(3): p. 381-390.

[3] Lortie, M. and R.E. Kearney, "Identification of time-varying Hammerstein systems from ensemble data," *Annals Of Biomedical Engineering*, 2001. **29**(7): p. 619-635.

[4] Baker, M., et al., "Time-varying Parallel-cascade System Identification of Ankle Stiffness from Ensemble Data," in *Proc. 26th Annual International Conf. of the IEEE EMBS*. 2004: San Francisco, CA, USA. p. 4688-4691.

[5] Kearney, R.E., R.B. Stein, and L. Parameswaran, "Identification of intrinsic and reflex contributions to human ankle stiffness dynamics," *IEEE Transactions On Biomedical Engineering*, 1997. **44**(6): p. 493-504.