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Identification of Hammerstein Systems Using Subspace Methods with Applications to Ankle Joint Stiffness

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***Abstract*— A Hammerstein system is a series connection of a static non-linearity followed by a linear dynamic system. The subspace method is an efficient alternate to the classic Prediction Error Method to identify linear time invariant systems, especially those with multiple inputs and/or outputs. Furthermore, the subspace method has been extended to identify block-structured, nonlinear systems including those with Wiener and Hammerstein structures. This paper reviews the extended subspace method for the identification of Hammerstein systems, and demonstrates how it can be used to estimate dynamic joint stiffness. Simulation results demonstrate that the algorithm estimates the linear and nonlinear components of the ankle joint stiffness accurately.**

***Keywords*—Hammerstein systems, subspace method, ankle joint stiffness**

I. Introduction

Subspace methods have been accepted as a good alternate to classic Prediction Error Methods (PEM) for the identification of linear time invariant (LTI) systems. Subspace identification methods for LTI systems can be classified into three groups: MOESP methods (Multivariable Output-Error State-sPace) [1-2]; N4SID methods [3-5] and CVA (canonical variate analysis) methods [6-7]. These methods estimate a state space model [8], of the form given in Equation 1, directly from input-output measurements.

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{w}(k) \end{aligned} \tag{1}$$

where $\mathbf{u}(k) \in \mathbb{R}^m$ and $\mathbf{y}(k) \in \mathbb{R}^l$ are vectors containing measurements, at discrete time *k*, of the *m* inputs and *l* outputs of the process; $\mathbf{w}(k) \in \mathbb{R}^l$ is an additive, zero mean, noise signal that is uncorrelated with the input; $\mathbf{x}(k) \in \mathbb{R}^n$ is the state vector of the process at discrete time *k* containing the values of *n* states, where n is the order of the system; $\mathbf{A} \in \mathbb{R}^{n \times n}$ is the system matrix that describes the dynamics of the system; $\mathbf{B} \in \mathbb{R}^{n \times m}$ is the input matrix that describes how deterministic inputs influence the states; $\mathbf{C} \in \mathbb{R}^{l \times n}$ is the output matrix that describes how the internal states are transformed to generate the output y_k; $\mathbf{D} \in \mathbb{R}^{l \times m}$, is the direct feed through term. Compared with the PEM methods, subspace methods are computationally efficient, do not require the model structure to be known *a priori,* and extend easily to multiple-input/multiple output (MIMO) systems. Since the LTI structure cannot describe the dynamics of some systems properly, the subspace method has been extended to identify block-structured, nonlinear systems, including those with Wiener [9-10] and Hammerstein [11] structures.

This paper reviews the subspace method for the identification of Hammerstein systems, which comprise the series connection of a static non-linearity and a linear dynamic system, and describes its application to dynamic joint stiffness. The paper is developed as follows: Section II reviews the MOESP subspace method for the identification of LTI systems; Section III describes how the method can be extended to identify Hammerstein systems; Section IV describes its use to estimate ankle joint stiffness; Section V presents simulation results demonstrating its application to open- and closed-loop experiments.

II. Subspace Method For LTI Systems

The subspace method used in this paper is the MOESP algorithm which is reviewed briefly here. For systems of the form given by Equation 1, the input-output signals can be constructed as:

$$\mathbf{Y}_{0,i,N} = \mathbf{\Gamma}_i \mathbf{X}_{0,N} + \mathbf{H}_i \mathbf{U}_{0,i,N} \tag{2}$$

where

$\mathbf{U}_{0,i,N}$ and $\mathbf{Y}_{0,i,N}$ are Hankel matrices, whose general form is

$$\mathbf{R}_{i,j,N} = \begin{bmatrix} r(i) & r(i+1) & \cdots & r(i+N-1) \\ r(i+1) & r(i+2) & & r(i+N) \\ \vdots & & & \vdots \\ r(i+j-1) & r(i+j) & \cdots & r(i+j+N-2) \end{bmatrix}$$

where *i* is the left upper entry of the Hankel matrix, *j* is the number of block rows, and N is the number of columns.

$$\mathbf{\Gamma}_i = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{i-1} \end{bmatrix}$$, is the extended Observability matrix

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$$\mathbf{H}_i = \begin{bmatrix} \mathbf{D} & 0 & & 0 & 0 \\ \mathbf{CB} & \mathbf{D} & & & \\ & & \ddots & & \\ \vdots & & \ddots & \mathbf{D} & \\ \mathbf{CA}^{i-2}\mathbf{B} & \mathbf{CA}^{i-3}\mathbf{B} & \cdots & \mathbf{CB} & \mathbf{D} \end{bmatrix}$$ is a Toeplitz matrix of impulse response elements

$\mathbf{X}_{i,N} = [\mathrm{x}(i) \quad \mathrm{x}(i+1) \quad \cdots \quad \mathrm{x}(i+N-1)]$ contains the internal states.

The first step of MOESP is to estimate a subspace containing information from only the zero-input response. This is achieved using orthogonal projection by LQ factorization [12]. Thus, the input and output Hankel matrices are stacked into a tall matrix, and then LQ factorization is applied to give.

$$\begin{bmatrix} \mathbf{U}_{0,i,N} \\ \mathbf{Y}_{0,i,N} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{11} & 0 \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} \quad (3)$$

Where the columns of $\mathbf{L}_{22}$ will span the subspace from the internal states. The SVD of matrix $\mathbf{L}_{22}$ is

$$\mathbf{L}_{22} = \boldsymbol{\mu\varsigma\upsilon}^T \quad (4)$$

Then $\hat{\boldsymbol{\Gamma}}$ is given by the column space of $\boldsymbol{\mu}$ in Equation 4; that is the first n columns of $\boldsymbol{\mu}$ where n is the order of the system. The system matrices, ***A B C D***, are then estimated using standard subspace methods

III. Hammerstein system Identification Using Subspace Methods

Figure 1A shows a SISO Hammerstein system comprising the series connection of a static non-linearity and a linear dynamic system

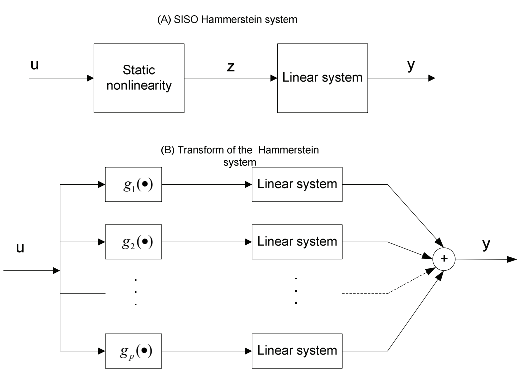

Figure 1 Transformation of a SISO Hammerstein system to a MISO linear system. A) The SISO Hammerstein system B) The equivalent MISO linear system formed using the terms of the basis function $g_i(\bullet)$ to generate a set of constructed inputs.

If the static nonlinearity can be approximated by a basis expansion $g(\cdot)$, then:

$$z_k = g(u_k, \tau) = \sum_{i=1}^{r} \tau_i g_i(u_k)$$

or

$$z_k = [\tau_1 \quad \cdots \quad \tau_r] \begin{bmatrix} g_1(u_k) \\ \vdots \\ g_r(u_k) \end{bmatrix} \quad (5)$$

where $g_i(u_k)$ are the terms of the basis function; τ is the parameter of the basis function; u_k is input to the nonlinearity and z_k its output.

The linear element can be modeled as the state-space system:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}z_k \\ \mathbf{Y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}z_k \end{aligned} \quad (6)$$

From Equation 5 it is apparent that the nonlinearity output is the product of a row vector, containing the nonlinear parameters, and a column vector, containing the kernel of the basis function. Defining the terms: $\tilde{\mathbf{B}} = [\mathbf{B}\tau_1, \quad \cdots \quad \mathbf{B}\tau_r]$, $\tilde{\mathbf{D}} = [\mathbf{D}\tau_1, \quad \cdots \quad \mathbf{D}\tau_r]$ and $U_k = [g_1(u_k), \quad \cdots \quad g_r(u_k)]^T$, allows the Hammerstein system to be rewritten as

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \tilde{\mathbf{B}}\mathbf{u}_k \\ \mathbf{Y}_k &= \mathbf{C}\mathbf{x}_k + \tilde{\mathbf{D}}\mathbf{u}_k \end{aligned} \quad (7)$$

Thus, once a basis function has been chosen, the nonlinear, SISO Hammerstein system can be described by a multiple input/single output (MISO) linear state space model and estimated using the MOESP algorithm [11].

IV. State Space Model for Ankle Joint Stiffness

A. State Space Model for Intrinsic Stiffness

Dynamic joint stiffness is used to study the mechanical behavior of the mechanisms acting about the ankle. It may be separated into two components: an intrinsic component due to the mechanical properties of the joint, passive tissue, and active muscle fibers; and a reflex component due to muscle activation in response to the activation of stretch receptors in the muscle. Kearney et al. [13] found that the parallel cascade model shown in Figure 2 described dynamic joint stiffness well.

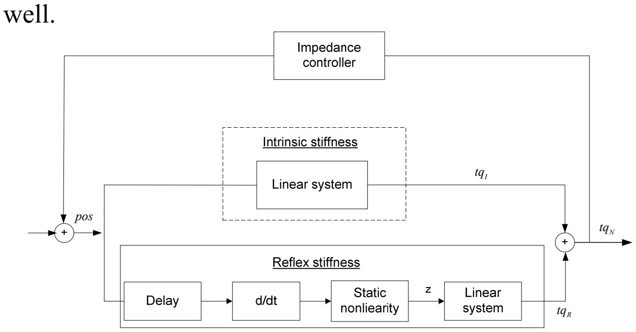

Figure 2 Parallel-cascade structure of ankle dynamics showing position (pos), intrinsic torque (tq_I), reflex torque (tq_R), and net torque (tq_N)

B. State Space Model for Intrinsic Stiffness

For perturbations about an operating point, stiffness has a linear relationship between position and torque that is described well by the second-order quasi-linear system of Equation 8.

$$\frac{TQ_I(s)}{POS(s)} = Is^2 + Bs + K \tag{8}$$

where TQ_I is the intrinsic torque, POS is the position, and I, B, and K are the inertial, viscous, and elastic parameters, respectively [13]. Alternatively, intrinsic stiffness can be described by Equation 9

$$tq_I(t) = \begin{bmatrix} pos(t) & vel(t) & accel(t) \end{bmatrix} \begin{bmatrix} K \\ B \\ I \end{bmatrix} \tag{9}$$

where $pos(t)$ is measured position, $vel(t)$ is the velocity and $accel(t)$ is acceleration. Thus, a state space model for intrinsic stiffness is:

$$\begin{aligned} \dot{\mathbf{x}}_I(t) &= \mathbf{A}_I \mathbf{x}_I(t) + \mathbf{B}_I \mathbf{U}_I(t) \\ tq_I(t) &= \mathbf{C}_I \mathbf{x}_I(t) + \mathbf{D}_I \mathbf{U}_I(t) \end{aligned} \tag{10}$$

where $tq_I(t)$ is the intrinsic torque, and $\mathbf{U}_I$ contains the constructed inputs $\begin{bmatrix} pos(t) & vel(t) & accel(t) \end{bmatrix}$. The $\mathbf{A}_I$, $\mathbf{B}_I$ and $\mathbf{C}_I$ matrices are all equal to zero, and so the model reduces to:

$$tq_I(t) = \mathbf{D}_I \mathbf{U}_I(t) \tag{11}$$

C. State Space Model for Reflex Stiffness

Reflex stiffness arises from muscle contraction in response to reflex activation from stretch receptors in the muscle. At the ankle, reflex stiffness can be modeled with a Linear-Nonlinear-Linear (LNL) block-structured model, comprising the series connection of a differentiator, a delay of about 40 ms [13], a static non-linearity, and a second-order low-pass system with transfer function:

$$H_{RS}(s) = \frac{TQ_R(s)}{VEL(s)} = \frac{g_R \omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} e^{-s\tau} \tag{12}$$

If the velocity is used as the input to the reflex stiffness, the reflex stiffness becomes a Hammerstein system that can be rewritten as

$$\begin{aligned} \mathbf{x}_R(k+1) &= \mathbf{A}_R \mathbf{x}_R(k) + \tilde{\mathbf{B}} \mathbf{U}_R(k) \\ tq_R(k) &= \mathbf{C}_R \mathbf{x}_R(k) + \tilde{\mathbf{D}}_R \mathbf{U}_R(k) \end{aligned} \tag{13}$$

Thus, the SISO Hammerstein model of reflex stiffness can be described by a linear, MISO state space model with inputs $\mathbf{U}_R(k) = \begin{bmatrix} g_1(vel(k)), & \cdots & g_p(vel(k)) \end{bmatrix}^T$.

D. State Space Model for Joint Stiffness

Although it is not possible to estimate separate state space models for intrinsic and reflex directly, it is possible to estimate a state space model for overall ankle dynamics because the sum of the torques from the intrinsic and reflex stiffness can be measured (i.e. $tq_N = tq_I + tq_R$). Thus, Equations 10 and 13 can be combined to give:

$$\begin{aligned} \mathbf{x}_R(k+1) &= \mathbf{A}_R \mathbf{x}_R(k) + \begin{bmatrix} 0 & \tilde{\mathbf{B}} \end{bmatrix} \begin{bmatrix} \mathbf{U}_I(k) \\ \mathbf{U}_R(k) \end{bmatrix} \\ tq_N(k) &= \mathbf{C}_R \mathbf{x}_R(k) + \begin{bmatrix} \mathbf{D}_I & \tilde{\mathbf{D}} \end{bmatrix} \begin{bmatrix} \mathbf{U}_I(k) \\ \mathbf{U}_R(k) \end{bmatrix} \end{aligned} \tag{14}$$

Identifying $\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{D}}$ does not estimate the intrinsic stiffness and the reflex stiffness directly. However, simulating the estimated system with the appropriate inputs permits the torque from the intrinsic and reflex stiffness to be estimated. Specifically, the output from the simulation with the input signal $\begin{bmatrix} \mathbf{U}_I(k) \\ 0 \end{bmatrix}$ gives an estimate of the intrinsic torque, tq_I. Similarly, the response to $\begin{bmatrix} 0 \\ \mathbf{U}_R(k) \end{bmatrix}$ gives an estimate of the torque from the reflex stiffness, tq_R.

V. Simulation Study

A. State Space Model for Reflex Stiffness

To test and validate the algorithm, simulated data were generated using Matlab's Simulink. Intrinsic stiffness was modeled as:

$$\frac{\theta(s)}{TQ_I(s)} = \frac{1}{0.015s^2 + 0.8s + 150} \tag{15}$$

where θ is joint angle, TQ_I is torque from the intrinsic stiffness. Reflex stiffness was described by a half-wave rectifier followed by a second order low pass filter as

$$\frac{TQ_R(s)}{V_R(s)} = \frac{3200}{s^2 + 80s + 1600} \tag{16}$$

where TQ_R is reflex torque, V_R is half-wave rectified joint angular velocity.

A pseudo random binary sequence (PRBS) was used as the position input. There was a 40 ms delay between the position signal and the velocity. The simulation lasted for 50 seconds. A Chebyshev polynomial [14] was used to describe the nonlinearity in the reflex stiffness. Velocity was delayed by 40 ms and used as the input signal to reflex stiffness. The first row was removed to improve the conditional number. The second row was also removed to avoid possible correlation with the velocity input to intrinsic stiffness. Thus the constructed input was:

$$U_k = \begin{bmatrix} T_3(x) & T_4(x) & \cdots & T_n(x) \end{bmatrix} \tag{17}$$

where the Chebyshev polynomials are given by:

$$\begin{aligned} T_1(x) &= 1 \\ T_2(x) &= v_k \\ T_n(x) &= 2 \cdot v_k \cdot T_{n-1}(x) - T_{n-2}(x) \end{aligned} \tag{18}$$

and v_k is the delayed velocity. The input to intrinsic stiffness, u_k, was constructed from the position, velocity and acceleration.

The percentage variance accounted for (VAF) was used to measure how well the identified torque predicted the true torque. The VAF between the true and identified torque was:

$$\text{VAF}\% = \left(1 - \frac{\text{variance}(y - y_{est})}{\text{variance}(y)}\right) \times 100\% \quad (19)$$

B. Identification of Ankle Stiffness in Open Loop

In the first simulation study, we identified the ankle joint stiffness in open loop. Thus, the impedance controller, in Figure 2, was set to zero so that changes in ankle torque had no effect on the position so it is an open loop problem [15]. Figure 3 shows the estimated torques and the measured torques. The estimated net torque fit the measured torque with a VAF of 96%. The estimated intrinsic torque fit the simulated intrinsic torque with a VAF 99%. The estimated reflex torque fit the simulated reflex torque with a VAF of 98%.

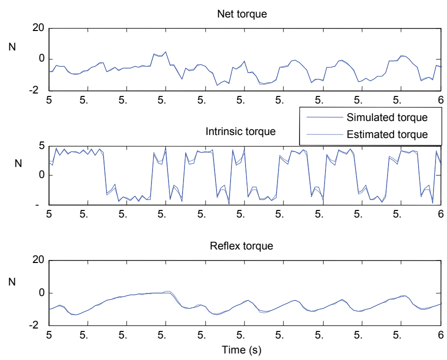

Figure 3 Simulated and estimated torques from the open loop simulation.

C. Identification of Ankle Stiffness in Closed Loop

Next we investigated the ability of the subspace method to identify ankle joint stiffness in closed-loop. The impedance controller was set to a second order low pass filter to simulate a compliant load comprising a mass, spring, and dashpot. In this case, ankle torque was fed back via the impedance controller to change the position and the identification becomes a closed loop identification problem. Applying standard, open-loop methods will give biased results [16]. However, modifying the method by using the previous inputs and outputs as instrumental variables eliminated the closed-loop effects and yielded accurate estimates.

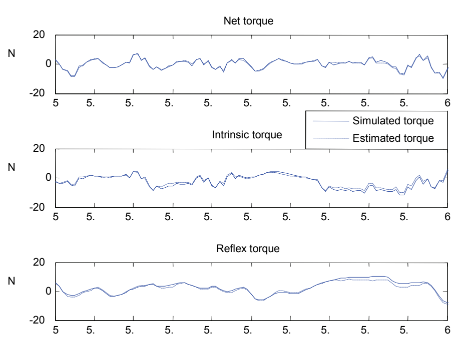

Figure 4 Simulated and estimated torques from the closed loop simulation.

Figure 4 shows the estimated torques and the simulated torques for the close-loop simulation. The estimated net torque fit the measured torque with a VAF of 95%. The estimated intrinsic torque fit the simulated intrinsic torque with a VAF 96%. The estimated reflex torque fit the simulated reflex torque with a VAF of 95%.

VI. Conclusion

In this paper, we reviewed the subspace method of identification a Hammerstein system. The nonlinear, SISO Hammerstein system is transformed to a linear, MISO system using a basis function. Then the MISO system is estimated using subspace method. To validate the method, we used a parallel cascade model for the ankle joint stiffness. An overall state space model is estimated directly from the constructed inputs and measured output. The intrinsic and reflex torques were estimated by simulating the estimated state space model with appropriate inputs. The simulation studies showed that the subspace method provides an accurate estimate of the ankle joint stiffness from the open loop data as well as from the closed loop data.

References

[1] Verhaegen, Michel and Patrick Dewilde. Subspace model identification part 1: The output-error state-space model identification class of algorithm, International Journal of Control, Vol. 56, pp:1187-1210, 1992.

[2] Verhaegen, Michel Identification of the deterministic part of MIMO state space models given in innovations form from input-output data. Automatica, Vol.30, pp: 61-74, 1994.

[3] Moonen, Marc, Bart De Moor, Lieven Vandenberghe and Joos Vandewalle, On- and off-line identification of linear state-space model, International Journal of Control, Vol.49, No.1, pp: 219-232, 1989.

[4] Van Overschee, Peter and Bart De Moor, N4SID: Subspace algorithms for the identification of combined deterministic and stochastic systems, Automatica, Vol.30, , pp: 75-93, 1994.

[5] Van Overschee, Peter and Bart De Moor. Subspace Identification for Linear Systems; Theory, Implementation, Applications. Dordrecht, The Netherlands: Kluwer Academic Publishers. ISBN 0-7923-9717-7. 1996.

[6] Larimore, Wallace E., System identification, reduced-order filtering and modelling via canonical variate analysis, In Proceedings of the American Control Conference, pp:445-451 San Francisco, California June, 1983.

[7] Peternell, K., W. Scherrer and M. Deistler, Statistical analysis of novel subspace identification methods, Signal Processing 52, pp:161-177, 1996.

[8] T. Kailath, Linear Systems, Prentice Hall, 1980.

[9] Westwick, David and Michel Verhaegen, Identifying MIMO Wiener systems using subspace model identification methods, Signal Processing, VOL.52, pp:235-258, 1996.

[10] Chou, C. T. and Verhaegen, Michel Identification of Wiener models with process noise In Proceedings of the 38th IEEE Conference on Decision and Control, Phoenix, Arizona, pp: 598-603, December 1999.

[11] Verhaegen, Michel and David Westwick Identifying MIMO Hammerstein systems in the context of subspace model identification method. International Journal of Control, Vol.63, pp:331-349, 1996.

[12] G. H. Golub and C. F. Van Loan, Matrix Computations, Second Edition, Johns Hopkins University Press, 1989, ISBN 0-8018-3739-1.

[13] R. E. Kearney, R. B. Stein, and L. Parameswaran, Identification of intrinsic and reflex contributions to human ankle stiffness dynamics, IEEE Trans Biomed Eng, Vol. 44, pp:493-504, 1997.

[14] Barrett, J.F., The use of functional in the analysis of non-linear physical systems, Journal of Electronics and Control, 15, pp: 567-615, 1963.

[15] Y. Zhao, D. T. Westwick, R. E, Kearney, Decomposition of a Parallel Cascade Model For Ankle Dynamics Using Subspace Method, Proceedings of American Control Conference, New York, USA, 2007

[16] Y. Zhao and R. E. Kearney, Closed-Loop System Identification of Ankle Dynamics with Compliant Loads, Proceedings of EMBS 2007, Lyon, France