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A Hybrid Extended Least Squares Method (HybELS) for Vestibulo-Ocular Reflex Identification

Atiyeh Ghoreyshi and Henrietta L. Galiana, *Fellow, IEEE*

***Abstract*—The Vestibulo-Ocular Reflex (VOR) plays an essential role in the majority of daily activities by keeping the images of the world steady on the retina when either the environment or the body is moving. The modeling and identification of this system plays a key role in the diagnosis and treatment of various diseases and lesions, and their associated syndromes. Today, clinical protocols incorporate mathematical techniques for testing the functionality of patients' VORs through the analysis of the patients' responses to various stimuli.**

We have developed a new tool for simultaneous identification of the two modes of the horizontal VOR, using a novel algorithm. This algorithm, HybELS (Hybrid Extended Least Squares), is a regression-based identification method tailored for hybrid ARMAX (AutoRegressive Moving Average with eXogenous inputs) models, which can also be used for the identification of other neural systems. In the context of the VOR, MELS (Modified Extended Least Squares) has been proposed previously for the identification of vestibular nystagmus dynamics, one mode at a time. It also involved searching for segment initial conditions to avoid biased results. Our hybrid approach identifies the two modes simultaneously, and does not require estimation of initial conditions, since it takes advantage of state continuity in the transitions between fast and slow phases. The results on experimental VOR in the dark show that HybELS outperforms MELS in several aspects: It proves to be more robust than MELS with respect to the system order used for identification, while resulting in more accurate estimates in almost all contexts as well. Furthermore, due to the hybrid nature of the method, its calculations are algebraically more compact, and HybELS turns out to be much less computationally expensive than MELS.

I. Introduction

THE oculomotor system plays an essential role in the majority of our basic daily activities, e.g. walking, driving, reading, etc. It keeps the images of the world steady on the retina, when either the environment or the body is moving, and also enables us to track visual targets, or switch between targets [17]. A few decades ago, in order to gain more insight into oculomotor subsystems, scientists proposed mathematical models, and were able to make predictions of how they would behave in different situations [1], [2], [3], [16]. Later, identification techniques helped them comment on which subsystems' degradation would yield the observed symptoms in patients [4]. Today, clinical protocols and commercial mathematical techniques are available to test the functionality of patients' oculomotor subsystems, e.g. Vestibulo Ocular Reflex (VOR), smooth pursuit, saccades etc., usually with specific tests for each [5].

In this work, we present a new tool for the simultaneous identification of the two modes of the horizontal VOR using system identification techniques. We make use of theories for hybrid systems, since the oculomotor system is a hybrid system which switches between fast and slow modes, producing ocular nystagmus [6], [7]. However, traditionally, this fact is ignored and the system is studied purely in one of the modes [1], [8]. This is usually done, in the slow phase case for example, by considering the envelope of the eye velocity response, and replacing fast phase segments with interpolated slow phase segments. It has already been shown by Galiana [9], [10] that while this approach can be a starting point for studying the system, it is by no means sufficient and if used alone, leads to biased estimates of reflex dynamics with weak clinical relevance.

In order to demonstrate the necessity of a hybrid approach to the problem of oculomotor system identification, the step response of a low-pass system is shown in Fig. 1 on the left. On the right, we can see the response of a hybrid system to the same input. This hybrid system's behavior is identical to the system on the left in one mode, and has a high-pass behavior in the second mode. One can see that at all switching instants, transient responses are evoked, and if one looks at the envelope of the response on the right, it does not correspond at all to the response of the non-switching system (This type of switching behavior is common in oculomotor responses e.g. the eye position response to a step head acceleration input). Therefore, using the envelope of the switching signal to identify the parameters of the low-pass mode of the system would clearly yield erroneous results. Unfortunately, the envelope of eye velocity in nystagmus is still used in the literature to identify slow phase characteristics [11].

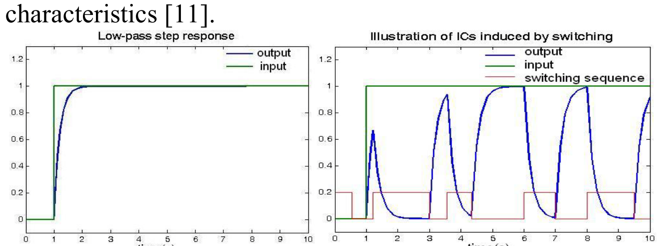

Fig. 1. Switching can induce initial conditions (ICs), enriching the response of a system. On the left, step response of a low-pass system is shown. On the right, the response of a hybrid system to the same input is depicted. This response is much richer due to switching.

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The authors are with the Biomedical Engineering department of McGill University, Montreal, Quebec H3A2B4 CANADA (phone: 514-398-4400, ext. 00431; e-mail: atiyeh.ghoreyshi@mail.mcgill.ca, henrietta.galiana@mcgill.ca).

In this work, we propose a new method (HybELS: Hybrid Extended Least Squares) for the identification of the VOR, which respects the hybrid nature of the system. We will compare the performance of our method with that of MELS, a state of the art method for identification of a selected VOR mode (slow or fast) [12]. We will present results on experimental data for the VOR in the dark.

II. Methods

A. System Identification – Our Approach

Our approach to the identification of the oculomotor system is a hybrid parametric approach. Due to the short length of data segments in ocular nystagmus, non-parametric identification of this system with current mathematical tools is not feasible [15]. Parametric identification methods have been previously used for hybrid system identification, in the context of the oculomotor system [9], [12]. We explore these methods further and develop HybELS as a hybrid regression-based batch method, and compare the results with those of MELS, most recently introduced for VOR dynamics identification.

B. HybELS

Assuming linearity and time-invariance, we can write the input-output equation of our hybrid system (in a specific mode) in the Laplace domain as:

$$Y(s) = H(s)U(s) \tag{1}$$

Or in the z-domain as:

$$Y(z) = \frac{(a_0 + a_1 z^{-1} + \cdots + a_m z^{-m})U(z)}{(1 + b_1 z^{-1} + \cdots + b_q z^{-q})} \tag{2}$$

Assuming zero initial conditions in a nystagmus segment, the equivalent of this equation in the discrete time domain is:

$$y(n) = \sum_{i=0}^{m} a_i u(n-i) - \sum_{j=1}^{q} b_j y(n-j). \tag{3}$$

If there are non-zero initial conditions, then the equation becomes:

$$y(n) = \sum_{i=0}^{m} a_i u(n-i) - \sum_{j=1}^{q} b_j y(n-j) + \sum_{l=0}^{q-1} \alpha_l \delta(n-l) \quad ,$$
$$y(i) = u(i) = 0 \; for \; all \; i < 0. \tag{4}$$

It is very important to consider the effects of non-zero initial conditions in the response, because as can be seen in Fig. 2, they can completely change the appearance of the response, especially here where segment duration can be short compared to the system's time constants. Hence ignoring initial conditions would lead to erroneous estimates of mode dynamics in any switched response.

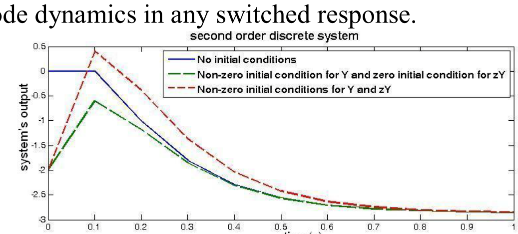

Fig. 2. Demonstration of initial condition effects on the response of a system.

If we further assume the presence of output noise, we will have:

$$z(n) = \sum_{i=0}^{m} a_i u(n-i) - \sum_{j=1}^{q} b_j z(n-j) + \sum_{p=1}^{q} b_p e(n-p) + \sum_{l=0}^{q-1} \alpha_l \delta(n-l) + e(n) \, , \tag{5}$$

where

$$z(n) = y(n) + e(n), \tag{6}$$

and e is the output noise. Notice that even though e is white noise at the output, it appears as colored in (5), when using the noisy output for regression.

In MELS, the above equation was written for all the segments of (pre-classified) data belonging to the same mode, and each mode of the system was identified by solving a regression problem, and iterating until convergence. As such, the parameters corresponding to the global dynamics of the system and the initial conditions for every segment were identified.

HybELS is developed considering the fact that all segments of data (slow or fast) are continuous; hence, instead of estimating the initial conditions at every segment, one can use the final states in a previous segment as the initial conditions for the next.

It is important to note that there exist different types of hybrid systems (Fig. 3). On the left, an example of a hybrid system is shown where the switching changes the whole subsystem to alternative control pathways. On the right, another example has been shown where the switching only changes the parameters of the system. While the former can have discontinuous states and dynamics, the latter has continuous states, but discontinuous dynamics.

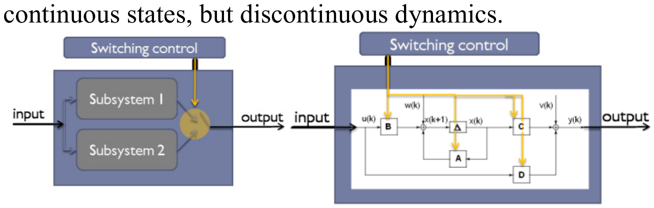

Fig. 3. Two possible configurations of hybrid systems.

Since our system is of the latter type [13], the output, eye position, and internal states remain continuous throughout the data record. Therefore, when identifying every segment, the history of the signal from the previous segment can be used instead of estimating initial conditions. This is demonstrated in Fig. 4.

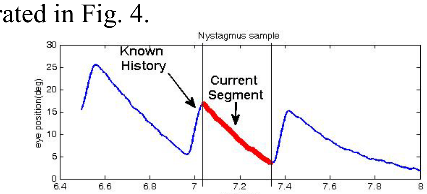

Fig. 4. Demonstration of how the history of the signal can be used instead of estimating initial conditions for each segment.

Therefore, instead of (4), we can write:

$$y(n) = \sum_{i=0}^{m} a_i u(n-i) - \sum_{j=1}^{q} b_j y(n-j) \quad ,$$
$$y(i), u(i) \; for \; all \; i < 0 \; substituted \; from \; previous \; segment \tag{7}$$

And with output additive noise:

$$z(n) = \sum_{i=0}^{m} a_i u(n-i) - \sum_{j=1}^{q} b_j z(n-j) + \sum_{p=1}^{q} b_p e(n-p) + e(n). \tag{8}$$

In order to find the parameters, we form the matrix Aj for the j[th] data segment:

$$Aj = \begin{bmatrix} u(js) & u(js-1) & \ldots -z(js-1) & \ldots e(js-1) & \ldots \\ u(js+1) & u(js) & \ldots \quad -z(js) & \ldots \quad e(js) & \ldots \\ u(js+2) & u(js+1) & \ldots -z(js+1) & \ldots e(js+1) & \ldots \\ \vdots & \vdots & \ldots \quad \vdots & \ldots \quad \vdots & \ldots \end{bmatrix} \tag{9}$$

We can then form the matrix equation:

$$\begin{bmatrix} z(0) \\ z(1) \\ \vdots \\ z(T) \end{bmatrix} = \begin{bmatrix} A1 & \emptyset \\ \emptyset & A2 \\ A3 & \emptyset \\ \emptyset & A4 \\ \vdots & \vdots \end{bmatrix} . \begin{bmatrix} \overrightarrow{\theta s} \\ \overrightarrow{\theta f} \end{bmatrix} + \begin{bmatrix} e(0) \\ e(1) \\ \vdots \\ e(T) \end{bmatrix} \tag{10}$$

where $\overrightarrow{\theta s}$ is the parameter vector corresponding to slow phases and $\overrightarrow{\theta f}$ is the parameter vector corresponding to fast phases, or

$$\vec{z} = \Psi.\vec{\theta} + \vec{e}, \tag{11}$$

where $\vec{\theta}$ is the lumped parameter vector containing the coefficients of both modes. We solve this equation for the parameter vector iteratively: We start with $\vec{e}$=0, solve for $\vec{\theta}$, update $\vec{e}$ with the residual of the regression; then with the new $\vec{e}$ in place, we solve for $\vec{\theta}$ again. We repeat this procedure until convergence, as shown in the flowchart in Fig. 5.

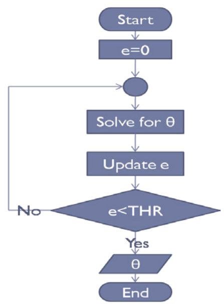

Fig. 5. HybELS flowchart.

In this process, we ensure that the residuals are zero-mean white Gaussian. An example of the residuals' distribution and spectrum are shown in Fig. 6. The number of steps needed in this example was 25.

It is important to emphasize here that, as seen above, HybELS uses a hybrid formulation that solves for all of the coefficients of the system at the same time. This is not only compact and efficient in computation, but is more importantly very robust, because of its comprehensive analysis of the system behavior as a whole.

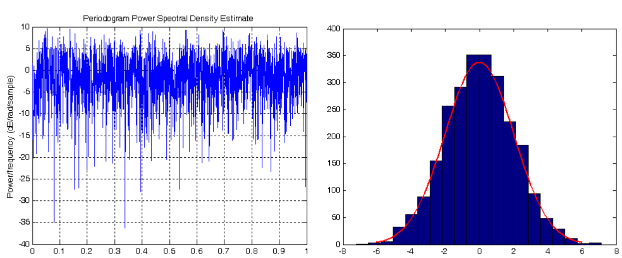

Fig. 6. Left: Power Spectrum Density of the residuals. Right: Amplitude Histogram of the residuals.

III. RESULTS

Our identification approach was first validated on simulated data to demonstrate its unbiased convergence to desired parameters [14]. We then compared the performance of linear HybELS with linear MELS on experimental data from VOR in the dark. As we will see shortly, the performances of HybELS and MELS in terms of identification are similar when the system order chosen for identification is low, but HybELS starts to outperform MELS as the order increases. The computational cost of HybELS compared to MELS is always lower by far.

Comparison with MELS

The VOR data is first classified into slow and fast phase intervals with a previously described algorithm [6]. A sample of the data is shown in Fig. 7.

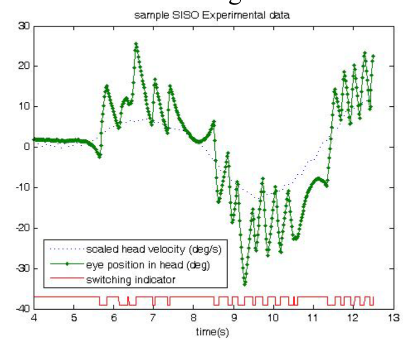

Fig. 7. Experimental VOR in the dark data.

To compare the performances of the two methods, we performed 20 pseudo Monte-Carlo trials as follows: At each trial, we picked a randomly chosen 10s record of data for identification, and a non-overlapping 5s record for validation. We then compared the results of the two methods when the order is set to one (Table I). As we can see, when we use low orders for identification (one pole in this case), both methods perform well in terms of prediction error, while HybELS is much faster.

TABLE I
COMPARISON OF MELS AND HYBELS ON PSEUDO MONTE-CARLO TRIALS

Identification System Order = 1	MELS	HybELS
Number of Trials	20	20
Sampling Rate	500Hz	500Hz
Identification (Training) Data Length	10s	10s
Validation Data Length	5s	5s
Slow-Phase Mean RMS Prediction Error	2.1°	1.5°
Fast-Phase Mean RMS Prediction Error	2.7°	2.4°
Mean Elapsed Execution Time [3.2GHz Intel]	.93s	.16s

As the identification order increases (two poles and a zero in this case), the number of parameters, and consequently the number of coefficients to be estimated increase tremendously for MELS, but not for HybELS. Doubling the order translates into adding one or two parameters to HybELS, but into more than doubling the number of coefficients for MELS; doubling the order in MELS means having to estimate one more initial condition for each segment, in addition to the dynamics-related parameters which have to be estimated. This and the fact that HybELS considers the system as a whole instead of isolated in one mode at a time, make HybELS faster, more robust, and more accurate than MELS in this context (see results in Fig. 8 and Table II). What is also interesting is that MELS performs even worse for the identification of fast phase dynamics given more degrees of freedom, while HybELS results remain robust.

It is important to mention that the same data set is used for first and second order identification. Identical segment initial conditions are used for both methods to ensure fair comparison. Also note that both methods require, and are sensitive to, a priori classification of the data.

TABLE II
COMPARISON OF MELS AND HYBELS ON PSEUDO MONTE-CARLO TRIALS

Identification System Order = 2	MELS	HybELS
Number of Trials	20	20
Sampling Rate	500Hz	500Hz
Identification (Training) Data Length	10s	10s
Validation Data Length	5s	5s
Slow-Phase Mean RMS Prediction Error	1.3°	1.1°
Fast-Phase Mean RMS Prediction Error	4.6°	1.8°
Mean Elapsed Execution Time[3.2GHz Intel]	2.1s	.21s

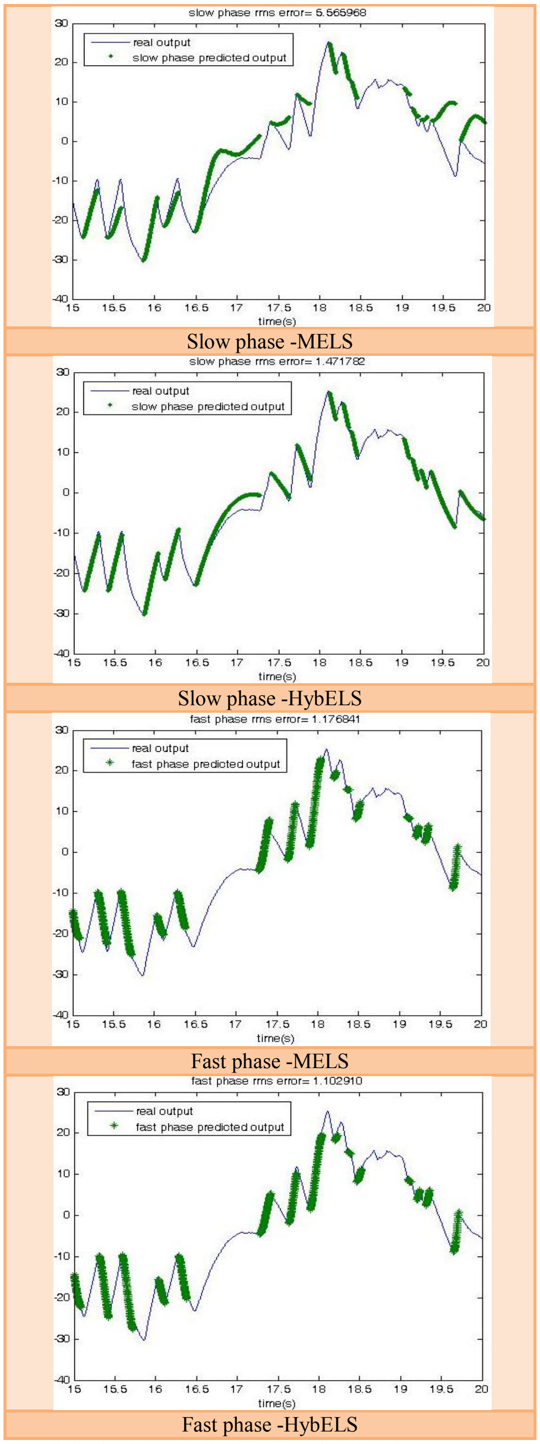

Fig. 8. Infinite-horizon predictions on validation data. System order=2.

IV. CONCLUSION AND FUTURE WORK

In this paper, we introduced HybELS, a Hybrid Extended Least Squares method for hybrid system identification when state continuity is preserved, with particular application to the VOR. We compared its performance with MELS, a state-of-the-art method, and showed with results on experimental data how HybELS outperforms MELS. This method is also extendable to include non-linearities and delays, and can be tailored for multiple-input cases, both of which can be developed as extensions to HybELS.

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