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Surrogate Data Approaches to Assess the Significance of Directed Coherence: Application to EEG Activity Propagation

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Abstract— **This paper addresses the topic of evaluating the significance of frequency domain measures of causal coupling in multivariate time series through generation of surrogate data. The considered approaches are the traditional Fourier Transform (FT) algorithm and a new causal FT (CFT) algorithm for surrogate data generation. Both algorithms preserve the FT modulus of the original series; differences are in the phase relationships, that are completely destroyed for FT surrogates and imposed after switching off the link over the considered causal direction for CFT surrogates. The ability of the algorithms to assess causality in the frequency domain was tested using the directed coherence as discriminating parameter. Evaluation on simulated multivariate linear processes and application over multichannel EEG recordings showed that the utilization of CFT surrogates improves specificity of the test for nonzero spectral causality, as FT surrogates may attribute to a direct coupling the presence of indirect connectivity patterns.**

I. Introduction

In neurophysiology, there is an increasing interest in assessing whether brain oscillations measured at different scalp or cortical locations can be related through identification of specific connectivity patterns. The concept of brain connectivity is typically explored in the frequency domain through multivariate time series analysis, measuring strength and direction of the causal relationships occurring among multichannel recordings of neurophysiological signals [1-3]. In particular, the directed coherence (DC) [1,4] is a factor in the decomposition of the traditional coherence function describing the causal linear relation between two time series in a multichannel data set.

The practical estimation of spectral causality measures such as the DC raises the problem of assessing the significance of the estimated coupling values. Since analytical evaluation of the confidence limit for testing for nonzero causality at predefined frequencies is hardly tractable, empirical criteria setting a fixed threshold on the DC values have been proposed [1,5]. This approach lacks of generality because the correct threshold for significance depends on the number and length of the considered signals and on the estimation parameters [6]. A better approach is to use the method of surrogate data [7], that consists in specifying a null hypothesis (here the absence of causal coupling over the specified direction), generating from the original time series a set of surrogate series that have all properties consistent with the null hypothesis in common with the original but are otherwise random, and testing the considered statistic (here the DC) against the null hypothesis through a comparison of its value on the original series and its distribution on the surrogate series. Among the many algorithms to generate surrogate data [8], Fourier transform (FT) surrogates [7], obtained by a phase randomization procedure applied independently to each series of the available multivariate data set, have been proven useful to assess the significance of the coherence function [9]. The FT algorithm has been proposed also for testing the significance of spectral causality measures in physiological applications [2,4,10]. Nevertheless, the fact that this algorithm destroys all phase relationships of the original time series leaves room for improvements aimed at obtaining a higher specificity in the significance assessment of spectral causality measures.

In this study we present a new algorithm to generate FT surrogate data for testing the significance of the DC. The performance of the new method is assessed, in comparison with that of the FT method, on simulated multivariate time series and on multichannel EEG recordings.

II. Methods

A. Fourier Transform Surrogates

Given a multivariate time series $\mathbf{Y}(n)=[y_1(n),...,y_M(n)]^T$, $n=1,...,N$, measured as a realization of length N of a multivariate process with M channels, we generated Fourier transform (FT) surrogates [7]: (i) applying to each series $y_m(n)$ the FT operator to obtain the complex functions $Y_m(f)=A_m(f)e^{j\varphi_m(f)}$; (ii) substituting the phases $\varphi_m(f)$ with independent realizations of uniform random variables $\phi_m(f)$ taken in the range $[0,2\pi)$; (iii) applying the inverse FT operator to get the surrogate series $\bar{y}_m(n)$, $m=1,...,M$. The multivariate FT surrogates preserve the power spectrum of the original series, being otherwise fully uncorrelated.

B. Causal Fourier Transform Surrogates

To test the existence of significant causality along a given direction of interaction, e.g., from channel l to channel m, we generated multivariate surrogate series for which the causal coupling is preserved in all directions but the l to m

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direction. This task was achieved first by fitting $\mathbf{Y}(n)$ with a multivariate autoregressive (MVAR) model:

$$\mathbf{Y}(n) = \sum_{k=0}^{p} \mathbf{B}(k)\mathbf{Y}(n-k) + \mathbf{W}(n) \qquad (1)$$

where p is the model order, $\mathbf{B}(k)$, k=0,1,...,p, are $M\times M$ matrices in which the element $b_{ij}(k)$ describes the interaction from $y_j(n-k)$ to $y_i(n)$ (i,j=1,...,M), and $\mathbf{W}(n)=[w_1(n),..., w_M(n)]^T$ is a vector of uncorrelated zero-mean white noise processes. Model identification was performed by the vector least squares approach, using the Cholesky decomposition to solve the least squares problem [11]. Subsequently, the coefficients representing the interactions along the causal direction under analysis (i.e. from l to m) were forced to zero: $b_{ml}(k)$=0, k=0,1,...,p. The resulting model was then fed with independent realizations of Gaussian white noise to get the multivariate series $\hat{\mathbf{Y}}(n) = [\hat{y}_1(n)...\hat{y}_M(n)]^T$. Finally, the FT operator was applied to the fitted multivariate series to obtain the functions $\hat{Y}_m(f) = \hat{A}_m(f)e^{j\hat{\varphi}_m(f)}$, the phases $\hat{\varphi}_m$ were retained and imposed as Fourier phases of FT surrogates for which the Fourier modulus was taken from the original series, and the surrogate series $\tilde{y}_m(n)$, m=1,...,M, were derived through inverse FT transformation. These causal FT (CFT) surrogates preserve the power spectrum of the original series y_i, as well as the causal interactions along all directions except the l to m direction.

C. Statistical Approach to Test for Nonzero Causality in the Frequency Domain

The existence of a significant causal coupling between any given pair of channels was assessed by the statistical hypothesis test associated with the method of surrogate data [7]. This approach is based on a null hypothesis to be rejected, a surrogate data set constructed in accordance with the null hypothesis, a discriminating statistic that has to be calculated on original and surrogate series, and a statistical test allowing to reject (if it is the case) the null hypothesis.

According to the procedures described above, the null hypotheses to which multivariate FT and CFT surrogates are consistent are those of an uncorrelated M channels Gaussian linear stochastic process and an M channels Gaussian linear stochastic process with absence of causal interaction from channel l to channel m, respectively.

As discriminating statistic we utilized the directed coherence (DC) [1,4], a frequency domain measure of the strength of the linear causal coupling among multivariate time series. The DC definition is based on modeling the multivariate process under analysis as a MVAR process as in Eq. (1) and then considering its input-output spectral representation: $\mathbf{Y}(f)=\mathbf{H}(f)\mathbf{W}(f)$, where $\mathbf{Y}(f)$ and $\mathbf{W}(f)$ are the FT of $\mathbf{Y}(n)$ and $\mathbf{W}(n)$, and $\mathbf{H}(f)$ is the $M\times M$ transfer matrix in the frequency domain. The transfer matrix can be expressed in terms of the MVAR model coefficients as:

$$\mathbf{H}(f) = (\mathbf{I} - \mathbf{B}(f))^{-1}, \quad \mathbf{B}(f) = \sum_{k=0}^{p} \mathbf{B}(k)e^{-j2\pi f k} \qquad (2)$$

($\mathbf{I}$ is the $M\times M$ identity matrix). Given this representation, the squared DC from channel l to channel m is:

$$\gamma_{ml}^2(f) = \frac{\sigma_{ll}^2 |H_{ml}(f)|^2}{\sum_{i=1}^{M} \sigma_{ii}^2 |H_{mi}(f)|^2} \qquad (3)$$

where σ_{ii} is the variance of the input noise w_i in Eq. (1). The set of the DC functions of Eq. (3) provides a frequency domain picture of the connectivity pattern of the multivariate process, with each $\gamma^2_{ml}(f)$, m,l=1,...,M, quantifying the normalized coupling strength from channel l to channel m as a function of frequency, being 0 when y_l does not cause y_m at frequency f, and 1 when the whole power of y_m at frequency f is due to the variability of y_l.

As statistical test, we performed a nonparametric test based on percentiles, rejecting with a 5% significance level the null hypothesis, and thus detecting the presence of a significant coupling from y_l to y_m at frequency f, when the DC $\gamma^2_{ml}(f)$ computed from the original series was higher than the 95th percentile of the DC distribution evaluated on the surrogate series.

III. Simulation Example

In this section we compare the performance of FT and CFT surrogates in detecting the causal coupling on simulated time series with known conditions of interaction.

The simulated multichannel time series consisted of a realization of N=1024 points of a MVAR process driven by M=3 uncorrelated white Gaussian noises. The MVAR process was shaped to have power spectrum of the individual time series similar to those of eyes closed EEG signals, i.e. with two main rhythms at f_α~10 Hz (alpha band) and f_β~25 Hz (beta band) superimposed to slower fluctuations (delta/theta,<8 Hz). The series y_1 was a fifth-order AR process with five poles $\pi_k=\rho_k e^{j\varphi_k}$, k=1,...,5: one real pole (ρ_1=0.8, φ_1=0) and two pairs of complex conjugate poles ($\rho_{2,3}$=0.94, $\varphi_{2,3}=\pm 2\pi f_\alpha/f_s$; $\rho_{4,5}$=0.9, $\varphi_{4,5}=\pm 2\pi f_\beta/f_s$; f_s=128 Hz is the simulated sampling frequency). The series y_2 and y_3 were generated to simulate an unidirectional propagation from channel 1 to channel 2 and a closed loop between channels 2 and 3. The generated process was:

$$\begin{aligned} y_1(n) &= \sum_{k=1}^{5} b_{11}(k)y_1(n-k) + w_1(n) \\ y_2(n) &= b_{21}y_1(n-1) + b_{22}y_2(n-1) + b_{23}y_3(n-1) + w_2(n) \\ y_3(n) &= b_{32}y_2(n-1) + b_{33}y_3(n-1) + w_3(n) \end{aligned} \qquad (4)$$

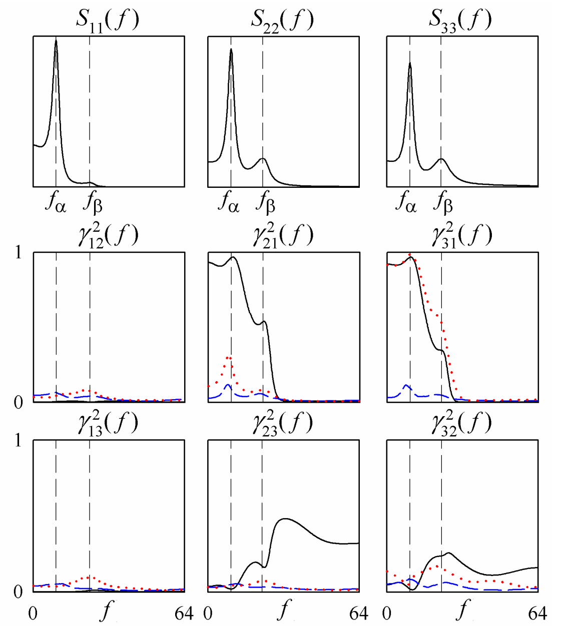

Fig. 1. Power spectrum of the series y_m ($S_{mm}(f)$), causal coupling from y_l to y_m ($\gamma^2_{ml}(f)$) estimated for the original time series (solid lines) and corresponding threshold for significance yielded by FT surrogates (blue dashed lines) and CFT surrogates (red dotted lines) for the simulated process of Eq. (4). Vertical dashed lines indicate the frequencies (f_α and f_β) of the main oscillations of the driven signal for the considered pairwise interaction, at which significance of the coupling is assessed.

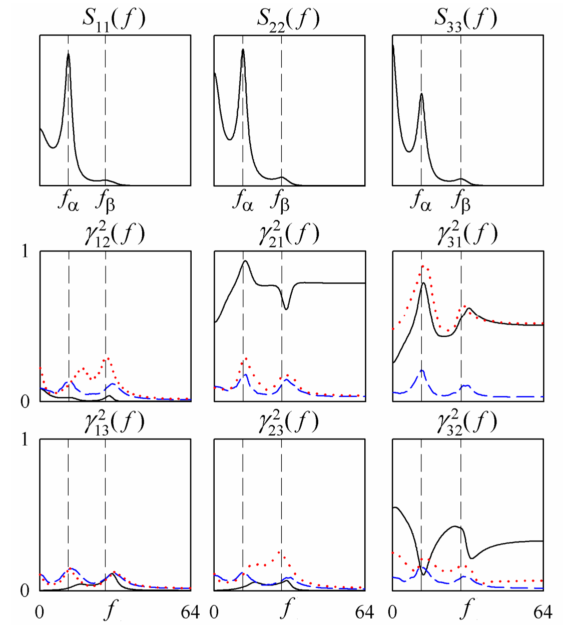

Fig. 2. Power spectrum of the series y_m ($S_{mm}(f)$), causal coupling from y_l to y_m ($\gamma^2_{ml}(f)$) estimated for the original time series (solid lines) and corresponding threshold for significance yielded by FT surrogates (blue dashed lines) and CFT surrogates (red dotted lines) for the EEG recordings of a representative subject. Vertical dashed lines indicate the frequencies (f_α and f_β) of the main oscillations of the driven signal for the considered pairwise interaction, at which significance of the coupling is assessed.

where b_{21}=0.3, b_{22}=0.5, b_{23}=0.6, b_{32}=-0.8, b_{33}=0.2, and $b_{11}(k)$ were derived by backward partial fraction expansion from the poles π_k, k=1,...,5.

Fig. 1 shows the spectral power and DC functions, along with their corresponding threshold for significance estimated by FT and CFT surrogates, obtained for the simulated process. According to both surrogate approaches, the DC detected the significant unidirectional link from y_1 to y_2 and the bidirectional link between y_2 and y_3. Indeed, the DC values $\gamma^2_{21}(f_\alpha)$, $\gamma^2_{21}(f_\beta)$ and $\gamma^2_{12}(f_\alpha)$, $\gamma^2_{12}(f_\beta)$ were respectively above and below the surrogate thresholds, while both $\gamma^2_{23}(f_\beta)$ and $\gamma^2_{32}(f_\beta)$ were above the thresholds. The two approaches provided a different interpretation of the link between y_1 and y_3: according to the imposed interactions, there was no direct connection from y_1 to y_3, but the threshold derived from FT surrogates was lower than $\gamma^2_{31}(f)$ both at f_α and at f_β, thus erroneously indicating significant causality; on the contrary, the CFT surrogate threshold encompassed $\gamma^2_{31}(f)$, detecting the absence of direct causal interactions.

IV. Application to EEG Data

EEG recordings were acquired (10-20 System, 256 Hz sampling rate, Fpz common reference) from fifteen healthy subjects (22-35 yrs) resting with eyes closed in the relaxed awake state. For each subject, the 19 acquired signals were bandpass filtered (FFT filter, 0.3-40 Hz), cleaned up of ocular and ECG artifacts by means of Independent Component Analysis [12] where necessary, and then downsampled to 128 Hz. Three EEG recordings representative of different cortical areas (y_1: occipital; y_2: central; y_3: frontal; electrodes Pz, Cz and Fz) were then selected for the analysis. A MVAR model (M=3 channels, fixed order p=5) was identified on synchronous stationary epochs of 8 s duration (N=1024 samples).

The spectral functions obtained for a representative subject are shown in Fig. 2. The EEG power spectrum displayed for all signals two main peaks in the alpha (f_α~10 Hz) and beta (f_β~29 Hz) bands. The corresponding DC functions evidenced a back-to-front propagation pattern, with a causal coupling that was high from the occipital to the central and frontal cortical areas and low in the opposite direction of propagation, documented respectively by high values of γ^2_{21}, γ^2_{31} and γ^2_{32} and low values of γ^2_{12}, γ^2_{13} and γ^2_{23}. With respect to FT surrogates, the use of CFT surrogates helped identification of direct connections within this connectivity pattern: e.g., the DC function γ^2_{31}, although very high, was deemed nonsignificant using CFT surrogates.

Results extended to the whole group are shown in Fig. 3. In both alpha and beta bands the DC function was markedly higher over each back-to-front direction than over the corresponding front-to-back direction ($\gamma^2_{ml}>\gamma^2_{lm}$ for $l<m$). Accordingly, the causal coupling was significant in a larger number of subjects over the back-to-front directions. The use of CFT surrogates evidenced that the back-to-front propagation of the information flow is not direct from the occipital to the frontal cortical areas, but seems mediated by the central area. Indeed, despite the high DC values from y_1

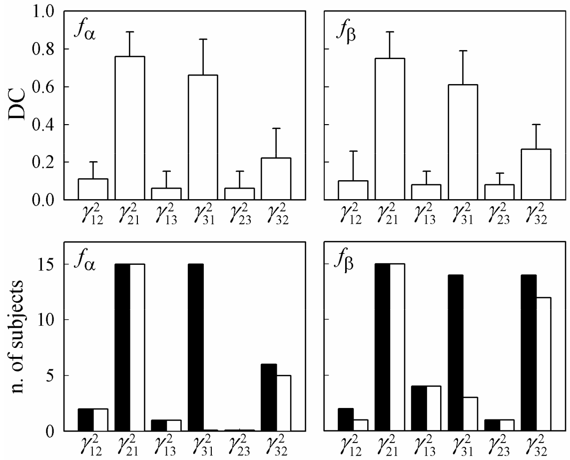

Fig. 3. Up: Directed Coherence (DC) from y_l to y_m (γ^2_{ml}), expressed as mean + SD over 15 subjects, estimated for EEG recordings in the alpha band (f_α, right) and in the beta band (f_β, right). Down: number of subjects for which the causal coupling resulted as statistically significant at f_α (left) and f_β (right) according to FT surrogates (black bars) and CFT surrogates (white bars).

to y_3 (high γ^2_{31}), the number of subjects in which this link was estimated to be significant using CFT surrogates was very low. On the contrary, traditional FT surrogates returned a misleading indication of significant directed link over this causal direction.

V. Discussion

In the present paper we propose an extension of the FT algorithm for generating multivariate surrogate data, devised to make it more accurate in the detection of the significance of causal measures of coupling. Our idea is based on the fact that the surrogate time series, to represent adequately the null hypothesis, have to reproduce as best as possible the properties of the original data but, at the same time, lack of the investigated property. With this in mind, we tried to obtain a tradeoff between the use of multivariate FT surrogates [13] and of independent realizations of univariate FT surrogates [2,4,7,10]: the first surrogates preserve the whole cross-correlation structure, thus being unsuitable to test for linear coupling; the second destroy any cross-correlation, thus being more random than required when testing for nonzero causality. The proposed CFT surrogates destroy the coupling only over the causal direction under analysis, and thus offer higher specificity in the test for nonzero causality. Examples of this advantage are shown by our simulation where a non-existent causal coupling is deemed significant using FT surrogates and nonsignificant using CFT surrogates. A possible drawback of the CFT algorithm is that accurate preservation of the causal coupling in the directions other than that under analysis depends on reliability of fitting the multivariate series with a MVAR model. However, this limitation is eased when CFT surrogates are used to test the significance of coupling measures which formulation is based on the same MVAR model. This was the case of the DC used as discriminating parameter in this work.

The application to EEG activity propagation suggested that CFT surrogates may support better than FT surrogates the utilization of the DC measure for elucidating specific causal connectivity patterns. Specifically, the DC evidenced an EEG information flow directed from the posterior to the anterior areas of the scalp, a result consistent with the notion that the alpha rhythm, which is predominant in eyes closed EEG, originates in the occipital area and then spreads to other brain areas [14,15]. With respect to FT, the use of CFT surrogates helped identifying direct connections within this propagation pattern.

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