

A New Method for Spatiotemporal Identification of Event-Related Potential Subcomponents

Hamid R. Mohseni, *Student Member, IEEE*, and Saeid Sanei *Senior Member, IEEE*

Abstract—In this study a novel method for tracking and separation of event-related potential (ERP) subcomponents from trial to trial is considered. The sources of ERP subcomponents are assumed to be electric current dipoles (ECD). The shape of each ERP subcomponent is also supposed to be monophasic wave and modeled using a Gaussian waveform. We are interested in the estimation and tracking of ERP subcomponent locations and parameters (amplitude, latency and width of each Gaussian waveform). Estimation of ECD locations, which have nonlinear relation to the measurement, is performed by particle filtering, and estimation of the amplitude is optimally estimated by a maximum likelihood approach, and finally estimation of latency and width of the Gaussian functions are given by Newton-Raphson technique. New recursive methods are introduced for both maximum likelihood and Newton-Raphson approaches to prevent the divergence of the filtering in the presence of very low signal to noise ratio (SNR). The proposed method was assessed using both simulated and real data and the results verified a successful deployment of the method in ERP analysis.

Index Terms—Event-related potentials, maximum likelihood estimation, Newton-Raphson technique, particle filtering.

I. INTRODUCTION

Event-related potentials (ERPs) with excellent temporal resolution are one of a number of physiological measures of brain activity [1]. Conventional methods for analyzing ERPs involve time-locked averaging over many trials. These approaches assume that the ERP parameters remain the same over time and the background EEG is a random process which is attenuated by averaging. While these procedures are widely employed in the psychological community, there is evidence that ERPs vary over time due to the changes in the degree of fatigue, habituation, or levels of attention [2].

Recent methods have been proposed to exploit spatiotemporal information of the ERP data from trial to trial (e.g. [3], [4]). The main drawback of these spatiotemporal methods is that the source locations are assumed to remain the same during the course of recording.

In this study we propose a novel method for separation of ERP subcomponents. ERPs are assumed to be superposition of some electric current dipoles (ECD) and their shapes are assumed to be monophasic waves and modeled by Gaussian waveforms. Amplitude, mean, and variance of each Gaussian waveform can be interpreted as amplitude, latency, and width of ERP subcomponents, respectively. The locations are estimated using particle filtering (PF). Many studies have proved that PF is one of the best methods when the relation between

the desired parameter (states) and measurement is nonlinear [5]. A closed-form solution for the amplitude is given using maximum likelihood approach. The solutions to the latency and width are given by maximum Likelihood method which is solved by Newton-Raphson technique. Very low SNR of some individual trials results in the divergence of filtering for estimation of amplitude, latency and width of the Gaussian waveforms. To compensate this failure, recursive methods are introduced which guarantee stability of the filtering across trials.

II. METHODS

In this section first the ERP modeling is presented and then recursive solutions to the estimation of model parameters are given.

A. Problem Formulation

Let the measured ERP $\mathbf{Y}_k \in \mathbb{R}^{L \times T}$ be a matrix composed of the potentials acquired from L electrodes with T time samples at k th trial. In addition, suppose ERP is generated from q ECDs which are specified by their three dimensional locations ρ_k^i , $i \in \{1, \dots, q\}$ at k th trial. The medium between the sources and the electrodes is assumed to be homogenous and the potentials at the scalp \mathbf{Y}_k be the superposition of the potentials from ECDs. Therefore we may write

$$\mathbf{Y}_k = \sum_{i=1}^q \mathbf{H}(\rho_k^i) \mathbf{a}_k^i \psi_k^i + \mathbf{N}_k \quad (1)$$

where $\mathbf{H} \in \mathbb{R}^{L \times 3}$ is the forward matrix and is a nonlinear function of the ECD locations. \mathbf{H} can be calculated in a spherical head model with three skull, scalp, and skin layers, or can be obtained using a realistic head model. In latter case, for each location a pre-calculated forward matrix \mathbf{H} is given. \mathbf{N}_k represent the Gaussian white noise (GWN) which is spatially and temporally uncorrelated with the source activities. In equation (1), $\mathbf{a}_k^i \in \mathbb{R}^{3 \times 1}$ is the amplitude of the i th ECD moment in x , y , and z directions, and $\psi_k^i = [\psi(1)_k^i \dots \psi(T)_k^i]$ represents the shape of the i th ECD moment. Each $\psi(t)$ is modeled using a Gaussian waveform as:

$$\psi(t)_k^i = \frac{1}{\sigma_k^i \sqrt{2\pi}} e^{-\frac{(t-\mu_k^i)^2}{2\sigma_k^i}} \quad (2)$$

Note that we assume the ECD amplitudes are different in x , y , and z directions, however, they have the same shape in three directions (i.e. same σ_k^i and μ_k^i in three directions). For simplicity and without loss of generality, we ignore the normalizing factor $\frac{1}{\sigma_k^i \sqrt{2\pi}}$ and assume that it is embedded in the amplitude vector \mathbf{a}_k^i . Although the real ERP subcomponents

This work is supported by the Schools of Engineering and Psychology, Cardiff University, Wales, UK. mohsenihr@cf.ac.uk

do not have an exact shape of Gaussian functions, this modeling allows a robust and fast estimation of peak parameters (latency and amplitude) with which the neurophysiologists are primarily concerned.

Our primary aim here is to recursively estimate the model parameters $\theta_k^i = \{\rho_k^i, \mathbf{a}_k^i, \mu_k^i, \sigma_k^i\}$, $i \in \{1, \dots, q\}$ at the k th trial based on the previous estimation of model parameters θ_{k-1}^i and available measurements \mathbf{Y}_k . Therefore, we assume that the evolution of the parameters is a Markovian process and does not vary extensively from trial to trial. This assumption has been exploited in many ERP analysis (e.g. [4], [6]). It can be also explicitly justified by Mocks, et al. observation [7] that consecutive responses from repeated stimuli vary slowly, since the brain state changes gradually over time. Although the responses across the entire experiment can differ significantly,

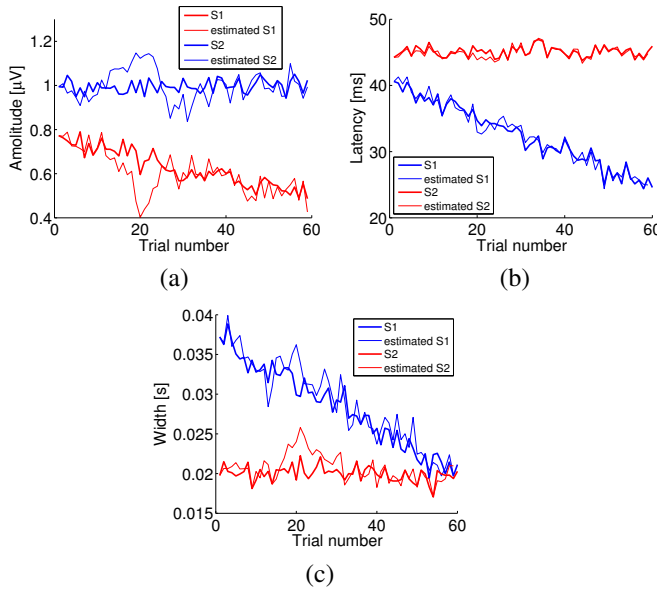


Fig. 1. An example of estimated amplitude, mean and variance of two ERP subcomponents using simulated data with SNR = -5dB, (a) amplitude, (b) mean, (c) variance.

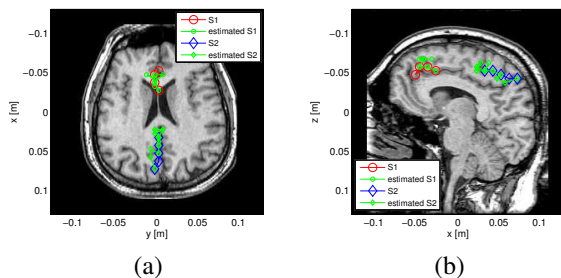


Fig. 2. Estimated location using the proposed method for the previous figure in (a) axial and (b) sagittal views.

B. Parameter Estimation

1) *Estimation of Source Locations:* Let the matrix of ECD locations be $\mathbf{R}_k = [\rho_k^1 \dots \rho_k^q] \in \mathbb{R}^{3 \times q}$. The source locations \mathbf{R}_k have nonlinear relation through forward matrix \mathbf{H} to

the measurements and if the real head model is used, no exact closed-form solution for \mathbf{H} exists. PF is an emerging methodology which can deal with nonlinearity of the system and non-Gaussianity of the posteriori distribution $p(\mathbf{R}_k | \mathbf{Y}_{1:k})$. In PF the posteriori distribution is approximated by discrete random measures defined by particles $\{\mathbf{R}_k^{(n)}, n = 1, \dots, N\}$ and their associated weights $\{w_k^{(n)}, n = 1, \dots, N\}$. The posteriori distribution based on these particles and weights is approximated as

$$p(\mathbf{R}_k | \mathbf{Y}_{1:k}) \approx \sum_{n=1}^N w_k^{(n)} \delta(\mathbf{R}_k - \mathbf{R}_k^{(n)}) \quad (3)$$

where $\delta(\cdot)$ is Dirac delta function. Suppose at trial k we want to approximate the posteriori distribution $p(\mathbf{R}_k | \mathbf{Y}_{1:k})$ subject to having $p(\mathbf{R}_{k-1} | \mathbf{Y}_{1:k-1})$. Then given the discrete random measure $\{\mathbf{R}_{k-1}^{(n)}, w_{k-1}^{(n)}; n = 1, \dots, N\}$ and the observation \mathbf{Y}_k we want to approximate $\{w_k^{(n)}; n = 1, \dots, N\}$. Using Bayes' rule and concept of importance sampling, the new weights are updated as follow [8]

$$w_k^{(n)} \propto w_{k-1}^{(n)} \frac{p(\mathbf{Y}_k | \mathbf{R}_k^{(n)}) p(\mathbf{R}_k^{(n)} | \mathbf{R}_{k-1}^{(n)})}{\pi(\mathbf{R}_k^{(n)} | \mathbf{R}_{k-1}^{(n)}, \mathbf{Y}_{1:k})} \quad (4)$$

where $\pi(\cdot)$ is the importance density. The choice of importance density is one of the crucial issues in designing the PF. In general, the closer the importance density to the actual posteriori distribution, the better the approximation is. The most popular choice of the importance density is $\pi(\mathbf{R}_k | \mathbf{R}_{k-1}^{(n)}, \mathbf{Y}_{1:k}) = p(\mathbf{R}_k | \mathbf{R}_{k-1}^{(n)})$. This implies that equation (4) reduces to

$$w_k^{(n)} \propto w_{k-1}^{(n)} p(\mathbf{Y}_k | \mathbf{R}_k^{(n)}) \quad (5)$$

where $p(\mathbf{Y}_k | \mathbf{R}_k^{(n)})$ is the likelihood function and is equivalent to the noise distribution $p(\mathbf{N}_k)$ which has been already assumed to be GWN.

2) *Estimation of Source Amplitudes:* In this section, we derive a maximum likelihood estimator for ECD amplitudes \mathbf{a}_i . It follows from (1) that the negative log-likelihood function of the observed data samples is

$$f(\theta_k; \mathbf{Y}_k) = \sum_{t=1}^T [\mathbf{Y}_k - \sum_{i=1}^q \mathbf{H}(\rho_k^i) \mathbf{a}_k^i \psi_k^i]^T \mathbf{Q} [\mathbf{Y}_k - \sum_{i=1}^q \mathbf{H}(\rho_k^i) \mathbf{a}_k^i \psi_k^i] + \ln(\mathbf{Q}) + \text{constant} \quad (6)$$

By equating the gradient of the function $f(\theta_k; \mathbf{Y}_k)$ with respect to parameter of interest \mathbf{a}_i to zero, estimations of the amplitudes are given by

$$\hat{\mathbf{a}}_k^i = \sum_{t=1}^T \mathbf{H}^\dagger(\rho_k^i) [\mathbf{Y}_k - \sum_{j=1, j \neq i}^q \mathbf{H}(\rho_k^j) \mathbf{a}_k^j \psi_k^j] / \psi_k^i(t) \quad (7)$$

where $\mathbf{H}^\dagger = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T$ is the pseudo-inverse of \mathbf{H} . The likelihood monotonically increases at each iteration and therefore convergence of the above algorithm to a local maximum is guaranteed. However, in the case of noisy individual trials and inaccurate estimates of other parameters, the amplitude may not be truly estimated. This may also cause divergence of the method in some high noisy trials. To prevent this failure, we assume that the evolution of the parameters are Markovian and then we may write

$$\hat{\mathbf{a}}_k^i = \hat{\mathbf{a}}_{k-1}^i + \lambda_a (\tilde{\mathbf{a}}_k^i - \hat{\mathbf{a}}_{k-1}^i) \quad (8)$$

where $0 < \lambda_a \leq 1$ is a forgetting factor. This recursive equation prevents sudden changes of amplitude because of some very low SNR of individual trials and by small enough values of λ_a guarantees the stability of the filtering.

3) *Estimation of ECD Shape Parameters:* ECD shape parameters include mean μ_k^i and variance σ_k^i of Gaussian templates. In general, the optimization problem in (6) does not appear to admit a closed-form solution similar to amplitude for mean and variance. Here, we use Newton-Raphson technique [9] to approximately solve this problem. Although PF can again be employed to estimate the nonlinear parameters μ_k^i and σ_k^i , PF requires extensive memory and computational time. Moreover, choosing a proper initial point for Newton-Raphson technique also can result in better estimation than the PF method.

In Newton-Raphson technique the following formulation is used to estimate the shape parameters $\gamma_k^i \in \{\mu_k^i, \sigma_k^i\}$ in each iteration

$$\gamma_{k+1}^i = \gamma_k^i - \lambda_\gamma \left[\frac{\partial^2 f}{\partial \gamma_k^{i2}} \right]^{-1} \frac{\partial f}{\partial \gamma_k^i} \quad (9)$$

for the same reason as we mentioned in the previous section, the forgetting factor $0 < \lambda_\gamma \leq 1$ is added to the original Newton-Raphson formulation to guarantee the stability of the filtering in the face of very low SNR. This equation needs the first and second order gradients of log-likelihood function $f(\theta_k; \mathbf{Y}_k)$ with respect to μ_k^i and σ_k^i . In the following, we drop the superscript i for the sake of convenience. The first and second order gradients have been calculated and simplified as

$$\frac{\partial f}{\partial \mu_k} = \frac{4}{\sigma_k^2} \sum_{t=1}^T (\mu_k - t) \psi_k(t) \alpha_k^T (\mathbf{Y}_k - \Psi) \quad (10)$$

$$\frac{\partial^2 f}{\partial \mu_k^2} = \frac{4}{\sigma_k^2} \sum_{t=1}^T \left(1 - \frac{2}{\sigma_k^2} (\mu_k - t)^2 \right) \psi_k(t) \alpha_k^T (\mathbf{Y}_k - \Psi) + \frac{2}{\sigma_k^2} (\mu_k - t)^2 \psi_k(t)^2 \alpha_k^T \alpha_k \quad (11)$$

$$\frac{\partial f}{\partial \sigma_k} = -\frac{4}{\sigma_k^3} \sum_{t=1}^T (\mu_k - t)^2 \psi_k(t) \alpha_k^T (\mathbf{Y}_k - \Psi) \quad (12)$$

$$\frac{\partial^2 f}{\partial \sigma_k^2} = \frac{4}{\sigma_k^4} \sum_{t=1}^T \left(3 - \frac{2}{\sigma_k^2} (\mu_k - t)^4 \right) \psi_k(t) \alpha_k^T (\mathbf{Y}_k - \Psi) + \frac{2}{\sigma_k^2} (\mu_k - t)^4 \psi_k(t)^2 \alpha_k^T \alpha_k \quad (13)$$

where $\Psi = \sum_{i=1}^q \mathbf{H}(\rho_k) \mathbf{a}_k \psi_k$ and $\alpha_k = \mathbf{H}(\rho_k) \mathbf{a}_k$. The latencies and widths of the ERP subcomponents estimated by the above formulation may not be the global minimums and they depend on the true estimation of initial points. The dependency of the results to the initial points is more sensitive in Newton-Raphson method than the PF and maximum likelihood methods employed for estimation of locations and amplitudes. The initial points as we will explain in the results section can be chosen according to the latency and width of fitted Gaussian waveforms to the ensemble average over all

trials.

4) *Overall Algorithm:* The aim of overall algorithm is to update the parameters recursively based on the available measurements. The pseudo-code of the method has been presented in Algorithm 1. In this method, each particle not only holds a parameter for location $\mathbf{R}_k^{(n)}$, but also holds parameters for amplitude $\mathbf{a}_k^{i(n)}$, variance $\sigma_k^{i(n)}$, and mean $\mu_k^{i(n)}$ of the Gaussian waveforms. Moreover, in this algorithm we assume that the initial points of the ECD locations are known.

Algorithm 1 Pseudo-code of the proposed method for tracking of ERP subcomponent parameters

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set  $k = 0$  and generate random numbers  $\mathbf{R}_0^{(n)}$  according to Gaussian distribution with mean in the known locations.
set  $\mu_0^{i(n)}$  and  $\sigma_0^{i(n)}$  equal to the parameters of fitted Gaussian functions to the average of ERPs over all trials.
for  $k = 1$  to  $K$  do
  - generate zero mean random noise  $\mathbf{w}_k^{(n)}$  with a priori covariance matrix  $\mathbf{Q}_w$  and set  $\mathbf{R}_k^{(n)} = \mathbf{R}_k^{(n)} + \mathbf{w}_k^{(n)}$ .
  - update new weights using  $w_k^{(n)}$  according to equation (5).
  - normalize the weights  $w_k^{(n)} = w_k^{(n)} / \sum_{n=1}^N w_k^{(n)}$ .
  - update  $\mathbf{a}_k^{i(n)}$  given  $\mathbf{R}_k^{(n)}$ ,  $\sigma_k^{i(n)}$  and  $\mu_k^{i(n)}$  and  $\mathbf{Y}_k$  for each particle using equations (7) and (8).
  - update  $\sigma_k^{i(n)}$  and  $\mu_k^{i(n)}$  given  $\mathbf{a}_k^{i(n)}$ ,  $\mathbf{R}_k^{(n)}$  and  $\mathbf{Y}_k$  for each particle using equations (9)-(13).
  - resample new  $N$  particles with replacement according to their importance weights  $w_k^{(n)}$  [5].
end for

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III. RESULTS

In this section, we apply the proposed method to synthetic and real EEG data to demonstrate its application in an empirical setting.

A. Simulated Data Results

We generate a set of EEG data containing ERP waves in the interval between 200ms and 500ms post-stimulus. The sampling frequency was set to 250Hz and the number of trials was set to 60. Two moving sources one in frontal and one in parietal sites were used for simulating ERP subcomponents. The shape of ERP subcomponents assumed to be Gaussian functions. The amplitude profile of the frontal source was assumed to decrease linearly, but its latency and width assumed to decrease linearly from trial to trial. The amplitude profile of the second source were assumed to approximately be constant, but its latency and width were assumed to decrease linearly across trials. GWN with different level was added to the amplitudes, the latencies and the widths of both sources. Spatially uncorrelated Gaussian noise was added to the simulated signals to achieve a realistic SNR levels of -5dB. In this example values of λ_ζ (ζ denotes a or γ) were set to 0.8. The simulated and estimated amplitude, latency and width of frontal (red lines) and parietal (blue lines) sources has been shown in Fig. 1. The simulated and estimated locations in axial and sagittal views for the same data has been depicted in Fig. 2. The algorithm exhibits acceptable performance in such a low SNR environment.

B. Real Data Results

Real data was obtained in an *odd-ball* paradigm in the Cardiff University Brain Research Imaging Center (CUBRIC). Participants heard in total 300 tones, 240 (80%) of which were frequent and 60 (20%) of which were infrequent. During acquisition, the frequency bandwidth of the linear bandpass filter was 0.03–40Hz and the sampling rate was 250Hz. An Fz (midline frontal) reference electrode was employed. EEG data were recorded with 25 scalp electrodes. In addition, horizontal and vertical electrooculogram were also recorded to identify eye blinks and movements.

A 200 ms pre-stimulus interval was used for baseline correction. Eye blinks were rejected using independent component analysis (ICA) [10]. We are interested in P300 whose origin and number of responsible sources is unknown. In some clinical literature for example in schizophrenic investigation [11] P300 is assumed to be specified by two sources in frontal and parietal sites. The P300 activity of the superior ECD corresponds mainly to the classical P3a and that of the basal oriented ECD to P3b [11]. The number of sources has extreme effect on the results. Increasing the number of sources exponentially increases the number of particles. For simplicity and better estimation, we assume only two sources exist.

Selecting the initial points is a crucial since it affects the behavior and convergence of the filtering. The initial point of location is assumed to be in frontal site for P3a and to be in parietal site for P3b. The data is average re-referenced and segmented around the P300 component. Therefore, epochs from 200ms to 500ms time-locked to stimulus onset for infrequent trials were extracted. To find the initial point of amplitude, latency, and width of each subcomponent, we assume that the locations are fixed and using the proposed method, the parameters are estimated for ensemble average over 60 trials. Since the average data suffers from less noise than single trial data, all values of λ_ζ were set to 1 and the algorithm was run until the results do not change.

The results are shown in Fig. 3. The amplitudes are the absolute value of the three dimensional source amplitude moments. These figures show that amplitude of the P3b is more stable than that of P3a in most trials. P3a amplitude is habituating from trial to trial while P3b approximately is constant. The latency of p3b is less than that of P3a (P3b occurs earlier than P3a) and also P3b latency is slightly more consistent than P3a latency during trials. Moreover, the width of P3b is larger and more stable than that of P3a. These observations can demonstrate that P3b is a stronger subcomponent than P3a in both amplitude and width. Fig. 4 depicts the location of P3a and P3b in axial and sagittal views. Red markers in frontal site denote the location of P3a and blue markers in parietal site denote the location of P3b subcomponents. The locations of P3b in these subjects are again more consistent than the location of P3a which is moving in the frontal site.

IV. CONCLUSION

In this study a method for separation of ERP subcomponent and tracking was proposed. A model based on ECD was given and its parameters were estimated using PF, maximum likelihood and Newton-Raphson techniques. In addition recursive methods for maximum likelihood and Neton-Raphson techniques were proposed to guarantee the stability of filtering in the case of very low SNRs.

Accurate extracting and tracking of single trial ERP has great benefit in several contexts. For example, for ERP researchers interested in using ERPs to isolate cognitive processes, the reliance on averaging introduces an inevitable degree of caution when making inferences about the onset times of processes. Caution is also necessary when inferring whether peak amplitude differences between averaged ERPs for different conditions do in fact reflect consistent peak amplitude differences at the level of individual trials.

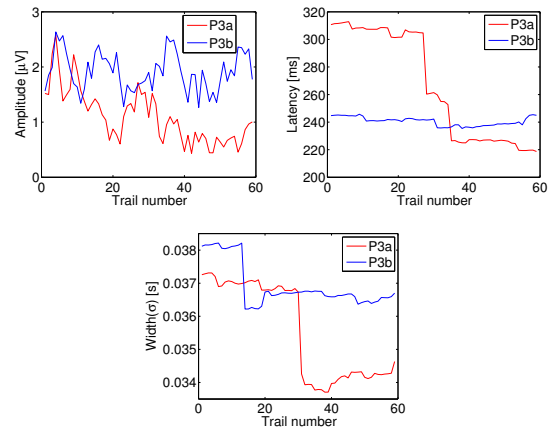


Fig. 3. Estimated amplitude, mean and variance in real data for P3a and P3b by red and blue lines respectively.

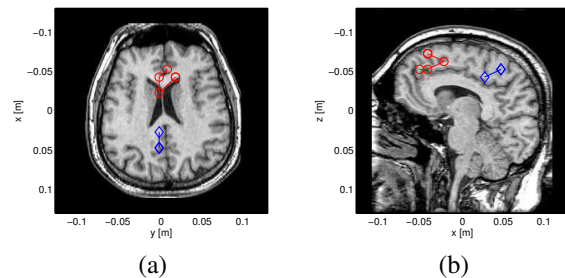


Fig. 4. Estimated location of P3a and P3b using red and blue markers in (a) axial and (b) sagittal views.

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