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A Simple Filter Circuit for Denoising Biomechanical Impact Signals

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Abstract— **We present a simple scheme for denoising non-stationary biomechanical signals with the aim of accurately estimating their second derivative (acceleration). The method is based on filtering in fractional Fourier domains using well-known low-pass filters in a way that amounts to a time-varying cut-off threshold. The resulting algorithm is linear and its design is facilitated by the relationship between the fractional Fourier transform and joint time-frequency representations. The implemented filter circuit employs only three low-order filters while its efficiency is further supported by the low computational complexity of the fractional Fourier transform. The results demonstrate that the proposed method can denoise the signals effectively and is more robust against noise as compared to conventional low-pass filters.**

I. Introduction

THE analysis of biomechanical signals is essential in understanding human motion. Of particular interest are the derivatives of these signals as they represent the velocity and acceleration of the corresponding body segments. However the process of computing these derivatives is not a straightforward task due to the fact that the acquired signals are contaminated by noise. Since noise is amplified during differentiation, a direct application of this operation to compute derivatives will always lead to severely distorted results. For this reason, noise removal must be applied prior to differentiation.

Schemes for kinematic data denoising to this day have mainly relied on digital filtering [1]. For example, the second-order bi-directional Butterworth filter (BW) [2] has been the standard method in many studies under the implicit assumption that signals at hand are stationary. However, the frequency content of biomechanical signals may undergo considerable changes especially when activities that involve impacts are considered since there is an abrupt transition from the low-frequency part of the movement (aerial or swing phase) into higher frequencies (impact phase) and vice versa. Conventional filtering in the Fourier transform (FT) domain cannot cater for these changes, therefore, it either under-smoothes or over-smoothes the displacement data [3].

To filter a given non-stationary signal effectively, the applied cut-off threshold should follow the time evolution of the signal's frequency content. The presented method, originally introduced in [4], achieves this by drawing upon the unique relationship between the fractional Fourier transform (FrFT) of a signal and its time-frequency (TF) representation to design a time-varying cut-off threshold for low-pass filtering. In this paper, we illustrate the differences of the method from classical low-pass filtering by contrasting their frequency responses in the TF plane. We then examine its denoising performance for different types of filters employed in the circuit. We further study its robustness against noise in comparison to conventional filtering by means of simulations. Section II provides a concise overview of the theoretical background and describes the algorithm. In section III, experimental results are provided. Conclusions are finally drawn in section IV.

II. Preliminaries

A. Theoretical Background

The ath-order fractional Fourier transform of the signal $s(t)$ may be defined for $0<|a|<2$ as [5]:

$$s_a(t_a) = \int B_a(t_a,t)s(t)dt \tag{1}$$

where $B_a(t_a,t)$ represents the kernel of the FrFT,

$$B_a(t_a,t) = \frac{\exp(-j\pi\,\mathrm{sgn}(\phi)/4 + j\phi/2)}{|\sin\phi|^{1/2}} \times \exp[j\pi(t_a^2\cot\phi - 2t_a t\csc\phi + t^2\cot\phi)] \tag{2}$$

where $\phi=a\pi/2$. Some essential properties of the FrFT are compactly provided as follows:

(i) The transformation is linear,

(ii) The transformation is additive in index (i.e. the a_2th-order FrFT of the signal $s_{a1}(t_{a1})$ is $s_{a1+a2}(t_{a1+a2})$,

(iii) $s_o(t_o)=s(t)$ and $s_1(t_1)=S(f)$,

where $S(f)$ is the FT of the signal. The FrFT is a generalization of the FT and converts the signal from the time domain to the ath fractional domain, t_a.

Another significant property of the FrFT is its relationship with TF representations [6], [7]. A fundamental TF representation is the Wigner distribution (WD). If $s(t)$ is the signal then its WD can be expressed as:

$$W_s(t,f) = \int s(t+\frac{\tau}{2})s^*(t-\frac{\tau}{2})e^{-j2\pi\tau f}d\tau \tag{3}$$

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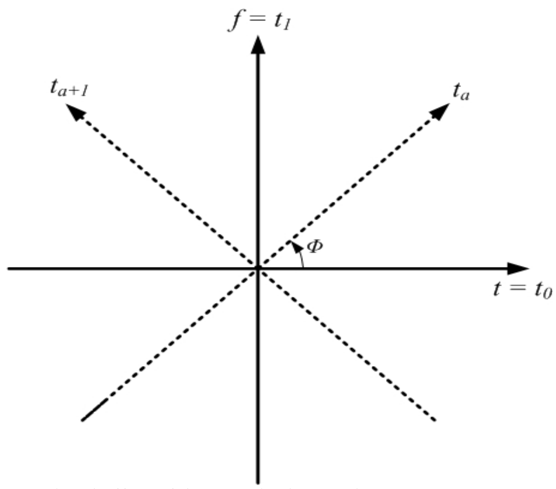

Fig. 1 Rotational effect of the FrFT in the TF plane.

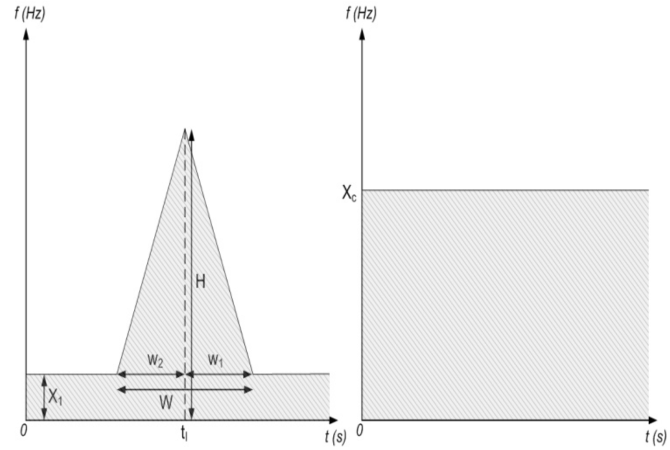

Fig. 2. (a) Designed time-varying cutoff frequency, and (b) low-pass cutoff threshold corresponding to conventional filtering. The frequency response equals to one inside the gray areas and zero otherwise. Only the first quadrant of the TF plane is shown.

where τ denotes the time lag. The WD of the *a*th order FrFT of a signal is expressed in a rotated system of reference as compared to the original *time* and *frequency* axes. The new axes correspond to the *fractional time* t_a and the *fractional frequency* t_{a+1} (Fig.1). As an example, in relation to property (iii) listed above, the FrFT of order 1, i.e. the classical FT, rotates the axes by 90 degrees.

B. *Description of the algorithm*

The rotational effect of the FrFT on the WD has an important consequence; the integral projection of the WD of the original signal onto the oblique axis t_a equals the squared magnitude of the *a*th-order FrFT of the signal [5]. For example, when $a=1$, this projection is the power spectral density of the signal. Equally, the cutoff frequency threshold of a Fourier-based low-pass filter may be visualized in the TF plane as a straight line parallel to the time axis (Fig. 2b).

By generalizing the concept of Fourier filtering to arbitrary fractional domains one has the flexibility to retain TF strips not necessarily parallel to the time axis. Furthermore, if this operation is successively applied then TF areas of different shapes can easily be singled out [8].

The proposed scheme consists of two distinct steps. The first is to determine a suitable time-variant cut-off frequency, and the second is to implement the corresponding low-pass filter by operating in consecutive fractional domains. The design of the time-variant cutoff threshold is greatly facilitated in the TF plane. Based on previous studies of impact signals [3], it was observed that the filtering boundary should extend toward higher frequencies in the impact region to capture the higher frequencies induced by the impact event, while a low cutoff is suitable at all other times. The simplest possible boundary that satisfies the above conditions is the one shown in Fig.2a. As opposed to conventional filtering where a single cutoff frequency is applied to the whole signal (Fig.2b), Fig.2a clearly provides a more appropriate threshold.

The presented time-varying cut-off threshold is controlled by four parameters. The cutoff frequency X_1 corresponds to the aerial phases of the signal. Point t_I represents the time of maximum acceleration, and the width W of the triangle refers to the duration of the impact phase. This consists of two separate time durations, W_1 and W_2, as shown in the figure. The height H of the triangle corresponds to the impact-induced expansion of the frequency content. The physical relevance of the above parameters to the actual motion enables the development of application-specific methods to estimate them, such as the empirical algorithms used in [4].

The simple triangle-shaped TF boundary of Fig.2a defines a time-varying cutoff frequency threshold with a low-pass filtering effect – if low-pass filters are used in the three domains determined by the three sides of the triangle. These are the ordinary frequency domain f ($=t_1$), and the two fractional ones, t_{a1} and t_{a2}, which are perpendicular to the right and left sides of the triangle, respectively. The intersections of the sides of the triangle with the specified axes t_{a1} and t_{a2} provide the low-pass cutoff values for each fractional domain.

To determine the appropriate fractional domains in which to filter, as well as the necessary cutoff values, we exploit the geometry of the designed boundary. Based on the values of the parameters H, W_1, W_2, t_I, and X_1 these can be calculated as follows (Fig. 3):

$$\phi_1 = 90 - \arctan\left(\frac{h}{w_1}\right) \quad and \quad X_{a_1} = w_1 \cos\phi_1, \tag{4}$$

where $w_1 = W_1/Ts$ and $h = H/Fs$, with Ts the sampling period and Fs the frequency step. Similarly, we obtain

$$\phi_2 = 90 + \arctan\left(\frac{h}{w_2}\right) \quad and \quad X_{a_2} = w_2 \cos\phi_2, \tag{5}$$

where $w_2 = W_2/Ts$. Having determined the required fractional orders and cutoffs one can design suitable multiplicative functions to carry on with the filtering, i.e. in the t_{a1} domain a function $g_{a1}(t_{a1})$ would be multiplied with $s_{a1}(t_{a1})$, then the result would be transformed into the t_{a2} domain and multiplied with a function $g_{a2}(t_{a2})$, followed by a final FrFT of the result back to the time domain t. Alternatively, one can transform the signal into the $(a_1\text{-}1)^{\text{th}}$ domain and convolve with $g_{a1-1}(t_{a1-1})$, where $g_{a1}(t_{a1})$ is the FT of $g_{a1-1}(t_{a1-1})$, and proceed similarly for the second domain. The latter approach allows us to choose from a wide range of available well-studied low-pass filters. The overall filter circuit is shown in Fig.4.

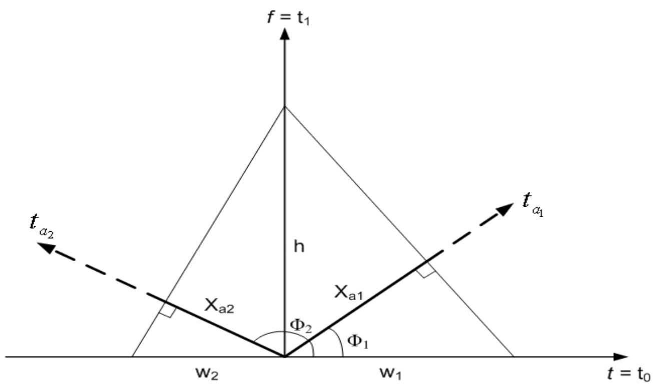

Fig. 3. The two fractional domains in which low-pass filtering takes place, and the corresponding cutoff thresholds.

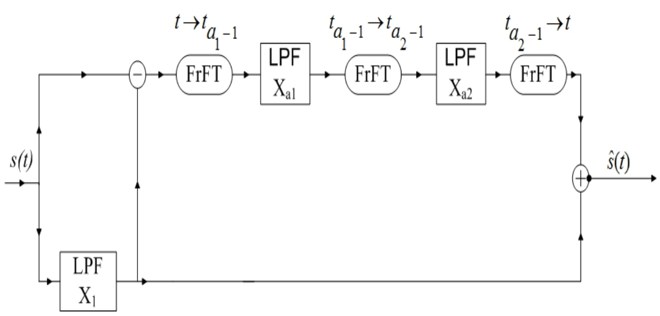

Fig. 4. Block diagram of the proposed filter circuit.

III. Experimental Results

The test signal employed here is the displacement data provided by Dowling [9], for which the separately measured acceleration $\alpha(t)$ is also available for comparisons. The motion involved a horizontally rotating pendulum that swung forward until it collided with a non-rigid barrier and then bounced back. The sampling rate was 512 Hz. The displacement signal was extrapolated on either edge to compensate for end-point distortions. Forward and reverse pass filtering was used to avoid time-shift distortions. The values for the parameters of the triangle were determined empirically [4] as: $X_1 = 12$ Hz, $W_1 = 0.111$s, $W_2 = 0.111s$, and $H = 36$ Hz. The accelerations $\hat{\alpha}(t)$ were computed using the second-order forward differences of the filtered signals.

The first experiment involved different types of popular low-pass filters, such as the Butterworth, Chebyshev (type I and II), and elliptical, to assess their effect on the result and their suitability for the filter circuit of Fig.4. Iterative calculations determined filter orders, as well as passband and stopband ripple allowances. The iterations aimed at minimizing an error function combining the RMS and absolute peak (AP) errors – at equal weights – between the derived and the reference accelerations. AP errors were calculated as $\left(\left|\hat{\alpha}(t_I)\right| - \left|\alpha(t_I)\right|\right) \Big/ \left|\alpha(t_I)\right|$. The results indicated that the Butterworth filter was most suitable for use with the presented scheme achieving RMS error equal to 14.5 and AP error equal to 0.002. Next was the elliptical filter with RMS error equal to 15.5 and AP error equal to 0.001. The RMS errors obtained for the Chebyshev type I and II filters were 15.8 and 19.9, respectively. The AP errors for the same filters were 0.002 and 0.012, respectively. The higher performance of the Butterworth filter can mainly be attributed to its smooth roll-off, which compensates for the sharp angles of the triangular boundary.

In the second experiment we examine the robustness of the overall scheme against noise. To this end, different levels of white Gaussian noise were added to the signal. The levels of noise were measured in terms of the signal-to-noise ratio (SNR). However, it should be noted that the signal already contained noise and no assumptions were made about its statistics. Table I presents the RMS and AP errors achieved after denoising with the presented scheme. For comparison, results obtained using a conventional Butterworth filter – a popular choice in Biomechanics – are also presented. The cutoff frequency of this filter, as well as its order, was determined so that the combined RMS and AP error was minimized. The listed RMS and AP errors are averages over 100 realizations of the noisy inputs.

The low-pass nature of the proposed scheme implies that noise lying below the cutoff frequency X_1 cannot be eliminated. Thus, there is a limit with respect to the minimum level of noise that the method can deal with. For the signal at hand, this was found to be equal to 40 dB SNR (Fig. 5). However, the time-varying cutoff threshold can protect the signal much more effectively than any conventional low-pass filter. To focus on the impact phase in particular (which consists of frequencies well above the X_1 value), we experimented with added noise of colored nature, i.e. noise residing above X_1. The results from this experiment are presented in Table II. As expected, the scheme could now cope with noise down to 0 dB SNR.

IV. Discussion

A simple filter circuit based on the concept of repeated FrFT filtering has been presented for the effective denoising of biomechanical impact signals. The TF visualization of the designed time-varying cutoff threshold shows that the circuit's frequency response is appropriate for the treatment of signals with distinct non-stationarities, as it caters for the temporal evolution of their spectrum. The shape of the designed cutoff was also found beneficial in protecting the signal against noise, as opposed to the flat cutoff frequency of conventional methods. It was also shown here that the

Butterworth filter is a more suitable choice for use in the presented circuit as compared to other well-known filters.

Our experimental results indicate that the proposed filter can efficiently remove noise from biomechanical impact data while preserving the higher frequencies attributed to the impact phase. The presented method could also be useful in a wide range of application areas, where signals with non-stationary characteristics are considered. It should also be stressed that the above advantages come at a very low computational cost, since the complexity of the FrFT is $O(N\log N)$ [10], same as that of the classical Fourier transform.

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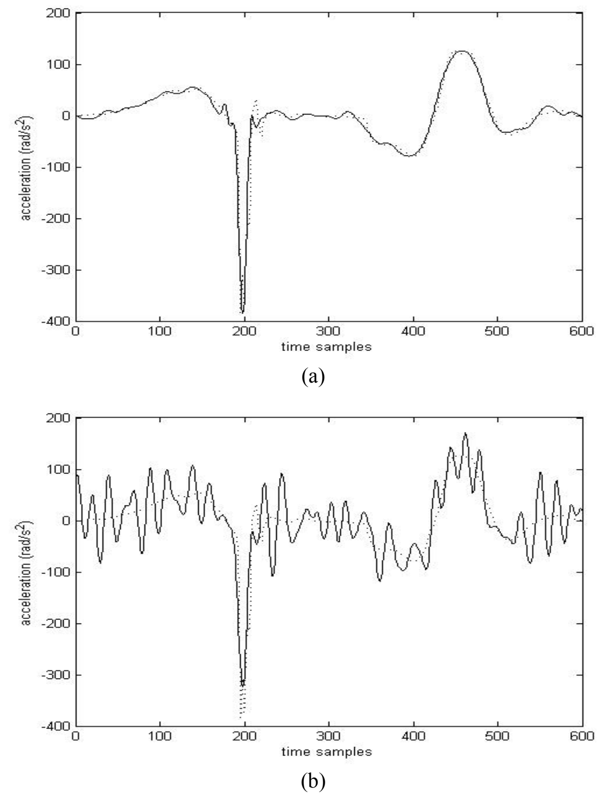

Fig. 5. Calculated acceleration after applying: (a) the proposed filter circuit (solid line) with added white noise of SNR = 40dB, and (b) the Butterworth digital low-pass filter (solid line) at the same SNR level. The reference acceleration measured by accelerometers is also shown (dotted lines).

TABLE I

RMS AND PEAK ERRORS OF THE CALCULATED ACCELERATIONS CORRESPONDING TO DIFFERENT LEVELS OF ADDED WHITE NOISE FOR DIFFERENT SNR VALUES DENOISED WITH THE PRESENTED FILTER CIRCUIT (USING BUTTERWORTH FILTERS) AND CONVENTIONAL LOW-PASS FILTERING

Method \ SNR(dB)		100	50	40	30	20	10	0
Proposed	RMSE	13.76	13.76	15.34	15.77	19.06	30.87	41.66
	Peak (%)	2.96	2.96	6.63	11.20	17.60	24.70	35.40
Conventional LP filtering	RMSE	34.70	34.61	34.21	34.92	43.16	64.65	163.13
	Peak (%)	9.91	10.33	15.36	29.15	42.75	47.80	63.68

TABLE II

RMS AND PEAK ERRORS OF THE CALCULATED ACCELERATIONS CORRESPONDING TO DIFFERENT LEVELS OF ADDED NOISE ABOVE X_1 WITH DIFFERENT SNR VALUES DENOISED WITH THE PRESENTED FILTER CIRCUIT (USING BUTTERWORTH FILTERS) AND CONVENTIONAL LOW-PASS FILTERING

Method \ SNR(dB)		100	50	10	0	-10	-20	-30
Proposed	RMSE	13.76	13.76	13.99	14.59	17. 81	21.91	28.27
	Peak (%)	2.96	2.96	3.08	9.51	14.07	15.82	34.04
Conventional LP filtering	RMSE	32.09	32.09	32.22	33.65	35.16	34.77	65.07
	Peak (%)	8.08	8.08	8.88	16.38	29.42	43.10	45.15