An Iterative Method for Indirectly Solving the Inverse Problem of Electrocardiography

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Abstract

Solution of the inverse problem of electrocardiography is an ill-posed problem and noise sensitive, leading to non-physiological solutions under noise conditions. We propose the use of an indirect iterative method for solving the inverse problem in terms of multiple moving dipoles.

A database of surface potentials generated by single dipoles was generated by solving the forward problem. The inverse problem was solved by iteratively searching the combination of surface potentials in our database most similar to the observed surface potentials.

Simulated surface potentials were used to test the algorithm. Single dipoles were accurately located in 92% simulations and in 90% simulations in which noise (SNR=6dB) was added. Double dipoles were accurately located in 78% simulations, and 72% when 6dB noise was added. The present method has shown to be accurate and robust against noise.

1. Introduction

The inverse problem of electrocardiography consists in the non-invasive determination of the electrical activity of the heart from measurements of potentials on the surface of the torso [1,3]. There are two alternatives for solving the electrocardiographic inverse problem based on different formulations of the intra-cardiac sources: (1) a distribution of electrical potentials on the epicardial surface [1] and (2) dipolar representations. Solutions based on the retrieval of the electrical activity on the surface of the epicardium have an ill-posed nature and high sensitivity to noise, leading to non-physiological solutions under low signal-to-noise ratios [2,3]. Solutions based on the simplification of the electrical activity of the heart into a limited number of electrical dipoles may be less accurate than those based on the retrieval of potentials on the entire epicardial surface but more easily interpreted for physicians. Restrictions on the number of electrical dipoles to be retrieved allow for reasonable computing times and physiologically possible solutions.

Different approaches have been proposed for the solution of the inverse problem of electrocardiography in terms of electrical dipoles. The single moving dipole model [2] is the most straightforward approach but restricted to a representation of the electrical activity of the heart as a unique and spatially localized wavefront. A multipolar approach [4] is more realistic, but more noise sensitive and susceptible to crosstalk errors between different dipoles [2]. The two moving dipole model offers a compromise between accuracy and crosstalk effects, although is restricted to a dipolar representation of the electrical activity of the heart.

In this work the inverse problem of electrocardiography is solved by an alternative indirect method based on a multipolar representation of the heart electrical activity. Surface potentials generated by multiple dipoles are iteratively combined to maximize the similarity between computed and recorded potentials. The performance of the algorithm is tested by localizing electrical dipoles from simulated surface potentials in an eccentric spheres model of the human torso.

2. Methods

2.1. Approach for ECG inverse problem

The present approach for indirectly solving the ECG inverse problem is based on an iterative search for the combination of surface potentials generated by single dipoles which best describes the recorded surface potentials. Surface potentials produced by single dipoles at multiple locations and orientations are computed by solving the forward problem with the boundary element method (BEM) [5-7] and constitute our surface potential database (SPDB). Potentials in the SPDB are combined and compared with the recorded potentials in terms of their relative difference measure (RDM*) and the combination of single dipoles with lower RDM* is selected.

2.2. Potential database generation

The torso model used to generate surface potentials (depicted in Figure 1) was based on the eccentric spheres model proposed in the literature [8] with inner organs

being modeled as spheres with homogeneous conductivities: blood $\sigma_1=0.006$ Ω^{-1}/cm, heart muscle $\sigma_2=0.002$ Ω^{-1}/cm, lungs $\sigma_3=0.0005$ Ω^{-1}/cm, muscle $\sigma_4=0.00125$ Ω^{-1}/cm, subcutaneous fat $\sigma_5=0.0004$ Ω^{-1}/cm and external air $\sigma_6=0$ Ω^{-1}/cm.

Spheres radius

r_1=4 cm
r_2=5 cm
r_3=11 cm
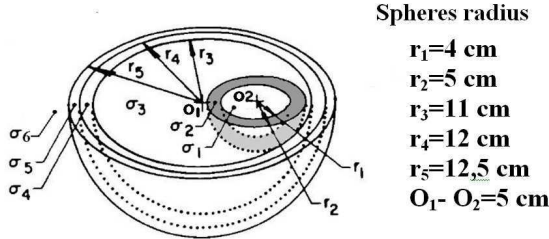r_4=12 cm
r_5=12,5 cm
O_1- O_2=5 cm

Figure 1. Spherical torso model employed [8]. The shaded volume conductor corresponds to the myocardium.

Surface potentials constituting the SPDB were computed by solving the forward problem on the surface of each sphere constituted by 162 nodes by means of the BEM for a set of electrical dipoles. Electrical dipoles with unitary amplitude and x, y or z orientation where located inside the myocardial volume at N_r, N_θ and N_φ discrete positions equally spaced in r, θ and φ respectively in a coordinate system centred at O_2. (i.e. for N_r=3, N_θ=9 and N_φ=8 the SPDB is constituted by potentials generated by unitary dipoles at 216 locations and 3 orientations for each location). Potentials generated by a single dipole with an arbitrary location and orientation could be calculated, therefore, as a linear combination of those obtained by unitary dipoles (x, y, z) at the same location.

2.3. Iterative search algorithm

Potentials on the surface of the outer sphere generated by multiple dipoles can be computed as a linear combination of potentials generated by single dipoles. Electrical dipoles generating a given potential distribution (V, the test or recorded potential) could be estimated by choosing the linear combination of potentials generated by single unitary dipoles (U, the estimated potential) which results in a higher similarity of both potential distributions.

An exhaustive search, testing all possible combinations of dipoles at any position and with any orientation, requires high computation times which make this approach impractical. For this reason, we propose the use of an iterative algorithm for minimization of the dissimilarity of surface potentials. Dissimilarity of surface potentials was measured as their RDM* value [5]:

$$RDM* = \sqrt{\frac{RDM^2 - (1-MAG)^2}{MAG}}$$

where RDM and MAG are defined as [6]:

$$RDM = \sqrt{\frac{\sum(V-U)^2}{\sum V^2}} \qquad MAG = \sqrt{\frac{\sum U^2}{\sum V^2}}$$

For the first iteration only linear combinations of potentials generated by single dipoles at the center of the myocardial wall are computed with steps in θ and φ equal to $\pi/2$ and possible weightings equal to -1, 0 and 1. The combination with lower RDM* is chosen and thus an initial guess for θ and φ coordinates for each searched dipole is obtained. Iteratively, search intervals in r, θ and φ are decreased and weighting coefficients ranging from -5 to 5 are applied to each unitary dipole. Weightings, and location of unitary dipoles that minimize RDM* for each searched dipole are chosen as the initial guess for subsequent iterations. As a consequence of the search strategy implemented, computing times will be dependent on the number of dipoles to be searched.

3. Results

3.1. Accuracy in single dipole location

Accuracy in the location of a single dipole is compromised by to factors: (1) selection of an iterative algorithm as opposed to an exhaustive search and (2) location of dipoles is not restricted to possible locations selected for the generation of the SPDB.

In order to quantify the errors introduced by the search strategy implemented, test potentials were obtained by solving the forward problem by BEM. A total of 1000 single dipoles at arbitrary locations among those stored in the SPDB (N_r=6, N_θ=17, N_φ=16) and with orientations among those that will be evaluated by the search algorithm were simulated. Surface potentials were used by the present iterative algorithm for the computation of the location and orientation of the electrical dipole which best matches with simulated potentials. Errors in location (ε_D) and orientation (ε_O) for each simulation were computed.

Location of single dipoles was accomplished without error in 85.8% simulations, whereas in the remaining cases the most proximal location possible was chosen. Estimation of location of single dipoles was accomplished with errors in location equal to 1.2±3.1 mm and errors in orientation equal to 4±11 °. Errors in orientation occurred for those dipoles whose location was not accurately determined and compensate for the error in location, resulting in low RDM* values.

3.2. Influence of SPDB resolution

Since indirect location of dipoles is restricted to the location of dipoles included in our database, the amount

of dipoles that constitutes our database may have a great influence on the performance of the presented algorithm. In order to quantify the influence of the database resolution in the performance of the algorithm, surface potentials generated by a single dipole were obtained by forward BEM computation for 1000 different dipoles at arbitrary location and orientation within the myocardial wall. Location and orientation of dipoles generating such surface potentials were obtained by the present algorithm with 3 different database sizes: (1) $N_r=3$, $N_\theta=9$, $N_\varphi=8$; (2) $N_r=6$, $N_\theta=17$, $N_\varphi=16$; (3) $N_r=11$, $N_\theta=33$, $N_\varphi=32$. Errors in location (ε_D) and orientation (ε_O) were computed. Also, the percentage of accurate dipole locations (ADL) was calculated as the percentage of dipoles located with errors under 10mm.

Table 1 shows the errors in dipole calculation for the three database resolutions tested. Notice that errors when dipole locations are not restricted to locations included in the SPDB are higher than those obtained when dipoles are located at positions included in the database for the same database size (5.4 mm vs. 1.2 mm). This error is due to the existent free space between the stored dipole locations in the SPDB and thus resulting locations are restricted to a subset of locations. In the best case, the algorithm will converge to the nearest r, θ, φ in the stored subset and thus, the larger the SPDB resolution, the smaller distance error is obtained. The distance error vs. percentage of occurrences presented a Rayleigh distribution, for the three database resolution, centred on 8 mm, 5 mm and 3 mm distance error for SPDB resolutions 1, 2 and 3, respectively.

Table 1. Performance for different database resolutions

SPDB resolution	1	2	3
ε_D (mean±SD)	9±5	5±3	4±2
ε_O (mean±SD)	29±21	18±15	14±12
ADL (%)	57.7	90.9	97.8

ε_D: location error (mm); ε_O:orientation error (º)

3.3. Influence of noise

In order to evaluate the influence of noise in the accuracy of the present method, white noise was added to surface potentials before iteratively computing dipole locations. Surface potentials generated by 1000 simulations of single dipoles at arbitrary locations and orientations were contaminated with white noise with different signal-to-noise ratios (SNR): 20 dB, 10 dB, 6 dB and 3 dB. Surface potentials with added noise were used to estimate dipole locations with the present algorithm by using a constant SPDB size ($N_r=6$, $N_\theta=17$, $N_\varphi=16$).

Table 2 shows the performance of our method under the presence of noise. For SNR higher than 6 dB, mean location errors were found to be lower than 10 mm and more than 90% dipoles were accurately located. For a SNR equal to 3 dB 82.0% dipoles were accurately located.

Table 2. Performance under noise influence

SNR	20 dB	10 dB	6 dB	3 dB
ε_D (mean±SD)	5±3	5±4	6±6	8±9
ε_O (mean±SD)	18±15	18±16	19±17	20±18
ADL (%)	91.8	91.2	90.0	82.0

ε_D: location error (mm); ε_O:orientation error (º)

3.4. Accuracy in double dipole location

Performance in the location of double dipoles was tested by locating two simultaneous dipoles inside the myocardial wall. A total of 1000 double-dipole configurations were tested by computing surface potentials by forward BEM, adding noise to surface potentials and iteratively search for the location and orientation of both dipoles by the presented method. Again SNR ratios equal to 20 dB, 10 dB, 6 dB and 3 dB were used and the same SPDB resolution used for previous simulations ($N_r=6$, $N_\theta=17$, $N_\varphi=16$). Mean errors in the location of the two simultaneous dipoles were computed.

Mean errors for double dipole location are shown in Table 3. Mean errors exceed 10mm even for low noise added and do not increase significantly for higher SNR. Crosstalk error may have a greater influence on the performance of the algorithm than addition of noise.

Table 3. Performance for double dipole location

SNR	20 dB	10 dB	6 dB	3 dB
ε_D (mean±SD)	12±18	17±23	16±23	16±23
ε_O (mean±SD)	20±16	20±17	22±18	22±20
ADL (%)	77.5	72.0	72.0	72.0

εD: location error (mm); εO:orientation error (º)

3.5. Computing time

Computing times were measured for the experiments described in sections 3.2 and 3.4 and compared to those required for an exhaustive search strategy. The processor used was an Intel Core2Duo-P8400, 2.26Ghz and 4GB RAM. Times required for an exhaustive search increase exponentially with SPDB resolution. Required computing times are greatly reduced by using an iterative search strategy, reduction more pronounced for large SPDB resolution (Table 4). Computing time is also dependent on the number of searched dipoles, increasing from 0.35s for localizing a single dipole for a medium SPDB size

$(N_r=6, N_\theta=17, N_\varphi=16)$ to 91 ± 18 s for a double dipole search and the same SPDB size. This computing time greatly reduces the required time for an exhaustive search $(8\cdot10^8\text{s})$.

Table 4. Computing times for the iterative and exhaustive search algorithms

SPDB resolution	1	2	3
Exhaustive search *	48	369	2628
Iterative search * (mean ± SD)	0.26±0.02	0.35±0.03	1.86±0.27

* seconds

4. Discussion

We have shown that the inverse problem in electrocardiography can be solved indirectly by comparison of recorded surface potentials to stored surface potentials generated by known sources. The accuracy in this indirect method will be dependent on the amount of surface potentials stored for comparison which limits the spatial resolution of the algorithm to the spatial resolution used for the construction of the database.

Although an exhaustive search of all combinations of stored potentials for solving the inverse problem is possible, the computing time of this approach is excessive. An iterative algorithm has been proposed as a more time-efficient approach, and tested on a database of simulated potentials computed for an eccentric spheres model.

The present method has shown to be accurate in the localization of single dipoles with intermediate database resolutions (1632 locations) even under the presence of considerable amounts of noise. However, accuracy of the iterative search strategy presented has shown to be inferior to that of the exhaustive search algorithm because the best match may not always be tested.

Limitations of the iterative search strategy presented are more evident when two simultaneous dipoles are searched. Accuracy in the presence of two simultaneous dipoles is notably reduced mainly due to crosstalk, which compromises the performance of the method to a higher extent than the presence of noise. The performance of the algorithm could be improved if search intervals for each parameter were not fixed and adapted to RMD* gradient, allowing larger search intervals until a moderate RMD* value is achieved.

The present method for indirectly solving the inverse problem in electrocardiography has been tested for an eccentric spheres model. Same methodology could be used for realistic torso models obtained by MRI image segmentation. Also, it has been tested for searching up to two dipoles, but could be applied for the search of any number of dipoles, although the computing time will rise together with the number of searched dipoles. In order to reduce computing times, initial conditions could be introduced into the model. For solving the inverse problem in electrocardiography during a ventricular activation, the use of the solution obtained for a preceding time instant could serve as an initial condition and thus speed up calculations.

5. Conclusion

A new iterative algorithm, based on comparison of stored surface potentials to recorded potentials has been implemented and tested to solve the inverse problem of electrocardiography, showing accurate results under noise conditions.

Acknowledgements

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