Estimation of Stress-Strain Relationships in Vascular Walls using Multi-Layer Hyperelastic Modelling Approach

Michel E Mickael[1], Abbas Heydari[2], Roger Crouch[1], Sherri Johnstone[1]

[1]School of Engineering and Computing Sciences, Durham University, UK, [2] School of Biological and Biomedical Sciences, Durham University, UK

Abstract

Determining the stiffness (or compliance) of biological vascular vessels is of importance when investigating pathological conditions, the design of stents, vascular grafts, distal anastomotic connectors in coronary artery bypass surgery, and understanding of biological pressure sensors. This communication is concerned with determining appropriate values of the material constants associated with a layered anisotropic hyperelastic constitutive model to estimate the mean stress for arterial and venous walls. Results show that the values of the material constants, determined from a constrained optimization approach, satisfying equilibrium, give rise to mean stress-strain states which are consistent with responses obtained from the standard averaged model.

1. Introduction

Understanding the stress-strain relationship for cardiovascular vessels could have a major impact on studying diseases such as arteriosclerosis and atherosclerosis. It could also help in designing vascular grafts. The authors are particularly interested in applying such information to investigate baroreceptors. These are pressure sensors that report blood pressure to the CNS. The process of their operation is divided into two main phases[1]. Firstly, blood pressure applied to the vascular wall is transferred into a strain which then controls the opening probability of many mechanosensitive ion channels. These are embedded into the vascular wall [1]. Their exact positions are disputed. Studying stiffness contributions from different layers of the arterial and venous walls, as well as the mean stress for the wall, would help in understanding the function and the process or the arterial wall, there have been various attempts to model the stress-strain profile using different assumptions. For example, one may use thin-walled cylinder theory to determine the average axial and circumferential stresses given the axial force and internal pressure. In this model, it is assumed that the strain (axial stretch and circumferential stretch) across the wall is constant. Using this simplified approach, the average stress-strain behaviour can be determined. We refer to this approach as the standard averaged model. Alternatively, one could begin with the assumption of a uniform strain field across the wall of the vessel, but now employ a constitutive model (appropriate for the wall material) to determine the stresses. The material constants satisfy structural equilibrium (that is, the sum of the stress-area products equals the applied forces). This is the approach presented here.

2. Modelling approaches

The arterial and venous wall consists of three concentric cylindrical layers; innermost layer; intima, middle layer; media and outermost layer; adventitia. The layers behave as transversely isotropic homogeneous nearly incompressible hyperelastic materials in which a strain-energy function, W, is assumed to exist [2]. The arterial wall extends passively, but the smooth muscle controls the active tension of the vessel. Forming a model for thin wall arterial response, Holzapfel et al [3] presented a two term strain-energy function that used experimentally obtained elastin and collagen responses to model passive extension. The effect of smooth muscle cells was neglected as it is thought that these do not contribute to the passive stiffness. Only elastin and collagen were considered as the constituents that act during the extension of the arterial wall. However, that model did not consider the responses for the individual layers constituting the arterial wall. von Maltzahn et al [4] measured experimentally the elastic properties of the media and adventitia. It did not include the role of the intima, which has been proven to be of significant importance [2]. In [5] Demiray and Vito used a two layer model, neglecting the role of the intima. The media was considered orthotropic, while the adventitia was considered isotropic. The relationship between the two layers and the whole structural stress was not presented. In [2] Holzapfel et al presented layer specific strain-energy equations assuming arterial layers. No mean relationship for the stress-strain response of the whole wall was given. The question addressed in this paper is, 'given the axial and circumferential stretches in arteries and veins, how can we develop a model to predict the overall stiffness?' A layered approach in conjunction with a hyperelastic anisotropic material model and thin wall theory was used. This model gives a basis for comparison between different arteries of different species and of different materials. It is assumed that the strain-energy function of the venous wall is of a similar form to that of the arterial wall [6]. The percentage thickness of each layer with respect to the total wall thickness is assumed to be the same for each vessel. Experimental findings by others have revealed the percentage thicknesses to be approximately 27%, 40 % and 33 % for the intima, media and adventitia respectively [2].

3. Experimental data

There are few papers reporting on the deformation of the arterial wall [2]. Literature containing experiments which compare the response of the whole wall and the single layer response are not available. Some of the data which exists is not in a form useful for the stress–strain analysis. For example, [7] investigates the static pressure-diameter relationship but does not show the axial force relationship with pressure. The arterial experimental data used here were extracted from an investigation [8]. That presented graphs representing the external diameter relationship versus luminal pressure together with the axial force relationship versus the luminal pressure for different arteries. Data for a coronary artery and a mammary artery have been chosen here. Different arteries were dissected from arteriosclerotic cadavers. Static pressure tests were then carried out after the application of an axial pre-stretch to pre-conditioned arteries.

The external diameter and corresponding axial force were then measured. Experimental data for the vena cava was sourced [6]. We digitised the data sets from those papers using 17 points across the pressure range and imported them into the models for this investigation.

4. Thin wall theory

Thin wall theory can be applied when the thickness/ radius ratio is less than a tenth. However, Holzapfel et al in [3] used thin wall theory to represent the circumferential and axial responses. The rationale for this was that as all collagen fibres are embedded in the tangential surface of the tissue, it can be assumed that there are no components in the radial direction [2]. In this case only circumferential and axial stresses become relevant. Thin wall theory offers a simple approximation for the relationship between mean circumferential and axial Cauchy stresses. Its named here the standard model.

5. Hyperelastic model

Here we describe the form of the hyperelastic constitutive equations. These are based on the theory presented by Holzapfel et al [2]. The equations are used to calculate the specific stress responses for each layer as functions of circumferential and axial stretch. Thin wall theory was applied to calculate the mean wall stress both circumferentially and axially. To find the relationship between the second Piola-Kirchhoff stress tensor, **S**, and Green-Lagrange strain tensor, **E** the concept of a strain-energy function, W is used.

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{E}} \qquad (1)$$

The Cauchy stress tensor, **σ**, can be calculated from the second Piola-Kirchhoff stress tensor, **S**, using the inverse Piola transformation. Using the Lagrangian multiplier, a relationship is derived between the total strain-energy and the volumetric, U(J) and isochoric components, W_{ic} ($\mathbf{E}$, A_1). Assuming the vascular walls to be incompressible, and that the total strain-energy is a function of the Green-Lagrange strain tensor representing one family of collagen fibres, thus:

$$W\,(\mathbf{E}_{1c}, A_1) = \mathrm{U(J)} + W_{ic}\,(\mathbf{E}_{1c}, A_1) \qquad (2)$$

$$A_1 = a_1 \otimes a_1 \qquad (3)$$

where a_1 is the angle components of one collagen fibre

$$\mathrm{U} = \mathrm{P(J-1)} \qquad (4)$$

where P has the units of hydrostatic pressure. Thus the Green-Lagrange strain-stretch relationship is given by

$$\mathrm{E}_i = \frac{1}{2}(\lambda_i^2 - 1) \qquad (5)$$

where λ_i is the principal stretch. For each layer, the strain-energy, W_{ic}, is further divided to two parts representing the response of elastin and collagen. The elastin strain-energy component $W_{iciso}(\mathbf{E}_{ic})$, is approximated to be

$$W_{iciso}\,(E_{ic}) = \frac{c_1}{2}\,(\mathrm{I}_1 - 3) \qquad (6)$$

where I_1 is the first invariant of stretch and c_1, is a material constant related to the elastin stress response.

The collagen component can be described by

$$\mathrm{W_{icaniso}}\,(\mathbf{E}_{1c}, \mathrm{A}_1) = \frac{\mathrm{k}_1}{\mathrm{k}_2}(\mathrm{e}^{\mathrm{q}} - 1) \qquad (7)$$

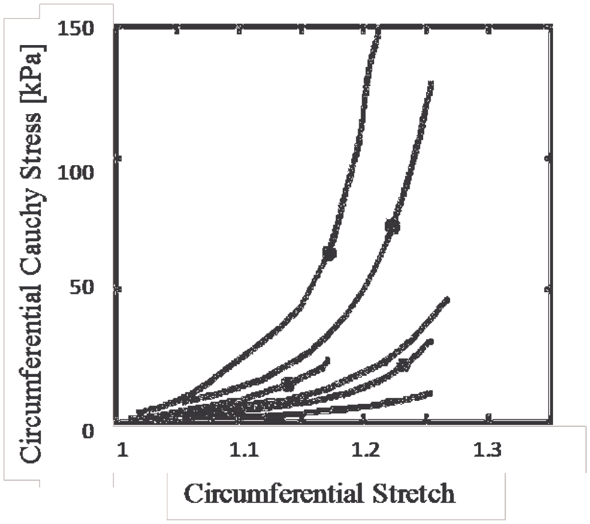

Figure1: Hyperelastic model estimations of circumferential stress-stretch for (a) coronary artery, for the intima (large open symbol), adventitia (middle open symbol) and meda (small symbol). An example of a Holzapfel et al coronary model response is indicated by the filled symbols for comparison [6].

where k_1, and k_2 are material constants related to the collagen stress response. q is a function of the dispersion factor, taken here to be equal one, and I_4 is the fourth stretch invariant. The dispersion factor value represents the amount of dispersion from the ideal alignment of the fibres [2]. A value of unity assumes that there are not any fibres oriented in the φ direction. For a value of zero the fibers are assumed to be isotropically oriented as presented by Demiray et al [5]. Thus for the intima (n)

$$q_n = k_{2n}(I_{4n} - 1)^2 \tag{8}$$

$$I_{4n} = \lambda_\theta^2 \cos^2(\varphi_n) + \lambda_z^2 \sin^2(\varphi_n) \tag{9}$$

Similar expressions can be obtained the media and adventitia. This paper proposes using stress equilibrium in the wall to calculate the mean wall Cauchy stress components, $\sigma_{t\theta}$ and σ_{tz} in the circumferential and axial directions. h is the wall thickness after deformation.

$$\sigma_{t\theta} = \frac{(\sigma_{\theta n} h_n + \sigma_{\theta m} h_m + \sigma_{\theta a} h_a)}{h} \tag{10}$$

$$\sigma_{tz} = \frac{(\sigma_{zn} h_n + \sigma_{zm} h_m + \sigma_{za} h_a)}{h} \tag{11}$$

6. Results and discussion

Optimisation of the hyperelastic model data was achieved using the Levenberg–Marquardt method with a root mean square error function provided by Matlab®. Parameter sensitivity analysis was investigated for the material parameters of the coronary artery. Figure1 presents the inferred layer circumferential Cauchy stress-stretch profiles for circumferential Cauchy stresses.

These stress-strain profiles confirm the layers have the same order of stiffness as Holzapfel et al [2] von Maltzahn et al [4] and Demiray et al [5] .

he model was constrained by relationships which can be explained by experimental evidence; these constraints are as follows:

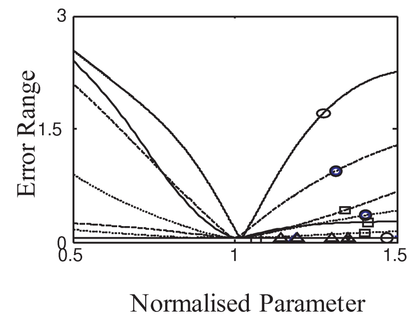

Figure 2: Parameter sensitivity of the coronary artery. A thick continuous line is used for the angle; a thick dashed line for k_2, a thin dashed line for k_1 and c_1 is drawn using a thin continuous line. The intimal layer is represented by a circle, the adventitial layer by a square and medial layer by a triangle.

Circumferentially and Axially:	$k_{intima} > k_{adventitia} > k_{media}$
For the intima:	$k_{axial} > k_{circumferential}$
For the media:	$k_{circumferential} > k_{axial}$
For the adventitia:	$k_{axial} > k_{circumferential}$
Fibre orientation:	$\varphi_{intima} > \varphi_{adventitia} > \varphi_{media}$

where k represents the stiffness. Material parameters presented by Holzapfel et al [2], were used as a guide to obtaining the material parameters used here (figure 2). The collagen material parameters are the most sensitive. In terms of layer response, the media possesses the lowest parameters value. The intima, being the stiffest layer has the parameters with highest sensitivity. The hyperelastic model (figure 3) has been shown to estimate the stress-strain profiles with root mean squared errors of 0.05.

This compares well with the Holzapfel et al [2] prediction of 0.07. At low pressure, the model fit was not so good. It is assumed that elastin is first stretched, while collagen attracts load at higher stretches. Improving the model of the elastin behaviour could increase the quality of the overall fit [9]. Collagen contribution to the mean stress is far greater than the elastin for all the three layers

[2]. This indicates that, in terms of load-carrying capacity, vascular vessels are collagen dominated.

Figure 4 shows that the vein is softer than the all arteries investigated both axially and circumferentially. Both the standard and hyperelastic models indicate that the maximum extension ratio for the vena cava is much bigger than that of the arteries (Figure 4). Venous parameters are lower than the arterial ones. These models ould be used to design parameters of synthetic stretch receptors and vascular grafts [6] and studying vascular diseases.

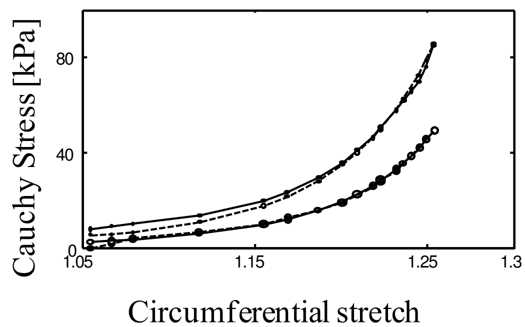

Figure 3: Comparison of the mean stress strain relationships for the axial (small symbol) standard model (dotted) and hyperelastic model (line).

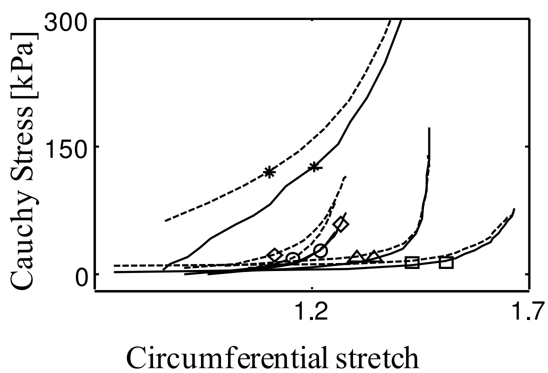

Figure 4: Comparison of mean stresses for abdominal aorta (star) [8], mammary (diamond), coronary (circle), and rat tail artery (triangle) [6] vena cava (square), in both axial (dashed line) and circumferential (continuous line) directions. Axial stretches are 1.66, 1.2, 1.1, 1.29, and 1.91 respectively.

Acknowledgements

The author would like to thank the Engineering and Physical Sciences Research Council (ESPRC) for funding this research (EP/F01189X/1) and also thank Dr Susan Pyner , Dr Ritu Kataky for their help and advice.

References

[1] Srinivasan R, Nudelman HB. Modeling the carotid-sinus baroreceptor, Biophys J. 1972: 12:9:1171-1182.

[2] Holzapfel GA, Sommer G, Gasser CT, Regitnig P. Determination of layer-specific mechanical properties of human coronary arteries with nonatherosclerotic intimal thickening and related constitutive modelling, Am J Physiol Heart Circ Physiol, 2005:289(5):H2048–H2058.

[3] Holzapfel GA, Eberlain R, Wriggers P, Weizsacker HW. New axisymmetrical membrane element for anisotropic finite strain. analysis of arteries,Comm. Num Methods in Eng.1996:12(8):507-517.

[4] Von Maltzahn WW, and Besdo D, Wiemer W. Elastic properties of arteries: a nonlinear two-layer cylindrical model, J Biomech, 1981:14: 389-397.

[5] Demiray H, and Vito R. A layered cylindrical shell model for an aorta, J Biomech, 1991:5(1):309–311.

[6] Desch GW,Weizsäcker HW. A model for passive elastic properties of rat vena cava, J Biomech, 2007:40(14):3130–3145.

[7] Gow BS, Hadfield CD. Elasticity of canine and human coronary arteries with reference to postmortem changes," Circ. Res, 1979:45(5):588-594.

[8] Van Andel C, et al., "Mechanical properties of porcine and human arteries: Implications for coronary anastomotic connectors," Ann. Thoracic Surg,2003: 76(1):58-64.

[9] Watton PN, Ventikos Y, Holzapfel GA. Modelling the mechanical response of elastin for arterial tissue," J Biomech.2009:42(9):1320-1325.

Address for correspondence.
Name: Dr Sherri Johnstone
Postal Address: School of Engineering and Computing Sciences, Science Site, South Road, Durham University, UK, DH1 3LE
Email: sherri.johnstone@durham.ac.uk