Moving Equivalent Multipoles Derived from the Body Surface Potential Map by Solving the Inverse Problem

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Abstract

The aim of this study is to determine instantaneous 3-D locations and moments of moving equivalent multipoles (EM) corresponding to the principal components (PC) obtained from body surface potential map (BSPM) of the CINC/Physionet Challenge 2007 database.

Moving EM (three dipoles and one quadrupole) were determined for the successive time windows of 10 ms in steps of 5 ms using multipole models in an unbounded and bounded spherical homogenous conductor. EM parameters were obtained in two steps: in the forward and inverse problem formulation, necessary for the reconstruction of the original BSPM signals.

The reconstruction errors in the region of the QRS complex were 1.0% for the first 3 biggest PC, and 6.2% and 5.7% for three dipoles and one quadrupole for the bounded and unbounded EM model, respectively. The corresponding RMS errors were 3.8, 21.5 and 19.8 μV.

1. Introduction

In the past, electrocardiographic (ECG) signals from the body surface has been used to determine equivalent multipoles (EM), equivalent dipoles (ED) or equivalent multipoles (EQ), that reproduced the body surface potential distribution with given accuracy [1-5] and served to provide some insight into the heart as a current generator.

Decomposition of ECG signals is another computational technique that has been often used to transform the multiple-lead ECG to identify dominant components of the recording [6], such as principal components (PC) obtained by the principal component analysis (PCA). When relating the properties of PC to those of EM, we speculated whether multipolar parameters can be expressed with the properties of the transformations matrices provided by PCA, which might enable to determine the optimal position of EM for each PC as a solution of the inverse problem.

Here we combine both methods to determine of EM signals corresponding to each PC signal of a given ECG segment and applied them for ECG signals obtained from body surface potential map (BSPM) of the CINC/Physionet Challenge 2007 database [7]. Our attention was focused in the error of determination, i.e. how accurately can EM signals reproduce the recorded signals after reconstructing them, using both the bounded and unbounded models.

2. Methods

2.1. Properties of PCA transforms

Let **A** be an MxN matrix with each column *i* corresponding to one of the N=71 measured ECG signals of the multi-lead surface ECG. Then the matrix **A** can be written as the product of an MxN column-orthogonal matrix **U**, with columns *k* representing PC signals, $\boldsymbol{U}_k(t)$, and NxN diagonal matrix **W** with positive or zero elements (the singular values), and the transpose of an NxN orthogonal matrix **V** [8,9]. The matrices **U**, **W** and **V** can be determined from the matrix **A** using singular value decomposition (SVD) algorithm, and when provided they enable the reconstruction of the original signals,

$$A_i(t_j) = \sum_k^N U_k(t_j) W_k V_{ik} \qquad (1)$$

where $\boldsymbol{A}_i(t)$ are the reconstructed signals at some observation location $\boldsymbol{r}_i$, $\boldsymbol{U}_k(t)$ are transformed signals, $\boldsymbol{W}_k$ are the singular values and $\boldsymbol{V}_{ik}$ are elements of the matrix that determine how much of each transformed signal to use to get the original signals.

It has been shown that nearly 99% of ECG signal energy can be represented in a 3D subspace, using only first three principal signals [6]. Hence, to reconstruct original ECG signals approximately it is usually enough to use only the first three PC signals, i.e. N=3 in (1).

2.2. EM models for the forward problem

Let us consider a dipole model for which dipole moment parameters and dipole spatial parameters can be separated. For such a model, the potential ϕ_{ik} at the observation location $\boldsymbol{r}_i$ due to a single dipole with the dipole moment $\boldsymbol{p}_k$ at the location $\boldsymbol{r}_k$ in a homogeneous medium is be given by

$$\phi_{ik} = \boldsymbol{p}_k \bullet \boldsymbol{g}_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k) \quad (2)$$

where $\boldsymbol{g}_{ik}(r_i, r_k)$ is a vector (g_x, g_y, g_z), containing spatial parameters of locations $\boldsymbol{r}_i$ and $\boldsymbol{r}_k$ only. For a dipole in an unbounded infinite homogeneous volume conductor (the unbounded dipole model), vector $\boldsymbol{g}_{ik}(r_i, r_k)$ is given by $\boldsymbol{g}_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k) = (\boldsymbol{r}_i - \boldsymbol{r}_k)/4\pi\sigma|\boldsymbol{r}_i - \boldsymbol{r}_k|^3$ [5] with σ as conductivity of the medium, whereas for a dipole in a homogeneous volume conductor bounded with the sphere of radius R, vector $\boldsymbol{g}_{ik}(r_i, r_k)$ is given by the expressions provided by Geselowitz and Ishiwatari [2]. The dipole vector $\boldsymbol{p}_k$ can be further separated into dipolar strength P_k and dipolar orientation $\boldsymbol{c}_k$, $\boldsymbol{p}_k = P_k\boldsymbol{c}_k$, where $\boldsymbol{c}_k$ is unit vector.

Next, consider a shorter segment of ECG signal in which dipole $\boldsymbol{p}_k$ is located at a fixed location $\boldsymbol{r}_k$ and with a fixed orientation in space $\boldsymbol{c}_k$, and whereas dipole strength P_k, is changing according to a time function $P_k(t)$. Thus, time dependent and spatial properties of dipole can be separated, $\boldsymbol{p}_k(t) = P_k(t)\boldsymbol{c}_k$.

For a quadrupole a similar relationship can be written by approximating it as a pair of anti-parallel dipoles, one at the location $\boldsymbol{r}_k+\boldsymbol{\varepsilon}/2$ and another at the location $\boldsymbol{r}_k-\boldsymbol{\varepsilon}/2$, where $\boldsymbol{\varepsilon}$ is vector at right angle to $\boldsymbol{r}_k$. In the limit, when $\varepsilon \to 0$, the potential ϕ_{Dik} due to quadrupole is approximated to $\phi_{Qik} = \phi_{Dik}(\boldsymbol{r}_k + \boldsymbol{\varepsilon}/2) - \phi_{Dik}(\boldsymbol{r}_k - \boldsymbol{\varepsilon}/2)$.

Hence, $\phi_{Qik} = \boldsymbol{Q}_k \bullet \boldsymbol{g}'_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k, \boldsymbol{\varepsilon})$,

where $\boldsymbol{g}'_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k, \boldsymbol{\varepsilon}) = \left(\boldsymbol{g}_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k + \boldsymbol{\varepsilon}/2) - \boldsymbol{g}_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k - \boldsymbol{\varepsilon}/2)\right)/\boldsymbol{\varepsilon}$

and $\boldsymbol{Q}_k = P_{Qk}.\varepsilon.\boldsymbol{c}_{Qk}$. Note that this definition allows the quadrupole potential to be written as a scalar product of two vectors.

It can be seen that expressions for potential due to dipole or quadrupole are similar: in both, dipolar of quadrupolar parameters are separated from the spatial ones. However, in case of dipole, its spatial part contains 3 coordinates only, and in case of quadrupole, there are 5 coordinates, as two additional are necessary for the location of the anti-parallel dipole.

2.3. Fitting of EM signals to PC signals

If two descriptions (1) and (2), one with PC and another with EM were identical, then the difference $\Delta\phi_i(t)$ between the EM reconstructed signal and that one using PC, should approach zero, $\Delta\phi_i(t) = \phi_i(t) - A_i(t) \to 0$.

Let us consider three PC with the biggest $\boldsymbol{W}_k$ and initially only the presence of dipoles. Then,

$$\Delta\phi_i(t) = \sum_{k=1}^{3}\left(P_k(t)\boldsymbol{c}_k \bullet \boldsymbol{g}_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k) - \boldsymbol{U}_k(t)\boldsymbol{W}_k V_{ik}\right) = 0 .$$

Since $P_k(t)$, and $U_k(t)$ are the only terms depending on time, they must be proportional. This leads to

$$\Delta\phi_i(t) = \sum_{k=1}^{3}\boldsymbol{U}(t)\boldsymbol{W}_k F_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k) \quad (3)$$

with the function

$$F_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k) = \boldsymbol{p}_{k0} \bullet \boldsymbol{g}_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k)/\boldsymbol{W}_k - V_{ik} \quad (4)$$

that relates linearly dipole moment parameters $\boldsymbol{p}_{k0} = P_{k0}.\boldsymbol{c}_k = (p_{x0}, p_{y0}, p_{z0})$ of each particular dipole to the transformation coefficients V_{ik}, obtained by SVD. Here, $\boldsymbol{p}_{k0}$ is characteristic for the whole time interval considered.

Assuming the presence of both dipole and quadrupole, function $F_{ik}(r_i, r_k, \varepsilon)$ can be written as

$$F_{ik}(\boldsymbol{r}_i, \boldsymbol{r}_k, \boldsymbol{\varepsilon}) = \boldsymbol{p}_{k0} \bullet \boldsymbol{g}_{ik}/\boldsymbol{W}_k + \boldsymbol{Q}_k \boldsymbol{g}'_{ik}/\boldsymbol{W}_k - \boldsymbol{V}_{ik} . \quad (5)$$

Sufficient condition for $\Delta\phi_i(t)=0$ to exist is that for any selected location $\boldsymbol{r}_i$ $F_{ik}(r_i, r_k) = 0$, since $\boldsymbol{W}_k$ and $\boldsymbol{U}_k(t)$ are both different from zero. If $F_{ik} \neq 0$, then the reconstructed ECG signal $\phi_i(t)$ in the forward problem formulation is different from that represented by PC by $\Delta\phi_i(t)$. Hence, function F_{ik} represents how much does each EM participates to the deviation of the reconstructed signal from the observed one, and hence tells on the accuracy of its determination.

Since decomposed signals are independent, it holds also for the dipoles representing them, hence their spatial parameters can be determined separately.

2.4. EM in the forward problem

To determine $F_{ik}(r_i, r_k)$ defined by (4) for a given set of 3 spatial dipole parameters $\boldsymbol{r}_k = (x_k, y_k, z_k)$, dipole moment parameters $\boldsymbol{p}_k$ have to be known. In the system with N=71 observed signals $\boldsymbol{A}_i(t)$, $F_{ik}(r_i, r_k)$ represents a set of 71 linear equations with three unknowns $\boldsymbol{p}_k = (p_x, p_y, p_z)$, which is over determined, and for which we wish to find the least square solution, i.e. to find $\boldsymbol{p}_k$ at given $\boldsymbol{r}_k$ that minimizes the objective function

$$\psi_k(\boldsymbol{r}_k) = \sum_i^N F_{ik}^2(\boldsymbol{r}_i, \boldsymbol{r}_k) . \quad (6)$$

When the presence of both dipole and quadrupole is expected, the a set of 3 spatial dipole parameters $\boldsymbol{r}_{Dk} = (x_{Dk}, y_{Dk}, z_{Dk})$ and a set 5 quadrupole parameters $\boldsymbol{r}_{Qk} = (x_{Qk}, y_{Qk}, z_{Qk}, \theta_{Qk}, \varphi_{Qk})$ is needed, where θ and φ are two spatial angles of the vector $\boldsymbol{\varepsilon}$, so that the scalar product of $\boldsymbol{c}_{Qk}.\ \boldsymbol{\varepsilon} = 0$.

2.5. EM in the inverse problem

To find $\boldsymbol{p}_k$ that best describe the observed surface potentials, an inverse algorithm using an optimization method can be applied using the objective function $\psi_k(\boldsymbol{r}_k)$.In calculation, $\boldsymbol{r}_{Dk} = (x_{Dk}, y_{Dk}, z_{Dk})$ and $\boldsymbol{r}_{Qk} = (x_{Qk}, y_{Qk}, z_{Qk}, \theta_{Qk}, \varphi_{Qk})$ are varied until reaching the optimal ones, $\boldsymbol{r}_D$ and $\boldsymbol{r}_{Qk}$, at the minimal value of the objective function $\psi(\boldsymbol{r}_{Dk}, \boldsymbol{r}_{Qk})$. For this purpose we applied the Simplex method using the Nelder-Mead algorithm[10], and then a gradient based method (Gauss-Newton)[9]), for which function derivatives were calculated from the function evaluations in the two adjacent points.

When the quadurpolar part is expected to be small, it can be solved in two steps. First, for each k it is solved for a dipolar part using $F_{Dik}(r_i, r_k)$ to determine dipolar parameters only. In the next step, $F_{Qik}(r_i, r_k, \varepsilon)$ is considered to be dependent on quadrupole only, and $\boldsymbol{V_{ik}}$ is replaced by the difference $\boldsymbol{V_{ik}} - F_{ik}(r_i, r_k)$,

$$F_{Qik}(\boldsymbol{r}_i, \boldsymbol{r}_k, \boldsymbol{\varepsilon}) = \boldsymbol{Q}_{k0}\boldsymbol{g}'_{\boldsymbol{ik}} / \boldsymbol{W}_k - \left(\boldsymbol{V}_{ik} - F_{Dik}\left(\boldsymbol{r}_i, \boldsymbol{r}_k\right)\right). \quad (7)$$

2.6. Determination of multiple EM from the BSPM recordings

To determine multiple moving EM we used BSPM of the CINC/Physionet Challenge 2007 database [7]. We treated 71 BSPM from the anterior and left lateral leads of the Dalhouise 120 torso leads. For the analysis of ECG signals, an adult torso was constructed using human anatomical data of the adult human chest, assuming homogenous and isotropic conductance. Then a sphere with radius of r_0=15 cm was fitted to the chest with the center at the level of the precordial lead V_3 in the transversal plane and 3 cm rightward and 3 cm backward from the center of the chest.

In a real situation, the thorax of a measured subject may not match that one of our model nor are the ECG electrodes always attached to the desired location, which is expected to increase the value of $\psi(\underline{\boldsymbol{r}_k})$. To decrease it we allowed for initial ECG lead locations in our torso model to be displaced slightly, but to remain on a spherical surface while keeping the dipole locations fixed, until reaching a new minimal $\psi_k(\underline{\boldsymbol{r}_k})$. For this purpose, we used an independent set of EDs belonging to the two biggest PC, those of the first and the second half of the complete QRS complex and of the T wave templates.

In calculations, all BSPM were first divided into short intervals (5 ms in the QRS, and 20 ms in the T wave region). ECG signals of these short segments were then decomposed to get PC, and the 3 PCs with the biggest $\boldsymbol{W}_k$ were then utilized to determine EM (three EDs and one EQ) belonging to each segment. The inverse problem was solved in the bounded and unbounded model representation, both with initially set and displaced lead locations.

Finally, decomposed signals as well as multiple EM belonging to each data set were served to reconstruct ECG signals, which were compared to the recorded signals in the region of the QRS complex. To estimate the accuracy of reconstruction, we used two errors, the mean relative absolute deviation and the RMS error.

The mean relative absolute deviation was defined as integral of absolute deviation of the recorded from the reconstructed signal over the interval of the QRS complex, normalized with the integral of the absolute value of the recorded signal over the same interval, averaged over all 71 leads. It was calculated for the PC presentation and EM presentation using both the bounded and unbounded model. We were interested how this measure decreases when increasing the order of approximation. In case of the PC representation, we studied the behavior of the successive summation from the 1st to the 3rd PC, and in the EM representation a successive summation of EDs and EQ obtained from the 1st PC to the ED obtained from the 2nd and 3rd PC. The RMS error was defined similarly, as the mean squared deviation of the reconstructed signal from the recorded one.

3. Results

Each set of BSPM data was represented with sequential arrays of EM (3 EDs and 1 EQ), each with its multipole moment and 3D location, thus providing additional information for the interpretation of ECG sources.

As an illustration, the sequence of ED during the QRS complex with the resolution of 5 ms for the case0001.dat is shown in Fig. 1, representing the ED of the 1st PC.

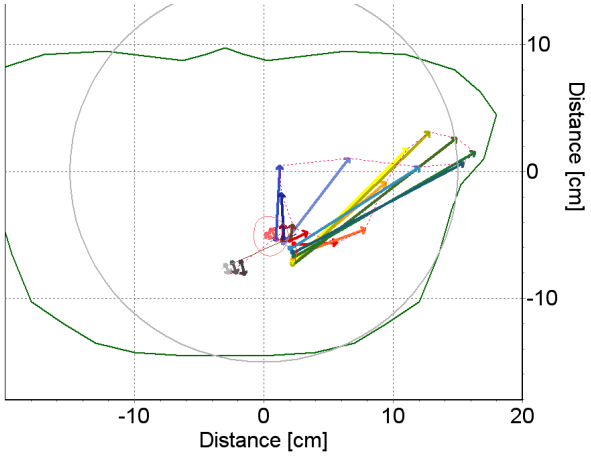

Figure 1. A sequence of EDs in the transversal plane with the contour of the thorax (green), and the spherical conductor (gray). Color code: initially red, then yellow, green, blue, black, and finally gray at the end of QRS.

This representation enabled a relatively precise reconstruction of all 71 BSPM used. In four BMPS cases, mean relative deviation of the reconstructed signal from the original BSPM in the region of the QRS complex were 0.01±0.002 for the first 3 biggest PC, 0.062±0.008 and 0.057±0.010 for 3 EDs and 1 EQ with displaced lead locations in the bounded and unbounded model, respectively. Displacing of leads had no significant influence on the spatial and multipole parameters of EM, but increased error roughly to 0.130 in both models. Detail contributions of particular PC and EM belonging to each particular PC are shown in Fig. 2 and in Table 1.

The corresponding RMS error values 3.8±0.7 μV, 21.5 ±4.0 μV and 19.8±1.7 μV for representation with the biggest 3 PC, and corresponding EM for both multipole models. Hence, no significant changes regarding error were found between the bounded and unbounded model.

Table 1. Accuracy of reproducing of measured ECG signals in the interval of the QRS complex as mean relative deviation between the reconstructed and recorded signals using incremental number of principal components or incremental number of EM in the bounded model with initially set (EM-u, uncorrected) and displaced lead locations (EM-c, corrected lead locations).

PC.		*PC1*		*+PC2*	*+PC2*
All models	Mean	**0.130**		**0.034**	**0.010**
	±SD	0.031		0.010	0.002
EM		*ED1*	*+EQ1*	*+ED2*	*+ED3*
EM-u	Mean	**0.229**	**0.192**	**0.137**	**0.130**
	±SD	0.045	0.030	0.018	0.017
EM-c	Mean	**0.163**	**0.146**	**0.070**	**0.062**
	±SD	0.022	0.027	0.007	0.008

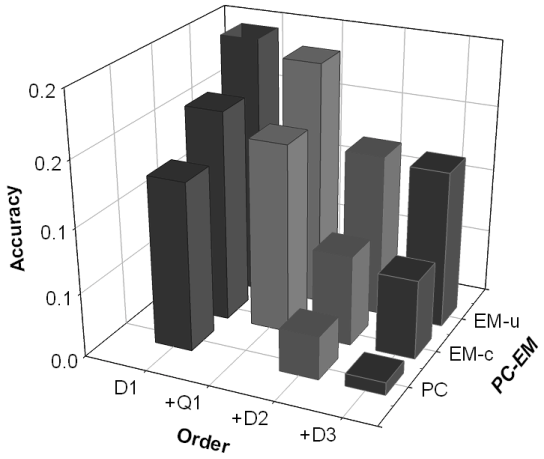

Figure 2. Accuracy of reconstruction of BSPM signals, estimated as mean relative deviation from the recorded signal. Order of approximation is in-creased by including 1st ED (D1), 1st EQ (+Q1), 2nd ED (+D2), and 3rd ED (+D3). The type of representation is shown by PC-EM either using PC or EM and in the bounded model with corrected (EM-c) and uncorrected lead locations (EM-u).

4. Discussion and conclusions

Our study of the four cases of BSPM showed that using 71 ECG leads it is possible to determine up to 3 EDs and 1 EQ belonging to 3 PCs with the biggest $\boldsymbol{W}_k$. Both error estimates, the relative deviation of the reconstructed signal and RMS errors, show that when using BSPM data with 3 PCs, which represent the reconstruction error of 1%, the representation using EM with corrected lead locations accounted for additional 5% error (total 6%), and with uncorrected one for additional 12%. Hence, in both models significant improvement was achieved using the correction with displaced leads, which did not change spatial and multipole moment parameters of EM substantially.

Studies using quadrupoles and the multi-lead ECG are rare in the literature. Trost et al [3] who used 16 leads and applied a model with one dipole with the fixed position and one quadrupole in 59 subjects, found the reconstruction error as big as 10%, similar as ours without correction of lead locations. They also reported that the inclusion of a quadrupole reduced the error by a factor about three. Our study shows much smaller influence of EQ (around 1%). We explain it by using 3 EDs instead of one, which represents the order of octopole and thus reduces thus the contribution of EQ [2].

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