33rd Annual International Conference of the IEEE EMBS
Boston, Massachusetts USA, August 30 - September 3, 2011

6 and 15 MeV photon spectra reconstruction using an unfolding depth dose gradient methodology

B. Juste, R. Miró, G. Verdú, S. Díez, J. M. Campayo

Abstract—**An accurate knowledge of the spectral distribution emission is essential for precise dose calculations in radiotherapy treatment planning. Reconstruction of photon spectra emitted by medical accelerators from measured depth dose distributions in a water cube is an important tool for commissioning a Monte Carlo treatment planning system. However, the reconstruction problem is an inverse radiation transport function which is poorly conditioned and its solution may become unstable due to small perturbations in the input data. In this paper we present a more stable spectral reconstruction method which can be used to provide an independent confirmation of source models for a given machine without any prior knowledge of the spectral distribution. This technique involves measuring the depth dose curve in a water phantom and applying an unfolding method using Monte Carlo simulated depth dose gradient curves for consecutives mono-energetic beams. We illustrate this theory to calculate a 6 and a 15 MeV photon beam emitted from an *Elekta Precise* radiotherapy unit using the gradient of depth dose curves in a cube-shaped water tank.**

I. Introduction

THE knowledge of the photon energy fluence spectrum produced by a medical linear accelerator (Linac) is extremely useful for precise dose calculations. The main goal of accurate dose calculation is delivering a prescribed dose to a target volume while minimizing radiation damage to the surrounding healthy tissues.

The major problem of direct determination of photon spectra lies in the very high photon flux emitted in each pulse, making the traditional pulse counting and integrating techniques impossible since single events cannot be discriminated, and it produces the detector saturation.

It is therefore necessary to establish a methodology which guarantees a realistic shape of the spectrum emitted by a linac. In this paper we present the robustness and practical utility of using depth dose measurements in a water phantom for estimating linac spectra.

The key is that a linear operator relates the spectrum to the depth dose measurements via a *Fredholm* integral equation. We illustrate in this paper the inherently ill-conditioned nature of the mathematical problem of deriving spectra from dose measurements. According to this, a novel approach of the depth dose methodology is presented, minimizing the ill-conditioned function. The original contribution of this work proposes a technique to significantly improve the accuracy of Bremsstrahlung spectra reconstructed from depth dose gradient data using inverse methods. While typical approaches directly use the measured depth dose curves, we show the advantage of using the gradient of these data for spectra reconstruction [1]. Regularization is introduced to alleviate the effects of noise due to measurement and computation.

Once the spectrum is reconstructed, a sophisticated Monte Carlo model of the accelerator unit head is used to simulate the water curves generated using the calculated spectrum by considering interactions with the target, collimators, jaws, etc. Results are compared with experimental results derived from water phantom measurements.

According to this, the present work is focused on explaining an alternative method to calculate a linear accelerator photon beam spectrum based on indirect measures. Specifically, the study has been focused on the reconstruction of a 6 and 15 MeV primary photon beam issued by the medical linear accelerator *Elekta Precise*, available at the *Hospital Clínic Universitari de Valencia, Spain*.

II. Methods and Materials

A. Spectrum reconstruction basic theory

1) Linear Problem

The attenuation coefficients of photons depend on their energy; therefore, the transmission at various depths can be related to the spectrum via a *Fredholm* integral equation, which theoretically can be solved exactly.

According to this, there exists a close relationship between the central axis depth dose curve in a water phantom *d(x)*, and the high energy photon beam emitted by a linac, $\chi(E)$. This relationship is based on the theory that the depth dose curve can be expressed as the linear superposition of contributions from a continuum of mono-energetic photon beams, each weighted by the output photon fluence at each energy *E*.

Depth dose, *d(x)*, (where *x* is the water phantom depth) can be expressed (Eq. 1) as a convolution of the incident energy spectrum $\chi(E)$ and a response matrix, $r_E(x)$, containing information of the depth dose curves for different consecutive mono-energetic beams:

Manuscript received June 20, 2011.

Rafael Miró (Associate Professor), Belén Juste (PhD Student), and Gumersindo Verdú (Full Professor) are with the Chemical and Nuclear Engineering Department at the Polytechnic University of Valencia, Camí de Vera s/n 46022 Valencia, Spain (phone: +34963877635, e-mail: bejusvi@iqn.upv.es, rmiro@iqn.upv.es, gverdu@iqn.upv.es).

Juan Manuel Campayo and Sergio Díez are radio-physicists at the Hospital Clínic de Valencia, Avda. Blasco Ibáñez. Valencia, Spain.

$$d(x) = \int_0^{E_{max}} r_E(x)\,\chi(E)\,dE \qquad (1)$$

In principle, this linear equation is invertible, so it should be possible to obtain the linac spectrum which produces the depth dose curve. However, the associated noise of the experimental dose curve measures and the nature of the problem convert this inverse problem in an ill-conditioned one. To solve the equation, *d(x)* can be discretized into *m* intervals, corresponding to the different depth positions to analyze. The dose at the i_{th} position is denoted by d_i and can be expressed as:

$$d_i = \int_{x_{i-1}}^{x_i} d(x)\,dx, i = 1, \dots, M \qquad (2)$$

Similarly, this can be applied to the spectrum, $\chi(E)$, obtaining the average energy for each energy spectrum interval:

$$\chi_j = \int_{E_{j-1}}^{E_j} \chi(E)\,dE, j = 1, \dots, N \qquad (3)$$

where χ_j is the photon spectrum with energy between E_{j-1} and E_j.

On the other hand, $r_E(x)$ can be approximated by a *M x N* matrix, *A*, whose elements are:

$$A_{ij} = \int_{x_{i-1}}^{x_i} \int_{E_{j-1}}^{E_j} r_E(x)\,dE\,dx \qquad (4)$$

It is established that A_{ij} is the deposited dose in a position *i*, for a given energy spectrum *j*.

This equation can be approximated to:

$$d_i = \sum_{j=1}^{N} A_{ij}\,\chi_j, i = 1, \dots, M \qquad (5)$$

where $\vec{\chi}=(\chi_1, \dots, \chi_N)$ is the original unknown beam and $\vec{d}=(d_1, \dots, d_M)^T$ is the depth dose curve.

The response matrix $A_{M\,x\,N}$, can be obtained simulating the corresponding depth dose curves for different mono-energetic incident beams with different E_i energies for *i=1,...,N* using the MCNP5 Monte Carlo method [2].

A response matrix (dim. 350x375) with energy range between 0-15 MeV (energy bin 40 KeV) and the depth stepped by *centimeter* were calculated by simulation taking the contribution of photons and electrons. This matrix is presented in Figure 1.

This matrix could be used for any reconstruction of spectra under 15 MeV as long as the experimental depth dose curve at that energy is known.

To test the reliability of this matrix in spectrum reconstruction, an unfolding procedure has been performed using the regularization tools of the *Hansen* Matlab® singular value decomposition unfolding toolbox [3].

As mentioned before, $\vec{d}=(d_1, \dots, d_M)^T$ is the depth dose curve experimentally acquired in a 50 cm x 50 cm x 50 cm cube shaped water tank irradiated by an *Elekta Precise* radiotherapy unit head, available at the *Hospital Clínic Universitari de Valencia, Spain*, using a 6 and 15 MeV photon beam and a 10 cm x 10 cm field size.

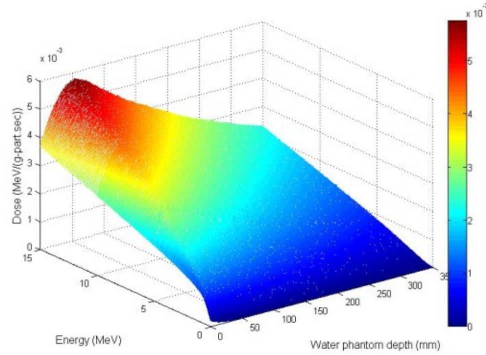

Fig. 1. Response matrix, A ϵ $R_{M\times N}$

2) Discretization of the problem

The discretization of the problem offers a linear system equation matrix:

$$d = A \cdot \chi \qquad (6)$$

$$\begin{aligned} d_1 &= a_1 \cdot x_1 + a_2 \cdot x_2 + \dots + a_M \cdot x_M \\ d_2 &= b_1 \cdot x_1 + b_2 \cdot x_2 + \dots + b_M \cdot x_M \\ &\vdots \\ d_N &= e_1 \cdot x_1 + e_2 \cdot x_2 + \dots + e_M \cdot x_M \end{aligned} \qquad (7)$$

where $A \in R_{M\times N}$, $\chi \in R_M$ and $d \in R_N$.

The original photon spectrum has been defined here (Eq. 6) as the solution vector χ.

3) Gradient methodology

The particularity of this paper is to significantly improve the Bremsstrahlung spectra reconstruction process from depth dose curves using depth dose gradients and its relation to the incident spectrum from a simple spatial relation derived from the standard *Fredholm* equation.

The derivative of the *Fredholm* equation generates another linear equation to be solved as an inverse problem to obtain the χ spectrum.

$$\frac{\partial d_E(x)}{\partial x} = d'(x) = \int_0^{E_{max}} \frac{\partial r_E(x)}{\partial x}\,\chi(E)\,dE \qquad (8)$$

As well as in the previous method, the inverse problem based on depth dose gradient curves to estimate the spectral distribution can be solved.

To perform the derivative of depth dose curves, each curve has been adjusted to a polynomial function 10th degree.

Since the inverse problem in terms of gradients (Eq. 8) is shown to be markedly less ill-conditioned than the usual inverse problem, the discretization has been applied to the gradient matrices.

$$d' = A' \cdot \chi \tag{9}$$

The gradient response matrix is presented in Figure 2.

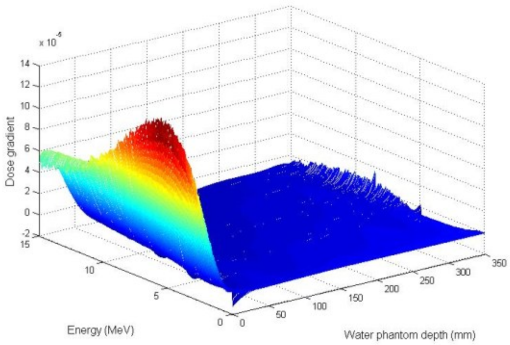

Fig. 2. Gradient response matrix, A ϵ $R_{M \times N}$

The discrete Picard condition of the gradient matrix is presented in the following figure (Figure 3). It can be seen that the Fourier coefficients $|u_i^T b|$ decay to a certain index i, from which the coefficients $|u_i^T b|$ become more stable. Therefore, the range where the Picard condition is satisfied is limited.

This Picard discrete condition allows concluding that although the matrix is ill-conditioned, the problem has a numerical solution. It also allows determining that there are 10 singular values that have useful information, in comparison to the rest that only add noise.

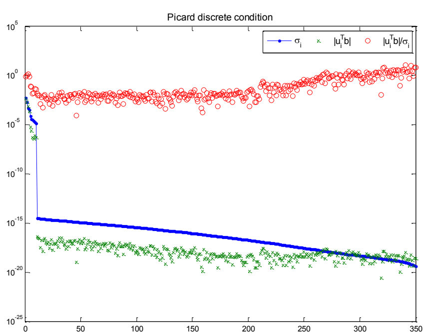

Fig. 3. Picard discrete condition of the gradient response matrix.

4) *Singular Value Decomposition*

The resulting equations system is ill-conditioned, therefore, it is necessary to apply sophisticated numerical techniques to solve this difficulty; concretely we have used the *Singular Value Decomposition* (SVD) methodology:

The SVD solves this equation (Eq. 6) as:

$$A' = \sum_{i=1}^{n} u_i \, \sigma_i v_i^T \tag{10}$$

Where u_i and v_i are the orthonormal singular vectors and σ_i the singular values.

$$A' = U \Sigma V^T \tag{11}$$

where U is an $M \times N$ matrix whose columns, u_j, $j = 1, \ldots, N$, are called the left singular vectors and they are orthonormal vectors. Σ is an $N \times N$ diagonal matrix whose elements, σ_j, are the singular values of A' and $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_N \geq 0$. V is an $N \times N$ orthogonal matrix whose columns, v_j, $j = 1, \ldots, N$, are the right singular vectors.

The SVD algorithm works very well for noise-free data. For data affected by noise, direct SVD reconstruction is extremely inaccurate (by many orders of magnitude), but there are several regularization schemes that improve its performance. We have analyzed the reliability of the *Truncated Singular Value Decomposition* (TSVD). With this numeric algorithm, an optimal solution, χ, can be obtained, generating a new response matrix, removing the parts of the solution corresponding to the smallest singular values. This method involves the selection of the cut-off value and the setting to zero of the diagonal elements of the pseudoinverse matrix that were smaller than this cut-off.

One way to determine the optimum truncation parameter, k, is to plot the L-curve, which consists of a relatively vertical segment and a relatively horizontal line in an ideal setting. For this curve, the 2-norm of the solution vector, $\|\chi\|$, is plotted versus the 2-norm of the residual vector, $\|A' \cdot \chi - d'\|$,, for different values of k. It is recommended to find the truncation parameter k, as the closest value to the maximum curvature point of the curve, that is, selecting the value corresponding to the L-shaped corner.

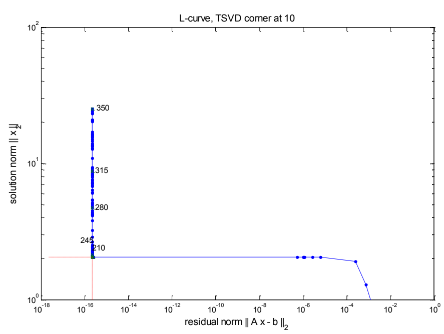

Fig. 4. L-curve of the gradient response matrix.

Figure 4 shows the corresponding L-shaped curve obtained applying the TSVD method for different values of k to the studied matrices. It can be observed that the corner corresponds to a value of $k=10$. For low values of k, an important loss of resolution occurs, whereas for high values of k, the noise of the unfolded spectrum increases.

B. *Monte Carlo simulated depth dose curves*

For the 6 and 15 MeV energy range we are considering, depth dose curves from mono-energetic spectra were calculated using the MCNP5 Monte Carlo code. The source surface distance (SSD) from a point source on the central axis to a homogeneous water phantom has been 100 cm. The used field size has been 10 cm × 10 cm. Each Monte Carlo simulation was run until the uncertainty in all evaluated points of computed depth dose curves was less than 0.3%.

To register the contribution of photons and electrons, the FMESH tally has been used with its respective flux-to-dose conversion factors, implemented as the DE/DF target in MCNP.

Concerning radiation transport, a detailed physics treatment for a coupled photon and electron mode simulation has been considered, taking into account the following physical processes: photoelectric effect with fluorescence production, Compton and Thomson scattering, and pair production in the energy range between 0.001 and 16 MeV. Photonuclear contribution has not yet been included in this study, it will be the goal of further investigations.

III. RESULTS

Figures 5 and 6 show the reconstructed photon spectra obtained using the improved depth dose gradient technique and the TSVD method for 6 and 15 MeV photon beams.

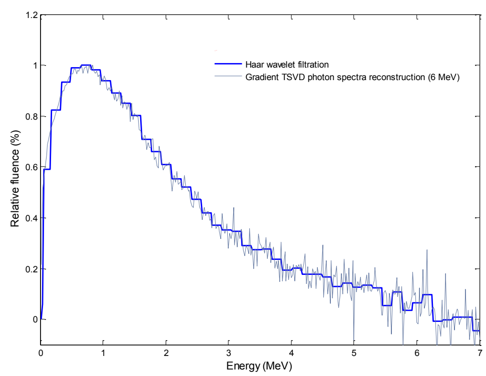

Fig. 6. Reconstructed 6 MeV spectrum obtained with the depth dose gradient technique using.

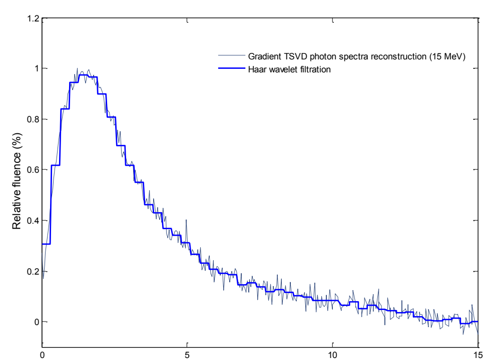

Fig. 7. Reconstructed 15 MeV spectrum obtained with the depth dose gradient technique using.

Both figures present the reconstructed spectrum along with a *Haar* (3rd level) filtered one using Wavelet tools.

To validate the reconstructed spectra we have used a complete Monte Carlo simulation in order to generate the depth dose curve using the reconstructed 6 and 15 MeV spectra as input source.

The generated depth dose curves have been compared with the real experimental data measured at the hospital, (figures 9 and 10), showing a root mean square of 1% and 3% correspondingly.

As it can be seen, the gradient technique gives a good representation of the depth dose curves and accurately tracks the build up and dose for higher depth, showing the robustness of this reconstruction methodology.

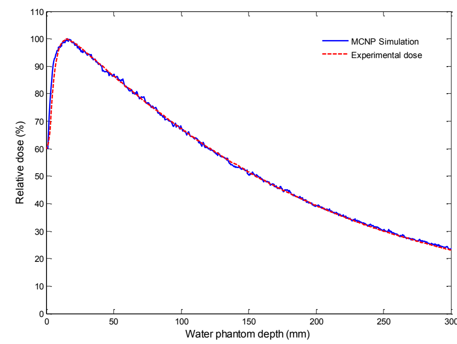

Fig. 9. Depth dose curve obtained with a 6 MeV photon spectrum.

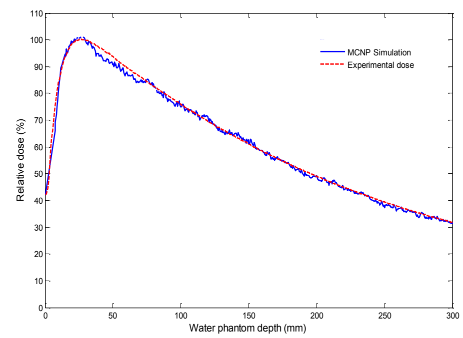

Fig. 10. Depth dose curve obtained with a 15 MeV photon spectrum.

IV. CONCLUSIONS

To characterize the primary photon spectrum emitted by a linac, the response function of the process was obtained. This function is approximated by a response matrix that contains all the required information to unfold the gradient of the depth dose curve measured experimentally in a water tank using simulated Monte Carlo curves.

An important problem in the unfolding process is to obtain the inverse of the response matrix. A pseudo-inverse matrix based on the TSVD method was used to obtain a good approximation of the inverse matrix. This pseudo-inverse matrix can be obtained easily and it permits an accurate and fast reconstruction of the primary spectrum when considering the contribution of photons and electrons to the dose.

REFERENCES

[1] P. M. Charland, L. D. Paniak, "Enhanced bremsstrahlung spectrum reconstruction from depth-dose gradients". Physics in Medicine & Biology,14, pp. 3245-3262, (2005)

[2] X-5 Monte Carlo team, "MCNP – A General Monte Carlo NParticle Transport Code, Version 5", LA-UR-03-1987, Los Alamos National Laboratory, April 2003.

[3] Hansen P.C., "Regularization Tools: A Matlab package for analysis and solution of discrete ill-posed problems", Numerical Algorithms, Vol. 6, pp. 1-35, (1994).