

# Variation of wave speed determined by the PU-loop with proximity to a reflection site

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**Abstract**— Wave speed is directly related to arterial distensibility and is widely used by clinicians to assess arterial stiffness.

The PU-loop method for determining wave speed is based on the water hammer equation for flow in flexible tubes and artery using the method of characteristics. This technique determines wave speed using simultaneous measurements of pressure and velocity at a single point. The method shows that during the early part of systole, the relationship between pressure and velocity is generally linear, and the initial slope of the PU-loop is proportional to wave speed.

In this work, we designed an in-vitro experiment to investigate the effect of proximity to a reflection site on the wave speed determined by the PU-loop through varying the distance between the measurement and reflection sites. Measurements were made in a flexible tube with a reflection site at the distal end formed by joining the tube to another tube with a different diameter and material properties. Six different flexible tubes were used to generate both positive and negative reflection coefficients of different magnitudes.

We found that the wave speed determined by the PU-loop did not change when the measurement site was far from the reflection site but did change as the distance to the reflection site decreased. The calculated wave speed increased with positive reflections and decreased with negative reflections. The magnitude of the change in wave speed at a fixed distance from the reflection site increased with increasing the value of the reflection coefficient.

## I. INTRODUCTION

Wave speed depends on both the inertial and elastic properties of the medium. The wave speed in an artery depends on the cross sectional area, elastic characteristics of the vessel and the density of blood [1]. Vessel distensibility is inversely proportional to the square of the wave speed [2]. It is for this reason that wave speed is now commonly used by clinicians as an indicator of cardiovascular disease and arterial stiffening. A decrease in distensibility due to stiffening of the arteries leads to an increase in wave speed [3].

Local wave speed refers to the determination of wave speed at the measurement site. Khir et al. proposed the PU-loop

method to determine the wave speed using single-point measurements of pressure and velocity. The method showed that during the early part of systole, it is most probable that only forward waves are present in the ascending aorta, the relationship between pressure and velocity is linear in the absence of reflections, and the initial slope of the PU-loop is directly related to wave speed [4]. In the coronary arteries where the PU-loop method fails because both backward and forward waves are generated during early systole, Davies et al. introduced another technique for determining local wave speed also based on the measurements of the pressure and flow at the same site [5]. Rabben presented a method for estimating wave speed from ultrasound measurements; the flow-area loop method, in which wave speed is estimated as the ratio between the change in flow to the change in cross-sectional area during the reflection-free period of the cardiac cycle [6].

It is generally known that any discontinuity in the properties of the artery will cause the wave fronts to produce reflected and transmitted waves according to the type of discontinuity, i.e. changes in area, local changes in the elastic properties of the arterial wall, or bifurcations. It has become common knowledge that arterial wave reflections contributes to the increase in systolic and pulse pressure as seen with ageing and in patients with hypertension [7], [8]. Reflections are also considered to be important in the management of clinical conditions of some cardiovascular diseases [9], [10].

Determination of wave speed using the PU-loop relies on the existence of a 'reflection-free' period, which may be too short if the measurement site is very close to the reflection site. Given the importance of local wave speed and wave reflection, we designed an in-vitro experiment to investigate the effect of reflections on the wave speed determined by PU-loop.

## II. MATERIALS AND METHODS

### A. Theoretical Methods

#### 1) PU-loop

The theoretical basis of this work is the solution of the one-dimensional equations for flow in an elastic tube using the method of characteristics. The method has been introduced by Khir et al., in 2001 [1].

The water hammer equation can be written as

$$dP_{\pm} = \pm \rho c dU_{\pm} \quad (1)$$

Where  $dP$  and  $dU$  are the changes in pressure and velocity respectively,  $\rho$  is density,  $c$  is wave speed and  $\pm$  indicates the forward and backward directions.

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Equation (1) describes the relationship between changes in the pressure and velocity, and plotting the measured pressure against the measured velocity over the cycle we obtain a PU-loop. During the very early part of the systole when only forward waves are expected to be present, the slope of the PU-loop should be linear. On arrival of the reflected waves, the linear relationship between pressure and velocity may no longer hold. If waves are running only in the forward direction, the expression of wave speed in terms of P and U is:

$$c = + \frac{1}{\rho} \frac{dP_+}{dU_+} \quad (2)$$

## 2) Reflection coefficient

The mathematical details involved in deriving the exact nature of the reflected waves is rather complex but the results are straightforward. The value of the reflection coefficient depends upon the area A and wave speed c upstream 0 and downstream 1 of the discontinuity. For arteries or flexible tubes where the velocity is generally much lower than the wave speed, the reflection coefficient R is

$$R = \frac{(A_0/c_0) - (A_1/c_1)}{(A_0/c_0) + (A_1/c_1)} \quad (3)$$

## B. Experimental Methods

The general experimental setup used in the study is shown in Fig. 1 and a description of the individual elements follows.

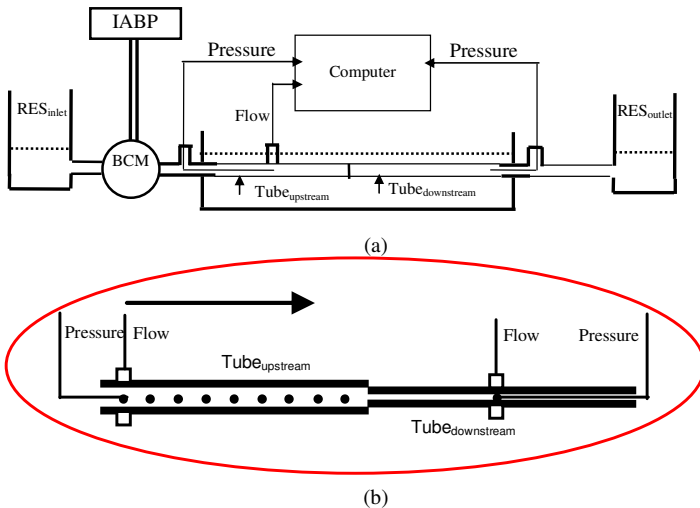


Fig. 1. A schematic diagram of the experiment set up in (a), and detailed diagram of the measurements and tubes (b).

### 1) Tubes

In this work we used one “mother” tube, which is 3 m long, 10 mm diameter and 1 mm wall thickness, flexible, made by silicone, and 6 “daughter” tubes of flexible, different sizes and materials, each 14 m long. This length was needed to ensure that the reflected wave arrives after the incident wave has passed at the inlet of the mother tube. The daughter tubes were connected with the mother tube to provide different reflection coefficients. The details of these tubes are shown in Table I. Each of the tubes is uniform in both dimension and mechanical properties along its length. The mother tube was fully merged into a water tank, where the water level was approximately 1 cm above the tubes. All tubes were kept in

the horizontal position. Daughter tubes A, B, C, and D were connected directly to the mother tube by overlapping the inlet of each over the outlet of the mother tube. Daughter tubes E, F were connected to the mother tube by overlapping the inlet of each over a short connecting tube, which in turn was connected to the mother, also through overlapping..

### 2) Pump

As shown in Fig. 1, BCM (Cardiacare, Minneapolis, MN, U.S.A) is a flexible diaphragm pulsatile left ventricle assist device (LVAD), which can be operated using an intra-aortic balloon pump (IABP). The inlet of BCM was connected with the inlet reservoir and the outlet of BCM was connected with inlet of mother tube. In this work, the BCM generates an approximate half-sinusoidal wave of 0.5 s.

TABLE I  
Daughter tubes properties and reflection coefficients

Set.	D <sub>in</sub> (mm)	h (mm)	E (MPa)	R
A	8	2	Silicone (2.901)	+0.34
B	8	1	Silicone (3.105)	+0.28
C	10	2	Silicone (2.918)	+0.12
D	12	1	Silicone (3.101)	-0.12
E	16.7	1.5	Rubber (4.475)	-0.39
F	21	1.5	Rubber (4.361)	-0.63

D<sub>in</sub>: Internal diameter, h: Wall thickness, E: Young’s modulus and R: reflection coefficient

### 3) Reservoirs

The inlet and outlet of each tube were connected to the inlet and outlet reservoirs, respectively. The height of the fluid in the reservoirs was adjusted to 10 cm above the longitudinal axis of the tube; producing an initial hydrostatic pressure of 1 kPa. The differences in mean transmural pressure for different-sized tubes were negligible.

### 4) Measurements

Simultaneous pressure and flow waveforms were measured at the same sites for each case, sequentially in time, every 10 cm along the mother tube. Due to the limit of the length of the pressure catheter, measurements could not be taken in the middle third of the mother tube. In the last 20 cm away from the reflection site, measurements were taken every 5cm. There are also one measurement of pressure and flow at 10cm downstream of the reflection site in every daughter tube. Pressure and flow were measured using an 8F tipped catheter pressure transducer (Millar Instruments Inc., Houston, Texas, USA) and ultrasonic a flow probe (Transonic System, Inc, Ithaca, NY, USA). External diameter and wall thicknesses of the tubes were measured by a digital caliber. All the data were acquired at a sampling rate of 500 Hz using Sonolab (Sonometrics Corporation, London, Ontario, Canada). Velocity is derived from the measured flow rate using the cross sectional area. The analysis procedure was carried out using programs written in Matlab (The Mathworks, MA, USA).

### III. RESULTS

#### A. Determination of wave speed

Fig. 2 shows the pressure and velocity waveforms and the corresponding PU-loop measured at the inlet of the mother tube.

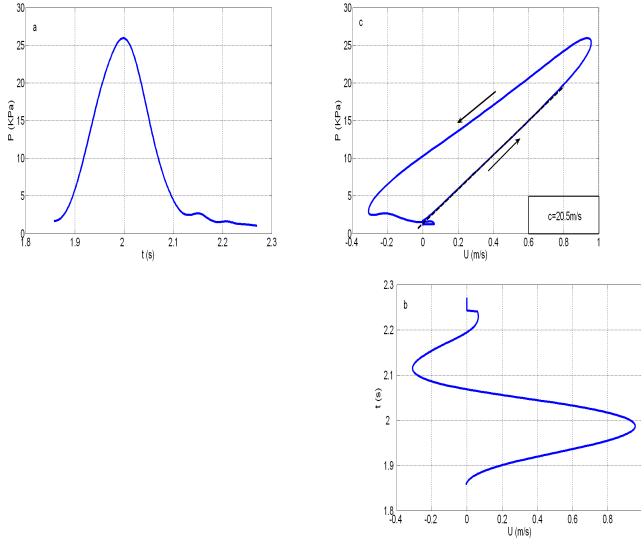


Fig. 2. Pressure (a), velocity (b) and PU-loop (c) measured at the inlet of the mother tube; 210 cm away from the reflection site. The dash line showed the slope of the loop in early systole, the arrows shown the direction of the loop. Wave speed determined is 20.5 m/s.

Local wave speeds measured along the mother tube are shown in Fig. 3 for set A. Wave speeds measured at the entrance of the mother tube were similar and constant within experimental error; approximately 20 m/s. The measured wave speed increased sharply as the measurement site approached the reflection site.

#### B. Reflection coefficients

In all cases we found that the measured wave speeds at the entrance of the tube were uniform. We therefore used these wave speeds and the wave speeds measured in the daughter tubes in equation (3) to calculate the reflection coefficients shown in Table I.

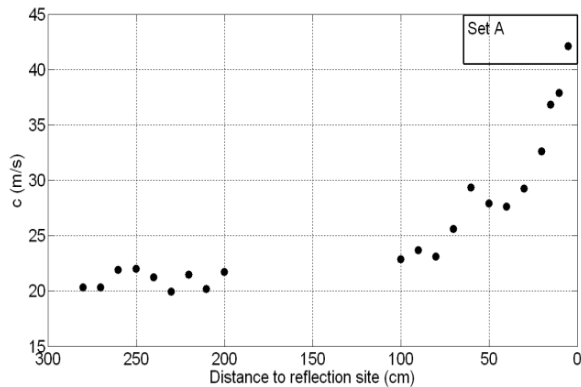


Fig. 3. Wave speeds determined by PU-loop at 21 positions along the mother tube of set A.

#### C. Effect of reflection on wave speed

Fig. 4 shows all the measured wave speeds determined by the PU-loop method along the mother tube for all reflection conditions. From this figure, we see that the measured wave speed determined at the entrance of the mother tube were very similar, around 20-22 m/s, for all cases. At the end of the tube, the measured wave speed increased or decreased sharply depending on the sign and magnitude of the reflection coefficient.

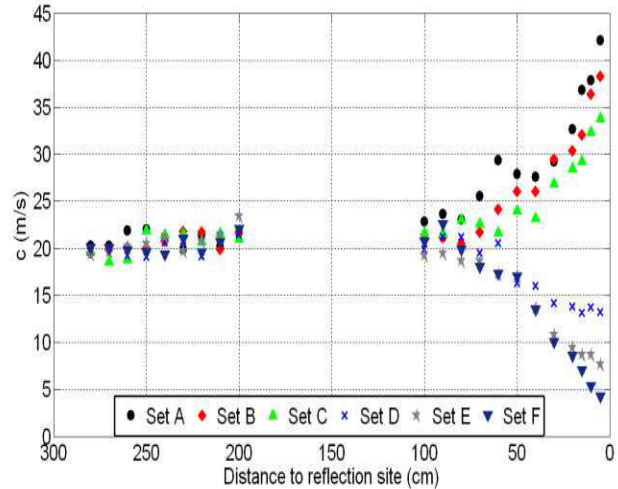


Fig. 4. Wave speeds determined in all sets.

### IV. CONCLUSIONS AND DISCUSSIONS

In this work, we designed an experiment to investigate the effect of reflection on the wave speed determined by the PU-loop method.

Pooling all of the measurements for the entrance of the tube, the wave speed measured for the mother tube was  $20.43 \pm 0.86$  m/s. In a previous experiment using tubing identical to the mother tube in a different setup without a fixed reflection site [11], the wave speed was measured to be  $20.22 \pm 1.28$  m/s. We therefore conclude that the wave speeds measured in the entrance of the mother tube were not affected by the reflection and that the standard deviation is a measure of the error in our measurements.

At the end of the mother tube, 6 different tubes were connected in turn to generate six different reflection coefficients. We chose 3 tubes which were smaller or thicker than the mother tubes in order to generate positive reflections and 3 bigger or thinner tubes to obtain negative reflections. From Table I and Fig. 4, we see that the positive reflections increase the measured wave speed when the measurement site is close to the reflection site; whereas negative reflections decrease the measured wave speed. We also see that the magnitude of the changes varies with the magnitude of the reflection coefficient for both positive and negative reflections.

Theoretically, in the absence of reflections, wave speed is determined from the slope of PU-loop during early systole. The arrival of reflected wave is expected to cause an

inflection point, although it is also possible the linearity of the initial part of the loop remains despite the arrival of reflected wave [12]. For example, in Fig. 5 we plot three PU-loops measured at three different sites: 80, 40, and 20cm away from the reflection site in set A. We see that none of the loops exhibit an inflection point, although the two loops corresponding to positions 40 and 20cm away from the reflection site have greater slopes indicating higher measured wave speed. We have also observed all wave speeds measured along the mother tube from its inlet to 80cm away from the reflection site were almost the same;  $\sim 20\text{m/s}$ . Assuming that wave speed determined at those sites was not affected by reflection, and observing the measured wave speed along the tube and closer to the reflection site than 80cm began to increase or decrease depending on the reflection coefficient (Table 1), suggests the reflected wave arrived earlier at those other sites and affected the slopes of their loops.

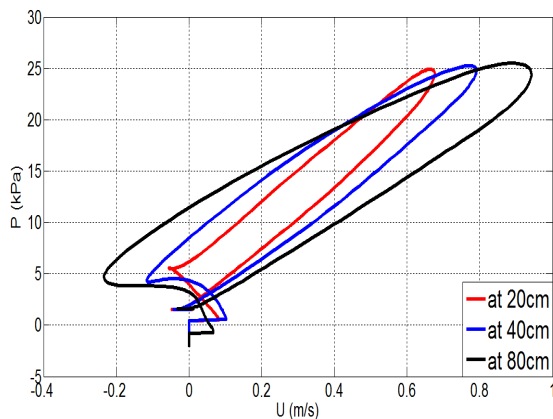


Fig. 5. PU-loops at different measurement site.

The PU-loop has been used to determine the wave speed in the ascending aorta of patients with cardiovascular disease [13], but is unsuitable for wave speed analysis in the coronary arteries, because coronary arteries are subject to influences from the aortic and microcirculatory ends [14]. This work suggests that nearby reflections would affect the wave speed determined by the PU-loop. The next step should be finding a way to correct the measured wave speed when local reflections are present. We are currently looking at the possibility of correcting the measured wave speeds using Taylor expansion. It can also be used to explain why we do not see a point of inflection when the rate of change of the input waveform is not too large. I.e. for small wave travel times, the local pressure is increased by a factor  $1+R$  with a correction term depending on the rate of change of the waveform.

## V. LIMITATIONS

In this study, we investigated the effect of reflection site on wave speed determined by the PU-loop in a flexible single tube formed by joining tubes together end to end. The arteries are much more complex so that experiments with bifurcations or tapered tubes will be necessary to further understand the effect of this phenomenon more fully in arteries.

In the present experiments, no measurements were taken in the middle part of the mother tube because of the limited length of the pressure catheter. This is not expected to influence the interpretations of the results as the measurement sites downstream from this region also give results indicating wave speed was not affected (shown in Fig. 4).

## VI. CONCLUSIONS

In conclusion, wave speeds determined by the PU-loop method would change when the measurement site is close to a reflection site. The measured wave speed will increase when there is a positive reflection, and, inversely, it will decrease when the reflection is negative. This effect increases with the magnitude of the local reflection. The magnitude of the change in wave speed at a fixed distance from the reflection site increased with increasing the value of the reflection coefficient.

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