Wave speed and intensity in the canine aorta: analysis with and without the Windkessel-wave system

Alessandra Borlotti *Member, IEEE-EMB* and Ashraf Khir

*Abstract***—The Windkessel model, coupled with the wave propagation theory, was applied to data measured in the ascending aorta of 11 anaesthetised dogs during total aortic occlusion at the thoracic and diaphragm levels. Wave speed and wave intensity were calculated using the measured pressure (P) and velocity (U), and separately using the pressure due to the wave (Pex) and U in the aorta approximately 1 cm distal to the aortic valve.**

Results show that wave speed, determined using the PU-loop method, is higher during thoracic than in diaphragm occlusion (p<0.001). On average wave speed calculated using P (c) is higher than that determined using Pex (cWK) in both occlusion sites (p<0.001). During aortic occlusion at the thoracic level, the intensity of backward waves was almost negligible using the Windkessel-wave system. Backward waves were observed during the occlusion at the diaphragm level, but their magnitude is lower compared to that determined with P. The Windkessel-wave system seems to reduce the magnitude of reflected waves during total aorta occlusion, notably if the occlusion sites are close to the ascending aorta.

I. INTRODUCTION

HE Windkessel model, proposed in 1899 by Frank [1], THE Windkessel model, proposed in 1899 by Frank [1],
shows the importance of the aortic compliance in turning the discontinuous cardiac output into a more steady pressure and flow in the downstream arteries, storing about 50% of the left ventricular stroke volume during systole and forward it to the peripheral circulation during diastole [2]. The Windkessel model describes very well the diastolic part of the pressure waveform, but is not particularly accurate for the systolic one because it does not take into account the wave contribution. In order to explain these two phases and the differences in shape of pressure and flow waveforms Wang *et al*. [3] proposed a new time domain approach that is a combination of the Windkessel model and the wave propagation theory; Windkessel-wave system. The authors considered the measured pressure in the aorta (P) as the sum of a reservoir pressure (P_r) and a pressure due to the waves that is termed excess pressure (P_{ex}) . The main findings of that study were: P_r is proportional to the aortic volume and Pex waveform has the same shape of the aortic flow.

Tyberg et al., calculated wave intensity analysis in the canine aorta using P_{ex} and velocity (U) [4], and have shown that there are not significant backward-traveling waves to the aortic root at basal conditions. However, Khir and Parker

Alessandra Borlotti, Brunel Institute for Bioengineering, Brunel University, Greater London, UK alessandra.borlotti@brunel.ac.uk

[5], calculated wave intensity at the canine aortic root using P and U, and reflected waves were evident also in control conditions. Therefore, the aim of this work is to quantify the differences between using the Windkessel-wave system and using the wave propagation theory only, on the wave speed and reflected waves. Total occlusions of the aorta at the thoracic and diaphragm levels were applied in 11 anaesthetised dogs. Wave speed and the backward waves were determined with and without the Windkessel-wave sysetm.

II. THEORETICAL CONSIDERATIONS

 Wang *et al*. [3] proposed that the measured ascending aorta pressure can be considered as the sum of a reservoir pressure (P_r) and an excess pressure (P_{ex}) , the Windkessel and wave components, respectively

$$
P(x,t) = P_r(t) + P_{ex}(x,t)
$$
 (1)

A. Windkessel theory

Variation of aortic P_r can be defined as

$$
\frac{dP_r(t)}{dt} = \frac{dP_r}{dV_r}\frac{dV_r}{dt} = \frac{Q_{in}(t) - Q_{out}(t)}{C}
$$
\n(2)

where V_r is the reservoir volume, Q_{in} is the aorta inflow, Q_{out} the outflow and C is the compliance of all the arterial tree. Q_{out} can be described by the follow equation

$$
Q_{out}(t) = \frac{P_r(t) - P_{\infty}}{R}
$$
 (3)

where P_{∞} is the asymptotic pressure of the diastolic exponential decay and R is the resistance of the peripheral systemic circulation. Substituting 3 in equation 2

$$
\frac{dP_r(t)}{dt} + \frac{P_r(t) - P_\infty}{RC} = \frac{Q_{in}(t)}{C}
$$
\n⁽⁴⁾

that leads to the general solution

$$
P_r(t) - P_{\infty} = (P_0 - P_{\infty})e^{-V_{RC}} + e^{-V_{RC}} \int_{t_0}^t \frac{Q_{in}(t)}{C} e^{-V_{RC}} dt
$$
 (5)

where t_0 and P_0 are time and pressure at the beginning of the ejection. Q_{in} was considered zero during the diastole.

B. Wave theory

Arterial wave propagation theory derived from the conservation of mass and momentum equations. Any disturbance inside the vessel generates wavefronts that travel in the forward and backward directions with speed c. Changes in pressure (dP) and velocity (dU) are related through the water hammer equation

Manuscript received April 15th, 2011.

Ashraf W. Khir, Brunel Institute for Bioengineering, Brunel University, Greater London, UK Ashraf.Khir@brunel.ac.uk

$$
dP_{\pm} = \rho c dU_{\pm} \tag{6}
$$

where ρ is the fluid density, c the wave speed and " \pm " refers to the wave direction. Khir *et al*. [6] introduced the PU-loop method for the determination of c based on equation (6).

Parker et al., introduced the wave intensity (dI) analysis [7], which is equal to dPdU and allows the separation between the forward and backward waves using:

$$
dI_{\pm} = \pm \frac{1}{4\rho c} \left(dP \pm \rho c dU \right)^2 \tag{7}
$$

III. METHODOLOGY

A. Experimental protocol

Experiments were performed in 11 anaesthetised mongreal dogs (average weight 22±3 kg). The experimental protocol is described in detail elsewhere [5]. Briefly, an aortic flow probe was placed around the ascending aorta, approximately 1 cm distal from aortic valve. Pressure in the same site was measured with a high-fidelity pressure catheter inserted from the right or left brachial artery. Snares were used to occlude the descending thoracic aorta at the level of the aortic valve and the lower thoracic aorta at the level of the diaphragm. Data were collected for 30s at two times: before occlusion and during the total occlusion.

B. Analysis

 Reservoir pressure was calculated from the measured pressure using a program written in Matlab (The Mathworks Inc, Natick, Massachusetts, USA) that finds the minimum mean-squared error between P and P_r (Figure 1) during the diastolic part of the cycle. Then, P_{ex} was obtained as the difference between P and Pr.

Wave speed in ascending aorta, was determined by the PUloop (c) and $P_{ex}U$ -loop (c_{WK}), before and during occlusion. Wave intensity analysis was performed with the measured pressure (dI) and separately with P_{ex} (dI_{wk}). Wave speed and the intensity of the reflected wave calculated with P and P_{ex} were averaged using three beats for each dog.

Figure 1. Typical waveforms of measured pressure (P, solid) taken in the ascending aorta during thoracic occlusion and the corresponding calculated reservoir pressure (P_r, dashed).

IV. RESULTS

A. Wave speed

Figure 2 shows the PU-loop and the $P_{ex}U$ -loop during thoracic and diaphragm occlusion. Wave speed was calculated from the slope of the linear part of the loop when it is most probable only unidirectional waves are present. In Figure 3 mean values of c and c_{WK} are reported before and during thoracic and diaphragm occlusion. c is higher than c_{WK} before and during the occlusion in both sites (p<0.001). c is 38% and 53% higher than c_{WK} before the occlusion and 98% and 89% higher during occlusion, all for thoracic and diaphragm respectively.

The difference between the two calculated wave speeds is higher during the occlusion than the control. As can be noticed from Figure 3, c during the thoracic occlusion is 46 $%$ higher than at control conditions. Surprisingly c_{WK} seems not to be affected by the occlusion.

Figure 2. A typical example of the PU-loop and PexU-loop during thoracic (a and b respectively) and diaphragm (c and d respectively) occlusion.

Figure 3. Mean values and standard deviation of wave speed in the ascending aorta before and during occlusion at the thoracic and diaphragm sites determined using the PU-loop (c) and the P_{ex} -loop (c_{WK}).

B. Reflected waves during aorta occlusion

In Figure 4 the absolute peak value of the backward wave intensity is shown during thoracic and diaphragm occlusion using P (dI.) and P_{ex} (dI_{WK}). During aortic occlusion at the thoracic level reflections were about seven times lower using P_{ex} than using P (0.96±0.94 vs. 6.88±6.04 W/m², p<0.001). Backward waves on average were reduced of about three times when the aorta was occluded at the diaphragm level $(2.68 \pm 2.20 \text{ vs. } 9.71 \pm 6.63 \text{ W/m}^2, \text{ p} < 0.001).$

Figure 5 and Figure 6 show the wave intensity, separated into its forward and backward components, before and during thoracic and diaphragm occlusion; calculated using P and P_{ex} . As can be seen from Figure 5b,c backward wave intensities are almost negligible in the ascending aorta before and during thoracic total occlusion, when using P_{ex} .

Figure 4. Mean values and standard deviations of the reflected waves during occlusion in both sites determined with the measured pressure (dI₋) and excess pressure (dIwk-).

V. DISCUSSION

In the present work the Windkessel model coupled with the wave propagation theory proposed by Wang et al. [3] was applied in order to study the aorta hemodynamics during total occlusion in two different sites. The Windkessel-wave system applied in this paper must not be confused with the lumped Windkessel model which may not to be efficient in describing fast dynamic and traveling waves [8].

Wang's system has been already applied to human [9] and animal data [3, 4, 10] in the ascending aorta as well as in other arteries [11]. Tyberg *et al*. [4] applied this model at the level of the aortic valve and they found that backward waves are not significant. We performed wave intensity analysis in the same site but in two particular hemodynamics conditions; total occlusion of the aorta at the thoracic and at the diaphragm levels. We expected to see predominant backward traveling waves in these two conditions, but we surprisingly found that they are particularly small when the Windkessel-wave system was applied. In particular our results show that reflection waves during aortic occlusion at the thoracic level are almost negligible and there is no difference in magnitude between control and during occlusion conditions. Also forward compression wave intensities decrease by 43% and 42% during thoracic and diaphragm occlusion respectively when the Windkesselwave system is applied. Moreover, c and c_{WK} calculated using the PU-loop and the $P_{ex}U$ -loop before and during the occlusions are significantly different in both sites.

Figure 5. Example of wave intensity using P (dI) and P_{ex} (dI_{WK}), separated in forward (dI₊ and dI_{WK+}) and backward (dI₋ and dI_{WK-}) components, before thoracic occlusion (a and b) and during occlusion (c and d).

Figure 6. Example of wave intensity using P (dI) and P_{ex} (dI_{WK}), separated in forward (dI₊ and dI_{WK+}) and backward (dI₋ and dI_{WK-}) components, before diaphragm occlusion (a and b) and during occlusion (c and d).

VI. CONCLUSION

We conclude that the application of the Windkessel-wave system in the present study tends to reduce the magnitudes of wave speed and the reflected waves in basal conditions. It can also lead to an unexpected reduction of the wave speed and reflected waves when the occlusion site is close to the measurement site. Further investigations are needed to explain the reasons of these findings.

REFERENCES

[1] K. Sagawa, R.K. Lie and J. Schaefer, "Translation of Otto frank's paper "Die Grundform des arteriellen Pulses" zeitschrift für biologie 37: 483–526 (1899)" J.Mol.Cell.Cardiol., vol. 22, pp. 253-254, 3. 1990.

[2] G.G. Belz, "Elastic properties and Windkessel function of the human aorta" Cardiovasc.Drugs Ther., vol. 9, pp. 73-83, 1995.

[3] J.J. Wang, A.B. O'Brien, N.G. Shrive, K.H. Parker and J.V. Tyberg, "Time-domain representation of ventricular-arterial coupling as a windkessel and wave system" American Journal of Physiology - Heart and Circulatory Physiology, vol. 284, pp. H1358-H1368, 2003.

[4] J.V. Tyberg, J.E. Davies, Z. Wang, W.A. Whitelaw, J.A. Flewitt, N.G. Shrive, D.P. Francis, A.D. Hughes, K.H. Parker and J.-. Wang, "Wave intensity analysis and the development of the reservoir-wave approach" Medical and Biological Engineering and Computing, vol. 47, pp. 221-232, 2009.

[5] A.W. Khir and K.H. Parker, "Wave intensity in the ascending aorta: Effects of arterial occlusion" J.Biomech., vol. 38, pp. 647-655, 2005.

[6] A.W. Khir, A. O'Brien, J.S.R. Gibbs and K.H. Parker, "Determination of wave speed and wave separation in the arteries" J.Biomech., vol. 34, pp. 1145-1155, 2001.

[7] K.H. Parker and C.J.H. Jones, "Forward and backward running waves in the arteries: Analysis using the method of characteristics" J.Biomech.Eng., vol. 112, pp. 322-326, 1990.

[8] N. Westerhof, J-W. Lankhaar and B. E. Westerhof, "The arterial Windkessel" Med Biol Eng Comput vol. 47, pp. 131-141, 2009.

[9] J.E. Davies, N. Hadjiloizou, D. Leibovich, A. Malaweera, J. Alastruey-Arimon, Z.I. Whinnett, C.H. Manisty, D.P. Francis, J. Aguado-Sierra, R.A. Foale, I.S. Malik, K.H. Parker, J. Mayet and A.D. Hughes, "Importance of the aortic reservoir in determining the shape of the arterial pressure waveform - The forgotten lessons of Frank" Artery Research, vol. 1, pp. 40- 45, 2007.

[10] J. Wang Jr., J.A. Flewitt, N.G. Shrive, K.H. Parker and J.V. Tyberg, "Systemic venous circulation. Waves propagating on a windkessel: Relation of arterial and venous windkessels to systemic vascular resistance" American Journal of Physiology - Heart and Circulatory Physiology, vol. 290, pp. H154-H162, 2006.

[11] J. Aguado-Sierra, J. Alastruey, J.-. Wang, N. Hadjiloizou, J. Davies and K.H. Parker, "Separation of the reservoir and wave pressure and velocity from measurements at an arbitrary location in arteries" Proc.Inst.Mech.Eng.Part H J.Eng.Med., vol. 222, pp. 403-416, 2008.