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Use of Genetic Algorithm for Selection of Regularization Parameters in Multiple Constraint Inverse ECG Problem

Alireza Mazloumi Gavgani, Yesim Serinagaoglu Dogrusoz, *Member, IEEE*

***Abstract*—Tikhonov regularization is one of the most widely used regularization approaches in literature to overcome the ill-posedness of the inverse electrocardiography problem. However, the resulting solutions are biased towards the constraint used for regularization. One alternative to obtain improved results is to employ multiple constraints in the cost function. This approach has been shown to produce better results; however finding appropriate regularization parameters is a serious limitation of the method. In this study, we propose estimating multiple regularization parameters using a genetic algorithm based approach. Applicability of the approach is demonstrated here using two and three constraints. The results show that GA based multiple constraints approach improves the Tikhonov regularization solutions.**

I. Introduction

Inverse problem of electrocardiography (ECG) can be described as the estimation of cardiac electrical sources (for example, the epicardial potential distributions) using the body surface potential measurements (BSPMs). The cardiac electrical source distributions provide useful information about the functioning of the heart. However, the inverse ECG problem is ill-posed; even small amount of noise in the measurements causes unbounded errors in the solutions. To overcome this drawback, various regularization methods have been applied in literature; Tikhonov regularization [1], truncated singular value decomposition [2], Bayesian Maximum *A Posteriori* (MAP) Estimation [3], Kalman Filtering [4], [5], [6] are some of the methods used for regularizing the inverse ECG problem.

In Tikhonov regularization, which is one of the most widely used approaches, one tries to reach a trade-off between a good fit to the measurements and the a priori information about the solution. This trade-off is achieved using a regularization parameter, which is usually found using the L-curve approach [7], [8]. In most of the previous applications of Tikhonov regularization, researchers used a single spatial constraint, such as the energy, the gradient, or the Laplacian of the epicardial potentials. But using a single constraint causes the solution to be effected by the choice of this constraint. One alternative is to use more than one constraint with the aim of combining advantages introduced by each constraint, and reducing the effects of their disadvantages.

Admissible solution method [9] and multiple constraints method [10] have been proposed to include more than one constraint in the solution. In the latter approach, each constraint is included in the cost function of the Tikhonov regularization via a separate regularization parameter. Although theoretically it is possible to include a large number of constraints in this approach, finding appropriate regularization parameters is a limitation of this study. Brooks *et.al.* used two constraints, and found the corresponding regularization parameters using L-surface, which is an extension of the L-curve method [10]. Adding a second constraint improved the results, but extending the L-surface to include more than two constraints is not a practical approach. In this study, we propose the use of Genetic Algorithm (GA) to estimate the optimum regularization parameters, then we solve the inverse ECG problem using the multiple constraints approach of [10]. GA has been used to estimate the epicardial potentials in the inverse ECG literature [11], [12]. However, we use GA to estimate only the regularization parameters; hence the number of unknowns decreases significantly.

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A. M. Gavgani is with the Electrical and Electronics Engineering Department, Middle East Technical University, Ankara, TURKEY (e-mail: alireza.gavgani@metu.edu.tr).

Y. S. Dogrusoz is with the Electrical and Electronics Engineering Department, Middle East Technical University, Ankara, TURKEY (e-mail: yserin@metu.edu.tr).

II. Methods

A. Problem Definition

The relation between the BSPMs and the epicardial potentials can be written as,

$$\mathbf{y}(k) = \mathbf{A}\mathbf{x}(k) + \mathbf{n}(k), \quad k = 1,2,...,T \tag{1}$$

where k is the time instant, $\mathbf{y}(k) \in \mathbf{R}^{\mathrm{Mx1}}$ is the vector of BSPMs, $\mathbf{x}(k) \in \mathbf{R}^{\mathrm{Nx1}}$ is the vector of epicardial potentials, $\mathbf{A} \in \mathbf{R}^{\mathrm{MxN}}$ is the forward transfer matrix, and $\mathbf{n}(k) \in \mathbf{R}^{\mathrm{Mx1}}$ denotes the measurement noise. The aim is to estimate $\mathbf{x}(k)$ from the torso potentials.

B. Regularization Approaches

1) Tikhonov Regularization: In this approach, the solution is found by minimizing the cost function:

$$\left\| \mathbf{A}\mathbf{x}(k) - \mathbf{y}(k) \right\|^2 + \lambda^2 \left\| \mathbf{R}\mathbf{x}(k) \right\|^2 \tag{2}$$

where $\mathbf{R}$ is the spatial regularization matrix and λ is the regularization parameter. The corresponding solution is then,

$$\hat{\mathbf{x}}_{\lambda}(k) = \left(\mathbf{A}^{\mathbf{T}}\mathbf{A} + \lambda^2 \mathbf{R}^{\mathbf{T}}\mathbf{R} \right)^{-1} \mathbf{A}^{\mathbf{T}}\mathbf{y}(k) \tag{3}$$

There are various methods proposed in literature to calculate an optimum λ value such as Composite Residual and Smoothing Operator (CRESO) [13], Generalized Cross-Validation (GCV) [14], and the most widely used L-curve approach [7], [8].

2) Multiple Constraints Approach: In this approach, the cost function in (2) is extended to include more than one spatial and/or temporal constraint. The new generalized cost function is [10]:

$$\left\|\tilde{\mathbf{A}}\tilde{\mathbf{x}}-\tilde{\mathbf{y}}\right\|^2+\sum_{i=1}^{k_s}\lambda_i^2\left\|\tilde{\mathbf{R}}_i\tilde{\mathbf{x}}\right\|^2+\sum_{j=1}^{k_t}\eta_j^2\left\|\tilde{\mathbf{\Gamma}}_j\tilde{\mathbf{x}}\right\|^2. \qquad (4)$$

Here, tilde over the vectors and matrices specifies the augmented problem, in which all equations in (1) for all time instants are combined into a single matrix equation as described in [10]. $\tilde{\mathbf{R}}_i$ and $\tilde{\Gamma}_j$ are the spatial and the temporal regularization matrices, and λ_i and η_j are the corresponding regularization parameters. The inverse problem is solved by the diagonalization method introduced in [10].

In [10], two spatial constraints, and one spatial - one temporal constraint were employed to solve the inverse problem. Corresponding regularization parameters were found from the L-surface approach. However, the L-surface approach could not be trivially extended to employ more than two constraints.

3) GA-based Regularization Parameter Estimation: GA is an optimization method that mimics the mutation and cross-over processes in biological organisms [15]. It starts with a population of randomly generated chromosomes (i.e. the regularization parameters), which is called the initial population. In each generation, chromosomes in the population are evaluated by first estimating the inverse solution for each chromosome, then calculating a fitness function for those solutions. Depending on their fitness function values, they are rated for their success as possible solutions. A new population of chromosomes is formed using random selection mechanisms crossover and mutation based on this evaluation. In our problem, real coded genetic algorithm [15] is used, which means that the chromosomes are modeled as real values, and real valued mutation and cross-over are used to obtain new generations. The initial chromosomes are selected randomly from a confidence region; the chromosome size depends on the number of constraints used in the solution. For example, for a solution with two constraints, the corresponding chromosome has two components, each corresponding to one of the two regularization parameters.

The fitness function is an important feature for the success of the GA. We modified (5) and used it as the fitness function in our GA method:

$$\frac{\left\|\tilde{\mathbf{A}}\tilde{\mathbf{x}}-\tilde{\mathbf{y}}\right\|^2}{\lambda^2\left\|\tilde{\mathbf{x}}\right\|^2}+\frac{\left\|\tilde{\mathbf{A}}\tilde{\mathbf{x}}-\tilde{\mathbf{y}}\right\|^2}{\eta^2\left\|\tilde{\mathbf{x}}\right\|^2}+\lambda^2\left\|\tilde{\mathbf{R}}\tilde{\mathbf{x}}\right\|^2+\eta^2\left\|\tilde{\Gamma}\tilde{\mathbf{x}}\right\|^2 \qquad (5)$$

This fitness function is defined for the case with one spatial and one temporal constraint, where λ and η are the corresponding regularization parameters. It can be easily modified to include any number of different constraints. We wanted to avoid solutions with very small norms, therefore we added divisions by $\|\tilde{\mathbf{x}}\|$ in the first two components of the fitness function; solutions with very small norms would produce a large value for $1/\|\tilde{\mathbf{x}}\|$, therefore the fitness function value would increase. On the other hand, solutions with large $\|\tilde{\mathbf{x}}\|$ values would also produce large fitness function values due to the last two norms. The chromosomes with large fitness function values would be considered as bad chromosomes. The algorithm would continue to search for the best chromosomes that generate the smallest fitness function values. When the fitness function reaches a pre-determined threshold value, the algorithm stops.

III. RESULTS

In this study, we used BSPMs simulated from epicardial potentials obtained from a ventricularly paced canine heart by R.S. MacLeod and his co-workers at the University of Utah Nora Eccles Harrison Cardiovascular Research and Training Institute (CVRTI) [16]. The epicardial measurements are taken from 490 points; sampling rate is 1000 Hz. The BSPMs are simulated by multiplying the epicardial potentials by a forward transfer matrix, and then adding Gaussian distributed noise. The forward matrices used in this study were computed using the Boundary Element Method (BEM). For the simulation of BSPMs, a realistic torso geometry including heart, lung and torso surfaces was used. The forward matrix used in the inverse solution is calculated from a homogeneous torso.

We simulated BSPMs at two different SNR values; 30 dB and 10 dB. We compared three different solutions: (i) Tikhonov regularization, (ii) Multiple constraints approach with two spatial constraints (energy and surface Laplacian), (iii) Multiple constraints approach with spatio-temporal constraints (the energy constraint, and the temporal smoothing constraint proposed in [10]). We used the correlation coefficient (CC) and the relative difference measurement star (RDMS) measures for quantitative comparison. In addition, the epicardial maps are compared visually using the map3d visualization software [17].

TABLE I
THE AVERAGE AND STANDARD DEVIATION VALUES OF CC AND RDMS FOR THE 30 DB SNR DATA.

30 dB SNR data	**CC** (avg ±std)	**RDMS** (avg ± std)
Tikhonov regularization	0.74± 0.17	0.65 ±0.17
Two spatial constraints	0.75 ±0.13	0.64 ±0.15
Spatio-temporal constraints	0.76 ± 0.11	0.62 ±0.13

TABLE II
THE AVERAGE AND STANDARD DEVIATION VALUES OF CC AND RDMS FOR THE 10 DB SNR DATA.

10 dB SNR data	**CC** (avg ±std)	**RDMS** (avg ±std)
Tikhonov regularization	0.58 ±0.27	0.82 ±0.24
Two spatial constraints	0.65 ±0.22	0.76 ±0.23
Spatio-temporal constraints	0.72±0.19	0.69±0.21

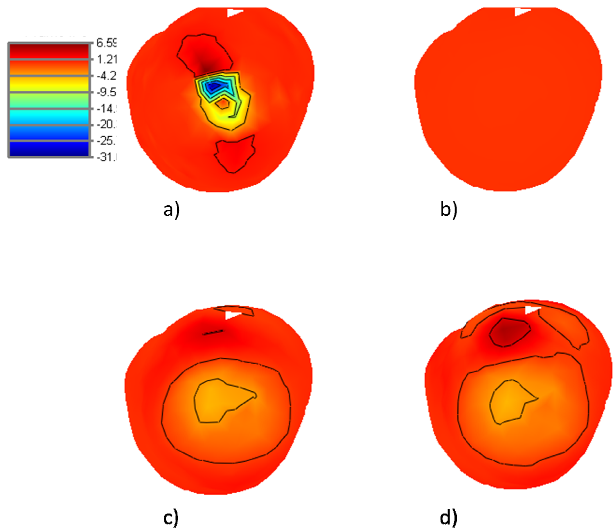

Fig. 1. Epicardial potential maps at the 6th time instant for the 30 dB SNR data. a) Real epicardial potentials, b) Tikhonov regularization, c) Two spatial constraints, d) Spatio-temporal constraints.

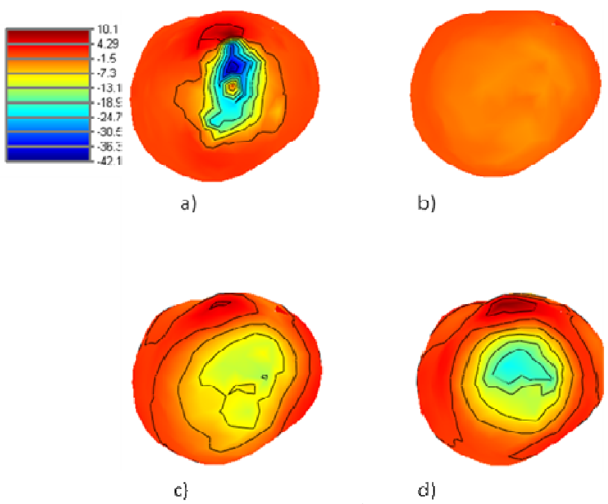

Fig. 2. Epicardial potential maps at the 17th time instant for the 10 dB SNR data. a) Real epicardial potentials, b) Tikhonov regularization, c) Two spatial constraints, d) Spatio-temporal constraints.

TABLE III

THE AVERAGE AND STANDARD DEVIATION VALUES OF CC AND RDMS FOR THE 10 DB SNR DATA, INCLUDING THE THREE-CONSTRAINT SOLUTION RESULTS.

10 dB SNR data	CC (avg ±std)	RDMS (avg ±std)
Tikhonov regularization	0.58 ± 0.27	0.82 ± 0.24
Multiple constraints (spatio-temporal constraints)	0.72 ± 0.19	0.69 ± 0.21
Multiple constraints (three constraints)	0.74 ± 0.18	0.65 ± 0.20

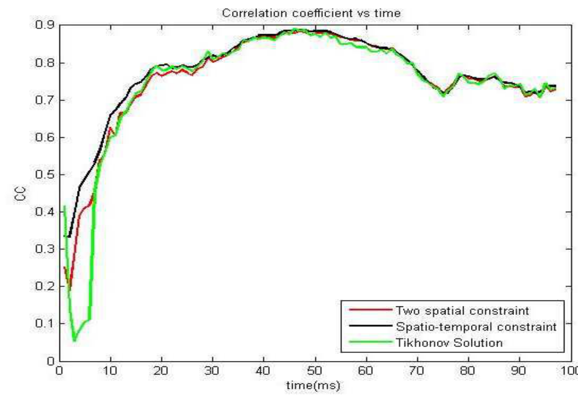

Fig. 3. Correlation coefficient values with respect to time for the 30 dB SNR data.

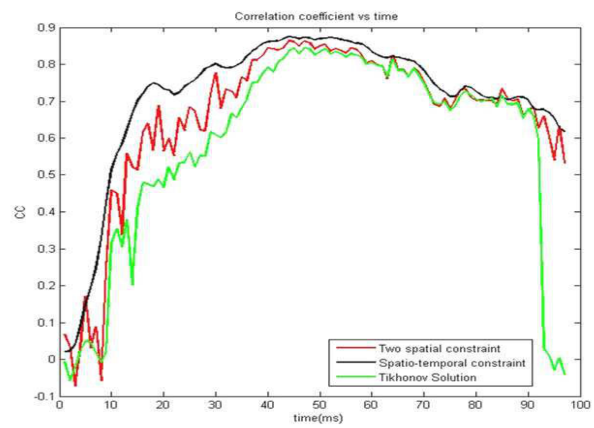

Fig. 4. Correlation coefficient values with respect to time for the 10 dB SNR data.

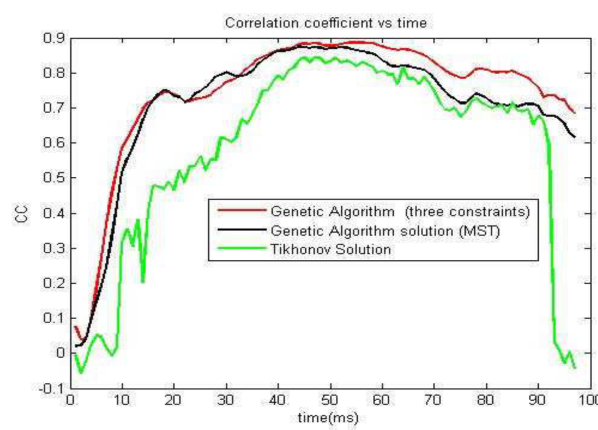

Fig. 5. Correlation coefficient values with respect to time for the 10 dB SNR data; in this figure, CC for the three-constraint solution is also included.

Fig. 1 and 2 show the real and the reconstructed epicardial maps, Fig. 3 and 4 show the CC values for each method plotted with respect to time, and Tables I and II list the averages and standard deviations of CC and RDMS values for each method over time, at 30 dB and 10 dB SNRs.

In order to show that the GA-based regularization parameter estimation method could easily be modified to include more than two constraints, we used all three constraints (two spatial, one temporal) simultaneously for the 10 dB SNR data. The results are presented in Fig. 5 and Table III.

Our observations are:

- Multiple constraints approach when the regularization parameters are found using the GA approach produces better results (*i.e.*, lower RDMS and higher CC values) compared to Tikhonov regularization. This observation is in agreement with the results presented in [10], demonstrating that GA can be used for regularization parameter estimation.
- Among the two multiple constraints approaches, employing spatio-temporal constraints yields better results in comparison to employing two spatial constraints.
- The improvements using the GA-based multiple constraints approach are more obvious when the SNR is lower.
- Multiple constraints approach can detect the earliest activated sites better than the Tikhonov approach. Fig. 1 and 2 are plotted at time instants at which the activated sites are visible in the two multiple constraints–solutions, but not in the Tikhonov solution, for 30 dB and 10 dB data, respectively. With the 30 dB SNR data, since the data is less noisy, these stimulation sites are reconstructed at an earlier time.
- Using three constraints (two spatial and one temporal) produces slightly better results compared to using only two constraints, especially after the 50th time instant.

IV. Conclusions

It was already demonstrated in [10] that using multiple constraints improves inverse solutions compared to Tikhonov regularization with a single constraint. However, the drawback was the complexity in estimating appropriate regularization parameters. Here we proposed a GA based algorithm to estimate multiple regularization parameters. We demonstrated the applicability of this approach using two and three constraints; however the nature of the algorithm allows for its extension into problems with higher number of constraints. The three constraint solution was included to show the applicability of the algorithm with higher number of constraints; however, we have not tried to find the best number and combination of constraints so that one can achieve significant improvements over the Tikhonov regularization results. Future work will include design and use of more suitable constraints to improve the solutions.

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