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Inverse Modeling for Heat Conduction Problem in Human Abdominal Phantom

Ming Huang, *Student Member, IEEE* and Wenxi Chen, *Member, IEEE*

Abstract **– Noninvasive methods for deep body temperature measurement are based on the principle of heat equilibrium between the thermal sensor and the target location theoretically. However, the measurement position is not able to be definitely determined. In this study, a 2-dimensional mathematical model was built based upon some assumptions for the physiological condition of the human abdomen phantom. We evaluated the feasibility in estimating the internal organs temperature distribution from the readings of the temperature sensors arranged on the skin surface. It is a typical inverse heat conduction problem (IHCP), and is usually mathematically ill-posed. In this study, by integrating some physical and physiological a-priori information, we invoked the quasi-linear (QL) method to reconstruct the internal temperature distribution. The solutions of this method were improved by increasing the accuracy of the sensors and adjusting their arrangement on the outer surface, and eventually reached the state of converging at the best state accurately. This study suggests that QL method is able to reconstruct the internal temperature distribution in this phantom and might be worthy of a further study in an anatomical based model.**

I. Introduction

BODY temperature is one of the most vital indices for human physiological condition. Deep body temperature is a reliable indicator of body functions, and its circadian rhythm contains abundant information, which reveals individual physiological states, and is important in patient monitoring and chronobiological studies [1].

Many different methods have been developed for measuring temperatures deep inside the body. Since the 1970s, methods that sought to measure the deep body temperature noninvasively have been proposed, such as zero-heat-flow method introduced by Fox *et al.* in 1971[2], [3], and dual-heat-flux method introduced by Kitamura *et al.* in 2009 [4]. The main problem of these existing methods is that, the deep body temperature is estimated without specific information of measurement location inside human body. Namely, the point where the temperature is measured remains unknown. We want to find out a new method, which can provide us more information about the temperature distribution inside human body.

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M. Huang is with the University of Aizu, Aizu-wakamatsu, Fukushima 965-8580, Japan (e-mail: d8112102@u-aizu.ac.jp).

*W. Chen is with the University of Aizu, Aizu-wakamatsu, Fukushima 965-8580, Japan (phone: 81-242-37-2606; fax: 37-2728; e-mail: wenxi@u-aizu.ac.jp).

There are two main characteristics in this kind of heat transfer: metabolic heat generation and thermal energy exchange between flowing blood and the surrounding tissues. This kind of heat transfer can be described by the bioheat equation with some anatomical and physiological restrictions [5]. Firstly, the geometric structure of the organs' spatial distributions is easily to be acquired by the means of such as magnetic resonance imaging (MRI) or X-ray computed tomography (CT). Secondly, the number of the organs inside human's abdomen is limited. Thirdly, according to physiology, the fluctuations of temperature values of the organs are torpid and are kept in a low level. The temperature level is maintained within a narrow range around 37°C [6]. These intrinsic properties provide us with some valuable information to simplify the inverse modeling.

This study concerned with both aspects of modeling, where the forward modeling refers to determining the heat state of a body or system given the geometry, boundaries conditions, and thermal properties. In terms of the inverse problem, the aim is to reconstruct the internal sources of heat from a set of measured data. This kind of problem is typically ill-posed [7].

In this paper, we want to incorporate the covariance of the solution, assumed to be known a-priori information, as long as the noise characteristics information of the temperature sensors and then utilize the quasi-linear (QL) method to assess the validity in estimating the temperature distribution in the internal boundaries from outer boundary temperature measurements.

II. Method

A. Forward Problem

The behavior of bioheat can be described by the bioheat equations below [8]:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + q_p + q_m \qquad T \in \Omega \qquad (1)$$

$$t = \alpha \qquad T \in \Gamma_D \qquad (2)$$

$$\vec{n} \cdot k \nabla T_s = h(T_{amb} - T_s) \qquad T \in \Gamma_N \qquad (3)$$

where ρ, c_p, and k are the density of the local tissues (kg/m^3), the specific heat of local tissues (J/kg·°C), and the thermal conductivity (W/m·°C), respectively. The terms q_p and q_m are the heat transfer rates due to the blood perfusion and metabolic procedure, respectively. T, t, T_{amb} and T_s are the temperature (°C) in torso domain, the internal boundaries temperature, the ambient temperature and the temperature on the outer boundary, respectively. Ω denotes torso domain, and Γ_D, Γ_N represent organs' surfaces (Dirichlet boundary) and body surface (Neumann boundary) respectively, h is the heat transfer coefficient with the ambient environment and $\vec{n}$ denotes the normal to the boundary.

To solve the inverse problem, we made the following assumptions: the heat flux flows out of the inner organs onto the skin. The heat effects due to blood perfusion and metabolism in the peripheral tissues and skin layer are neglected. The model stays steady under the state of thermal equilibrium. Every organ inside the abdomen is isothermal, so we could focus on the boundary temperature of each organ only. The bioheat equations are simplified as follow:

$$\nabla \cdot (k\nabla T) = 0 \qquad T \in \Omega \tag{4}$$

$$t = \alpha \qquad T \in \Gamma_D \tag{5}$$

$$\vec{n} \cdot k\nabla T_s = h(T_{amb} - T_s) \qquad T \in \Gamma_N \tag{6}$$

The 2-dimensional mathematical model we built is shown in fig. 1. The small ellipses were employed to simulate the main organs such as liver, stomach and so on. t_1-t_6 represent the thermal independent internal boundaries, while the points P1-P12 denote the temperature sensors on the outer boundary.

In forward problem, we could determine the heat state of the model uniquely with the corresponding boundaries condition and thermal properties of the model. We utilized the finite element method (FEM) for the discretization and forward model computation.

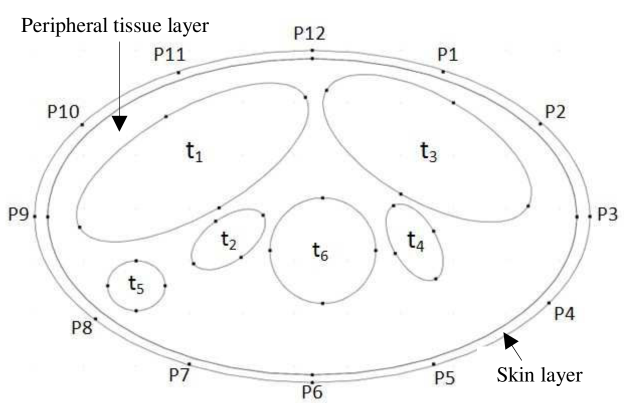

Fig. 1. Geometry of the two dimensional mathematical model of the abdominal phantom. The peripheral tissue layer and skin layer are considered to be homogeneous respectively. An internal boundary from t_1 to t_6 corresponds to stomach, left kidney, liver, right kidney, spleen and spine respectively.

B. Inverse Problem

In this study, we assumed that the temperature measurements were carried out on the outer boundary with temperature sensors, the unknown parameters were the temperature values of the internal organ boundaries. The inverse problem was described as follow: from the temperature readings acquired by the sensors arranged on the outer boundary, we tried to estimate the temperature value of each internal boundary.

For an ill-posed problem, a subtle variation of the measurement could be magnified on the solution [9]. It is the reason why we need the regularization for the inverse problem. The "regularization" is a class of techniques, in which by integrating some a-priori information of the model to constrain the original ill-posed problem to yield somewhat better posed problems.

By discretizing the model with FEM, we can describe the forward modeling with the transfer function below:

$$\boldsymbol{T_s} = f(\boldsymbol{t}) \tag{7}$$

$f(\boldsymbol{t})$ represents the process of forward model computation and outer boundary temperature values' extraction. After the model's discretization, $\boldsymbol{T_s}$ turns out to be an m-dimensional (m is the number of the sensors) vector consisted of temperature readings from sensors, while $\boldsymbol{t}$ is an n-dimensional vector consisted of internal boundaries temperature values.

The QL method is a method of inverse modeling to identify an unknown parameter set [10]. By combining a-priori information of the model, QL method treats the inverse problem from a statistical point of view, where the transfer function is successively linearized as below:

$$f(\tilde{\boldsymbol{t}}_{i+1}) \approx f(\tilde{\boldsymbol{t}}_i) + \mathbf{H}_i(\tilde{\boldsymbol{t}}_{i+1} - \tilde{\boldsymbol{t}}_i) \tag{8}$$

The vector $\tilde{\boldsymbol{t}}_i$ here represents the present estimation of the vector of internal boundaries temperature values, while the $\tilde{\boldsymbol{t}}_{i+1}$ represents the next estimation of the vector. $\mathbf{H_i}$ is an m-by-n sensitivity matrix that linearizes the transfer function about $\tilde{\boldsymbol{t}}_i$, $H_{p,q} = \partial f(\tilde{\boldsymbol{t}})_p / \partial \tilde{\boldsymbol{t}}_q (\tilde{\boldsymbol{t}} = \tilde{\boldsymbol{t}}_i)$. $H_{p,q}$ is the element in the p^{th} row and the q^{th} column of $\mathbf{H_i}$. The objective function for quasi-linear method is:

$$s = (\boldsymbol{T_s} - f(\tilde{\boldsymbol{t}}_i))'\mathbf{R}^{-1}(\boldsymbol{T_s} - f(\tilde{\boldsymbol{t}}_i)) + (\tilde{\boldsymbol{t}}_i - \mathbf{X}\beta)'\mathbf{Q}^{-1}(\tilde{\boldsymbol{t}}_i - \mathbf{X}\beta) \tag{9}$$

Weight matrices **R** and **Q** are the a-priori information about the model. **R** is the m-by-m covariance matrix of measurement errors. **Q** is the n-by-n covariance matrix of the solution. **X** is an n-by-1 vector of one; β is a known mean of the unknown parameter set. By invoking a local linearity to approximate the transfer function, QL method finds the next estimation iteratively:

$$\tilde{\boldsymbol{t}}_{i+1} = \mathbf{X}\beta + \mathbf{QH}_i'\boldsymbol{\xi} \tag{10}$$

ξ and β are found by solving the system of equations

$$\begin{bmatrix} \mathbf{H}_i\mathbf{QH}_i'+\mathbf{R} & \mathbf{H}_i\mathbf{X} \\ (\mathbf{H}_i\mathbf{X})' & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi} \\ \beta \end{bmatrix} = \begin{bmatrix} \boldsymbol{T_s} - f(\tilde{\boldsymbol{t}}_i) + \mathbf{H}_i\tilde{\boldsymbol{t}}_i \\ 0 \end{bmatrix} \tag{11}$$

Fig. 2 shows the flowchart of the QL method. In the forward modeling, what we need is the real temperature values on internal boundaries, the weighted matrices **R** and **Q**. Inside the QL method, the initial guess of $\tilde{\boldsymbol{t}}$ is necessary. Note that, in the initial process (i=0), only the first term of (10) is calculated because that in the initial process β is not available. Subsequently, program runs into the iterative procedure to search for the solution. The value of ε in the criteria is 0.001 here.

C. Implementation of Model Simulation:

The sensitivity matrix H was employed to quantify the effects of the unknown (boundary temperature values) variations on the results (the temperature values of the measuring sites) [11]. This matrix was computed with the adjoint sensitivity analysis procedure (ASAP), which, in this

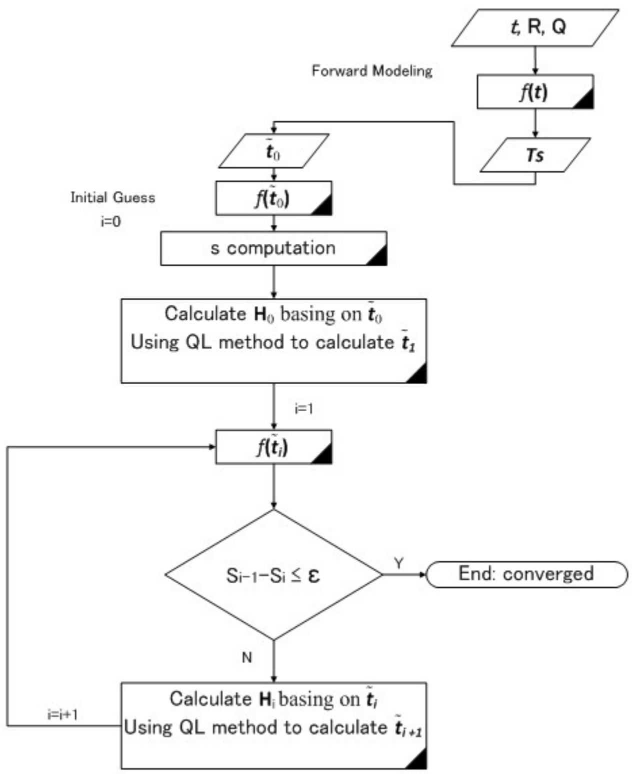

Fig. 2. The flowchart of the QL method. This algorithm is implemented with COMSOL-MATLAB linkage.

study, could be derived by the embedded adjoint method algorithm in COMSOL [12]. The simulation was programmed and performed with COMSOL-MATLAB linkage.

This simulation focused on the effects of the noise characteristics and arrangement (quantity and location) of the temperature sensors on the inverse solution. The noise characteristics could be introduced by diagonal matrix R, where the diagonal elements are identical and considered as square of systematic error of the sensors, three different values 1, 0.25, and 0.01 ($(^0C)^2$) were used. Meanwhile, three types of sensors arrangement were employed. The first type used four sensors (P3, P6, P9 and P12). The second type used eight sensors (excluding P1, P4, P7, P10) and the third used all 12 sensors. Q is generated basing on the physiological assumption that each organ that we are concerning is thermal independent and their temperature fluctuation is normally distributed with 1.0^{0}C standard deviation (SD) and 37.0^{0}C mean value.

The simulations were organized as follow: given **R** and the sensors' arrangement (termed as experiment condition), for each simulation, we preset the vector ***t***. As for the generation of ***t***, the temperature value of each internal boundary was changed respectively with an interval of 0.5^{0}C from 36.0^{0}C to 38.0^{0}C while the temperature values of others were kept at 37.0^{0}C. As a result, for each given experiment condition, thirty times of simulation were carried out.

III. Results of Numerical Simulations

Fig. 3 shows the results of the simulations, which is the summary of results from 270 times of simulation. We marked a specific experiment situation with the notation: (noise characteristics, sensors arrangement) in fig. 3. The six bars of each cluster denote the average of the absolute errors between the inverse solutions and the real values of the corresponding internal boundary over thirty times with specific experiment condition. The corresponding standard deviations of the errors are presents with small bars on top.

The results were divided into 3 groups and data of the same subfigure were acquired under the situation that sensors' noise characteristics are identical. For each internal boundary, generally speaking, the solutions were improved with the increasing of the sensor's accuracy. On the other hand, with identical sensor specification, deploying more sensors on the outer boundary will help to get a better solution. These improvements are especially obvious in the results of boundaries t_1 and t_3.

For boundaries t_4 and t_6, it is not so helpful by merely increasing the sensor accuracy with four-sensor arrangement. We got an acceptable result when we increased the sensor quantity from four to eight and improve the sensor accuracy at the same time. Extreme situations were found in t_2, we

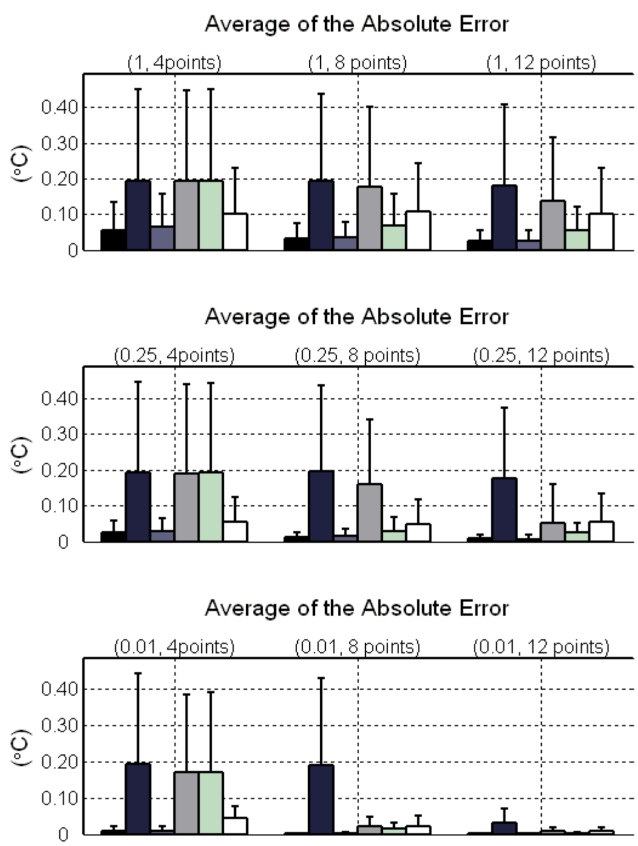

Fig. 3. A summary of the simulation results. The six bars from left to right in each cluster are the average values of absolute error from t_1 and t_6 over thirty times of simulations. Small bars on top presents the SD of the absolute error of the corresponding internal boundary.

could solely get a fine solution with twelve sensors and the best sensor accuracy.

IV. Discussion

The aforementioned contrast of fitting results may stem from the geometric construction of the model in which the parts with boundaries t_1 and t_3 occupy the largest areas and are close to the outer boundary. Hence, their temperature variations can be reflected on temperature measurements remarkably. However, as for the parts with boundaries t_4 and t_6, they are not so large as the previous two parts and, on the other hand, t_4 locates deep inside the phantom and is sandwiched by t_3 and t_6. For these reasons, we cannot trace their variations only with four sensors. It is even harder to get a fine solution for t_2, for the reason that t_2 is blocked by t_5 from the outer sensors, so that its variation has little effect even on the nearest sensor P8. Boundaries t_2, t_4 are employed to simulate the small organs locating in the central area of human abdomen, such as the kidneys. The results show that, it is possible to trace their temperature variation closely. Nevertheless, sensors with high precision and a reasonable arrangement are necessary. Among the results of the simulations, the maximum error of t_2 can be reduced to about 0.15 ^{0}C when twelve sensors with 0.1 ^{0}C systematic error were used. In terms of a noninvasive method, it is a satisfactory error level for the common resolution for temperature sensor is 0.1 ^{0}C. To improve the solutions, especially in an anatomy-based model where the organs possess anisotropic physical properties and irregular boundaries, it is better to place the sensors according to the actual heterogeneous characteristic of the organs rather than to place them evenly.

The current QL method can solve for the temperature values of internal boundaries with specific locations inside the model. However, in reality the actual cross-sectional geometry for individual is unique so that we can only give an approximate description of the temperature distribution. However, for parts such as t_1 and t_3 the deviation with the actual value is not as visible as parts such as t_2 and t_4.

The major computational burden for QL method is the generation of $\mathbf{H}$. Theoretically speaking, to generate $\mathbf{H}$, n+1 times of forward model runs is necessary. Compared with Monte-Carlo-based algorithms, which needs a large number of samples to find out the solutions, QL method seems more suitable for this specific task. In the process of simulations, we have tried to increase the upper limit of temperature variation to 45 ^{0}C, QL method was able to search for the solutions effectively and the initial guesses have little effect on the solutions.

Human vascular system plays an important role in systematic thermoregulation. It helps the vital organs to keep their temperature steady. In our model, by generally considering each organ as a heat source combining the effect of blood perfusion and metabolism, each internal boundary is considered isothermal. Meanwhile, the perfusion of the blood on the skin layer is neglected. This disposal gives an overall description of the organ's temperature and is coincide with the purpose of this study, which is to validate the feasibility of using the QL method in deep body temperature estimation theoretically.

For real human abdomen, the organs are not always isothermal and have irregular boundaries. To get a more precise model, we need to redivide the geometry into a series of finer subareas. For example, we can redivide the part t_1 into four or eight subareas. Whereas physiology based a priori information is still unavailable and a further partition of the model will naturally increase the computational costs in the computation of sensitivity matrix. In the further study, we would like to test the computation capability of QL method in an anatomy-based model with finer spatial partition.

V. Conclusions

In this study, the QL method is proposed to study inverse problem in bioheat heat conduction. Attributing the sensor's noise level and their arrangement as the major factors affecting the inverse solution, we carried out numerical simulations in a mathematical model. Basing on the statistical meaning of the a-priori information reflected by weighted matrices $\mathbf{Q}$ and $\mathbf{R}$, we got a series of results that demonstrate the dependent relationships among estimation performance, measurement accuracy and arrangement scheme of thermal sensors, QL method could be used to search for the accurate solution in the inverse problem.

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