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# A Chance-Constrained Approach to Preoperative Planning of Robotics-Assisted Interventions

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*Abstract*— **In this paper, a planning strategy for robotics-assisted interventions is formulated in terms of uncertainty at the task level. The proposed formulation attempts to increase the chance of success by maximizing robustness with respect to the task uncertainty. It is assumed that the instrument tip pose has a Gaussian distribution in the vicinity of the desired task frame, and the planner is formulated as a chance-constrained programming problem in terms of the chance of collisions and joint limit violations based on the inverse kinematics of the arms. The proposed objective function addresses the robustness as well as the performance of the robotic arms. As an illustrative example, the planning strategy is implemented for LIMA harvesting in minimally invasive coronary artery bypass with the *da Vinci* robot.**

## I. INTRODUCTION

In the last few years, emerging technologies have increased the feasibility and popularity of robotics-assisted surgical procedures. Intraoperative complexities suggest that the chance of success in robotics-assisted interventions (RAI) can be improved by preoperative planning. Issues such as intraoperative collisions, mechanical joint limits and singularities can reduce the success rate of RAI. In order to adapt to anatomical differences, any planning strategy has to take into account patient-specific preoperative data. Normally, such data can be acquired from imaging modalities such as Computed Tomography (CT) or Magnetic Resonance Imaging (MRI). To complicate matters, the intraoperative geometry of the patient can significantly deviate from what is expected based on preoperative data. Tissue deformations due to tool/tissue interactions, physiological motions, and altered geometry due to chest insufflation and lung collapse can introduce inaccuracies in the model acquired from preoperative data. Additionally, one should consider the inherent uncertainty in surgical tasks. In other words, even assuming that there is no uncertainty in the desired task frame (which is normally attached to an anatomical feature), the task implementation is not accurately known.

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The formulation proposed in this paper attempts to address uncertainty at the task level. As a matter of fact, it is possible to treat the contributions of the above sources of uncertainty as a single entity represented by the task uncertainty. The proposed framework attempts to find an optimum plan that maximizes the robot performance with the maximum tolerance with respect to the task uncertainty. The tolerance is defined in terms of collision and joint limit avoidance using the robot inverse kinematics. The planner is formulated as a chance-constrained programming problem and an efficient sampling-based technique for solving the problem is proposed.

The paper is organized as follows: In Section II a brief literature review is provided. The problem formulation is presented in Section III followed by the results of a case study in Section IV, and concluding remarks are given in Section V.

## II. RELATED WORK

A number of studies on preoperative planning of RAI have been reported in the literature. The design of an interactive 3D virtual environment for assisting the surgeon in assessing port locations on the patient's chest wall was reported in [1]. A planning strategy, formulated as an optimization problem for robotics-assisted minimally invasive cardiac surgery with the *da Vinci* robot and based on preoperative CT images was reported in [2]. This work addressed robot requirements such as visibility, dexterity and collision avoidance. Other planning strategies based on the patient's preoperative data for robotics-assisted surgery were reported in [3]–[5]. However, uncertainty was not explicitly addressed in any of the above studies. In [6], a deterministic approach for preoperative planning under geometric uncertainty was proposed. The problem was formulated as a semi-infinite programming problem that only considered uncertainty in the position of the wrist and was relatively computationally demanding. An integrated intraoperative planning and control strategy for the DLR MIRO system was reported in [7] that can handle intraoperative geometric uncertainty. While interesting, it must be noted that the robots that are currently used for RAI do not provide such functionality for safety reasons.

## III. PROBLEM FORMULATION

Every surgical procedure can be expressed in terms of several surgical tasks that can be represented by a number of task frames inside the surgical cavity. Normally, these task frames can be fixed to certain anatomical features, and therefore surgical tasks can be uniquely described with respect to the patient's anatomy. While the correlation between the task and the position of a specific surgical feature is easily understood, the link between the instrument orientation and the target may not be as obvious. However, a task can only be uniquely expressed with a position and an orientation.

In practice, surgical gestures (subtasks) are subject to spatio-temporal uncertainties. For different surgeons with different levels of skill and experience, the uncertainty can become more significant. The main objective of this planner is to find a plan that can maximize the tolerable motion uncertainty. In other words, this paper attempts to determine what is the maximum uncertainty for which the chance of collisions and joint limit violation remains sufficiently small.

RAI usually require three arms, including two arms for carrying the right and left instruments, as well as one arm for carrying the endoscope. Let $\mathbf{x}_\mathrm{r} \in \mathbb{R}^6$ and $\mathbf{x}_\mathrm{l} \in \mathbb{R}^6$ be the 6D pose of the right and left instrument tips, respectively. In this paper it is assumed that the desired task frames for the right and left instruments are given as $\hat{\mathbf{x}}_\mathrm{r} \in \mathbb{R}^6$ and $\hat{\mathbf{x}}_\mathrm{l} \in \mathbb{R}^6$, and the pose of the instruments in the vicinity of the task frames is represented by Gaussian distributions $\mathbf{x}_\mathrm{r} \sim \mathcal{N}(\hat{\mathbf{x}}_\mathrm{r}, \mathbf{Q}_{\mathrm{x,r}})$ and $\mathbf{x}_\mathrm{l} \sim \mathcal{N}(\hat{\mathbf{x}}_\mathrm{l}, \mathbf{Q}_{\mathrm{x,l}})$.

In order to increase the chance of success for completion of a given surgical task, the robustness of the plan with respect to the uncertainty in the task has to be maximized. In this paper, the proposed planner attempts to find an optimal plan that has the minimum susceptibility with respect to the lack of information about the task. With a gesture modeled by a Gaussian distribution, this is equivalent to maximizing a norm of the covariance of the task. Equivalently, this can be accomplished by minimizing a norm of the inverse of the covariance matrix, known as the information matrix.

### A. Objective Function

While it is desired that the robustness of the plan is maximized, the task performance must also be taken into account. Therefore, the objective of this planner has to be articulated in terms of two criteria: robustness and performance. As mentioned in the previous section, robustness with respect to task uncertainty can be maximized by minimizing a norm of the information matrix. In practice, for a given task and in the vicinity of a task frame, the endoscope remains still; therefore, henceforth the pose of the endoscope is treated as a deterministic variable.

Let a procedure be represented with $N$ discrete task frames represented by $\mathbf{x}_\mathrm{r}(k) \sim \mathcal{N}(\hat{\mathbf{x}}_\mathrm{r}(k), \mathbf{Q}_{\mathrm{x,r}}(k))$ and $\mathbf{x}_\mathrm{l}(k) \sim \mathcal{N}(\hat{\mathbf{x}}_\mathrm{l}(k), \mathbf{Q}_{\mathrm{x,l}}(k))$ for $k = 1,..,N$. The robustness criterion can be expressed as:

$$\min_{\Pi} \Sigma_{k=1}^{N} \left( \log(|\mathbf{Q}_{\mathrm{x,r}}(k)|^{-1}) + \log(|\mathbf{Q}_{\mathrm{x,l}}(k)|^{-1}) \right), \quad (1)$$

where $|\cdot|$ is the matrix determinant operator, $\log(|\mathbf{Q}_\mathrm{x}|)$ is the Shannon entropy and $\Pi$ is the vector of planning parameters.

In order to address the task performance, we propose a new measure. This measure has physical interpretations and naturally fits into the multi-criteria objective function. Given the task frames with Gaussian distributions, assume that the propagated distribution of the joint vectors can be approximated by Gaussian distributions $\mathbf{q}_\mathrm{r}(k) \sim \mathcal{N}(\hat{\mathbf{q}}_\mathrm{r}(k), \mathbf{Q}_{\mathrm{q,r}}(k))$ and $\mathbf{q}_\mathrm{l}(k) \sim \mathcal{N}(\hat{\mathbf{q}}_\mathrm{l}(k), \mathbf{Q}_{\mathrm{q,l}}(k))$ where $\mathbf{q}_\mathrm{r} \in \mathbb{R}^6$ and $\mathbf{q}_\mathrm{l} \in \mathbb{R}^6$ are the right and left instrument arm joint vectors. Note that these distributions depend upon the configuration of the arms as well as the distribution of the task frames. In general, it is desired that for a given displacement in the pose of the end effector, the joint displacement is minimized. This can be expressed in terms of the ratio of the joint covariance norm to the pose covariance norm as:

$$\min_{\Pi} \frac{1}{N} \Sigma_{k=1}^{N} \left( \log\left( \frac{|\mathbf{Q}_{\mathrm{q,r}}(k)|}{|\mathbf{Q}_{\mathrm{x,r}}(k)|} \right) + \log\left( \frac{|\mathbf{Q}_{\mathrm{q,l}}(k)|}{|\mathbf{Q}_{\mathrm{x,l}}(k)|} \right) \right), \quad (2)$$

and the resulting multi-criteria objective function can be rendered as:

$$\min_{\Pi} \begin{pmatrix} \frac{1}{N} \Sigma_{k=1}^{N} \left( \log(|\mathbf{Q}_{\mathrm{x,r}}(k)|^{-1}) + \log(|\mathbf{Q}_{\mathrm{x,l}}(k)|^{-1}) \right) \\ \frac{1}{N} \Sigma_{k=1}^{N} \left( \log\left( \frac{|\mathbf{Q}_{\mathrm{q,r}}(k)|}{|\mathbf{Q}_{\mathrm{x,r}}(k)|} \right) + \log\left( \frac{|\mathbf{Q}_{\mathrm{q,l}}(k)|}{|\mathbf{Q}_{\mathrm{x,l}}(k)|} \right) \right) \end{pmatrix}. \quad (3)$$

### B. Constraints

Due to the stochasticity of the task, it is reasonable to articulate the constraints probabilistically. In order to ensure that the chance of collisions and joint limit violations will not exceed a small threshold, the following constraints have to be considered:

$$P(\tilde{\mathbf{x}}(k) \in \mathbb{C}_{\mathrm{free}} | \Pi) > 1 - \varepsilon \quad \text{for} \quad k = 1, ..., N, \quad (4)$$

$$P(\mathbf{q}_\mathrm{r}(k) \in \mathbb{Q}_{\mathrm{valid,r}} | \Pi) > 1 - \varepsilon \quad \text{for} \quad k = 1, ..., N, \quad (5)$$

$$P(\mathbf{q}_\mathrm{l}(k) \in \mathbb{Q}_{\mathrm{valid,l}} | \Pi) > 1 - \varepsilon \quad \text{for} \quad k = 1, ..., N, \quad (6)$$

$$\mathbf{q}_\mathrm{e}(k) \in \mathbb{Q}_{\mathrm{valid,e}} \quad \text{for} \quad k = 1, ..., N, \quad (7)$$

where $\mathbf{q}_\mathrm{e} \in \mathbb{R}^4$ is the endoscope arm joint vector, $0 < \varepsilon \ll 1$, $\tilde{\mathbf{x}}(k) = \begin{pmatrix} \mathbf{x}_\mathrm{r}(k) \\ \mathbf{x}_\mathrm{l}(k) \\ \mathbf{x}_\mathrm{e}(k) \end{pmatrix}$ is the augmented vector of the poses, $\mathbb{C}_{\mathrm{free}}$ is the collision-free subset of the Cartesian space that can be defined in terms of the minimum distances between the arm links, $\mathbf{d}$, i.e., $\mathbb{C}_{\mathrm{free}} = \{\tilde{\mathbf{x}} | \mathbf{d} > 0\}$, $\mathbb{Q}_{\mathrm{valid}} = \{\mathbf{q} | \underline{\mathbf{q}} < \mathbf{q} < \bar{\mathbf{q}}\}$ is the valid subset of the joint space determined by the joint limits, and $P(\cdot)$ is the probability operator.

In fact, it is desired to express the above constraints as functions of the pose vectors. The collision-free subset of the Cartesian space can be described as $\mathbb{C}_{\mathrm{free}} = \{\tilde{\mathbf{x}} | \mathbf{g}(\tilde{\mathbf{x}}) > 0\}$ where $\mathbf{d} = \mathbf{g}(\tilde{\mathbf{x}})$, $\mathbf{g} : \mathbb{R}^{16} \to \mathbb{R}^{56}$ (see [6] for the geometric modeling of the *da Vinci* arms).

Additionally, the valid subset of the joint space can be described in terms of the pose of the end effector, i.e., $\mathbb{Q}_{\mathrm{valid}} = \{\mathbf{q} | \underline{\mathbf{q}} < \mathbf{h}(\mathbf{x}) < \bar{\mathbf{q}}\}$ where $\mathbf{q} = \mathbf{h}(\mathbf{x})$ with $\mathbf{h}_\mathrm{r} : \mathbb{R}^6 \to \mathbb{R}^6$, $\mathbf{h}_\mathrm{l} : \mathbb{R}^6 \to \mathbb{R}^6$ and $\mathbf{h}_\mathrm{e} : \mathbb{R}^4 \to \mathbb{R}^4$ represent the inverse kinematic mappings for the right, left and the endoscope arms, respectively.

### C. Chance-Constrained Programming Problem

A natural way of solving the above constrained optimization problem is the use of a probabilistic sampling method, which is a computationally expensive approach. Alternatively, the chance constraints can be approximated and replaced by a set of deterministic constraints. The latter method requires that the distributions (or at least the first two moments) of the constraints are known. Nevertheless, the constraints are generally nonlinear functions of the tip pose and in general their distributions are unknown and usually non-Gaussian. A trivial workaround is a local linear approximation of the constraint functions and estimations of the mean and variance of the constraints using the constraint function Jacobian. However, this method usually fails to reveal the true statistical properties of the constraint when the function is highly nonlinear [8]. Furthermore, an analytic expression of the Jacobian of the constraint function may not be known, and the numerical calculation of the Jacobian of the constraints may be computationally demanding.

As an alternative approach, the *unscented* transformation, proposed by [8] is employed for estimating the statistics of the constraints by fitting a Gaussian distribution on the transformed samples. Consider a nonlinear mapping of the stochastic variable $\mathbf{x}$ with a Gaussian distribution given as $\mathbf{y} = \mathbf{f}(\mathbf{x})$ where $\mathbf{f} : \mathbb{R}^m \to \mathbb{R}^n$. Assume that a set of *Sigma* points are selected as $\mathbf{X} = \{\hat{\mathbf{x}}, \hat{\mathbf{x}} \pm [\sqrt{\alpha \mathbf{Q}_\mathrm{x}}]_i\}$ for $i = 1, \ldots, m$ where $\alpha$ is a scalar, and $[\cdot]_i$ represents the $i^{\mathrm{th}}$ column of the argument. The mean and covariance of $\mathbf{y}$ can be estimated as:

$$\hat{\mathbf{y}} = \Sigma_{i=0}^{2m} W_i \mathbf{Y}_i, \tag{8}$$

$$\mathbf{Q}_\mathrm{y} = \Sigma_{i=0}^{2m} W_i (\mathbf{Y}_i - \hat{\mathbf{y}})(\mathbf{Y}_i - \hat{\mathbf{y}})', \tag{9}$$

where $\mathbf{Y}_i = \mathbf{f}(\mathbf{X}_i)$ and $W_i$ are scalar weights that are selected such that the statistics of the Sigma points and $\mathbf{x}$ are identical. The interpretation of this transformation is that instead of propagating the mean and covariance of $\mathbf{x}$, the Sigma points are propagated and the resulting mean and covariance are estimated by fitting a Gaussian distribution to the propagated points.

Once the means and the covariance matrices of the constraints are estimated, the chance constraints can be reduced to deterministic constraints. For $\varepsilon = 0.02$, the constraints (4)–(7) can be given as:

$$2\Big(\mathrm{diag}(\mathbf{Q}_\mathrm{d}(k))\Big)^{\frac{1}{2}} < \hat{\mathbf{d}}(k), \tag{10}$$

$$\left|\hat{\mathbf{q}}_\mathrm{r}(k) - \frac{\bar{\mathbf{q}}_\mathrm{r} + \underline{\mathbf{q}}_\mathrm{r}}{2}\right| + 2\Big(\mathrm{diag}(\mathbf{Q}_{\mathrm{q,r}}(k))\Big)^{\frac{1}{2}} < \frac{\bar{\mathbf{q}}_\mathrm{r} - \underline{\mathbf{q}}_\mathrm{r}}{2}, \tag{11}$$

$$\left|\hat{\mathbf{q}}_\mathrm{l}(k) - \frac{\bar{\mathbf{q}}_\mathrm{l} + \underline{\mathbf{q}}_\mathrm{l}}{2}\right| + 2\Big(\mathrm{diag}(\mathbf{Q}_{\mathrm{q,l}}(k))\Big)^{\frac{1}{2}} < \frac{\bar{\mathbf{q}}_\mathrm{l} - \underline{\mathbf{q}}_\mathrm{l}}{2}, \tag{12}$$

$$\left|\mathbf{q}_\mathrm{e}(k) - \frac{\bar{\mathbf{q}}_\mathrm{e} + \underline{\mathbf{q}}_\mathrm{e}}{2}\right| < \frac{\bar{\mathbf{q}}_\mathrm{e} - \underline{\mathbf{q}}_\mathrm{e}}{2}, \tag{13}$$

for $k = 1, \ldots, N$, where $\mathbf{d} \sim \mathcal{N}(\hat{\mathbf{d}}, \mathbf{Q}_\mathrm{d}(k))$ along with the joint distributions are estimated by (8) and (9).

Note that in order to ensure task feasibility, additional constraints have to be taken into account. For instance, in order to ensure that the targets are reachable, the approach angles of the instruments must make an acute angle with the normal vector at the site. Finally, the resulting problem is rendered as an ordinary nonlinear programming problem with the objective function given in (3) and subject to the constraints (10)–(13) along with the additional deterministic constraints.

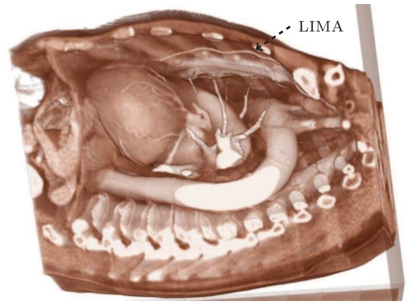


Fig. 1. The left internal mammary artery is located on the chest wall close to the sternum, extended from the first rib to the sixth rib.

In the following section, the efficacy of the proposed formulation for preoperative planning of RAI is demonstrated through an illustrative example.

## IV. A Case Study

The optimal placement of the *da Vinci* robotic arms for maximizing the chance of success when harvesting the Left Internal Mammary Artery (LIMA) in minimally invasive coronary artery bypass is considered. In order to provide an alternative blood supply for the blocked coronary artery irrigating the myocardium, the LIMA is taken down from the chest wall and is sutured to the artery on the heart surface. The LIMA is situated on the chest wall close to the sternum and usually extends from the first rib to the sixth rib (see Fig. 1). Therefore, harvesting the LIMA requires high maneuverability over a wide range of space inside the chest cavity. The maneuverability can be diminished by lower wrist dexterity, extracorporeal obstacles and joint limits. In order to increase the chance of success, it is essential that uncertainty at the task level is taken into account.

The location of the access ports on the rib cage, as well as the relative orientation of the slave arms with respect to the patient's body are incorporated into the vector of planning parameters, $\Pi$. Given the preoperative images of a patient, the location of the LIMA on the sternum can be accurately identified (see Fig. 1).

From the complexity point of view, LIMA harvesting is a relatively simple task and can be easily represented by a minimal number of task frames. The desired task frame for harvesting can be empirically determined by *in vivo* observation of the tool gestures, or more accurately, can be determined by statistical analysis of the observed tool gestures. The task consists of pulling the tissue with the left instrument while cutting the tissue along the artery using the electrocautery on the other instrument [9]. For simplicity, a diagonal constant covariance matrix is assumed for both arms, i.e., $\mathbf{Q}_{\mathrm{x,r}} = \mathbf{Q}_{\mathrm{x,l}} = \begin{pmatrix} \sigma_\mathrm{p}^2 \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma_\phi^2 \mathbf{I} \end{pmatrix}$, where $\sigma_\mathrm{p}^2$ and $\sigma_\phi^2$ are the position and orientation variances, respectively.

The constrained nonlinear programming problem was solved using **fmincon** in MATLAB, and the results are illustrated in Figs. 2 and 3. Fig. 2 shows the *da Vinci* arms

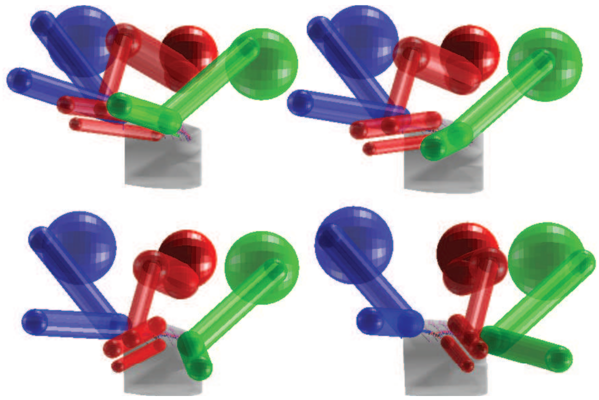

Fig. 2. The *da Vinci* arms while reaching the task frames for LIMA harvesting; the arms (modeled by geometric primitives for collision detection) are placed as recommended by the planner.

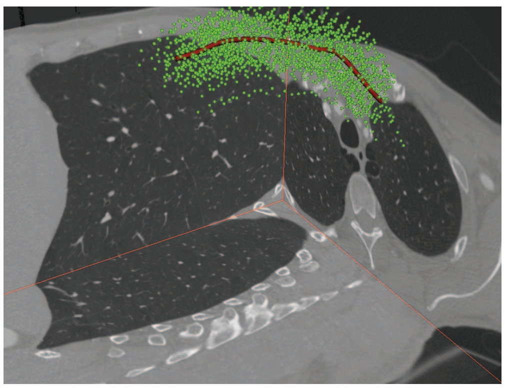

Fig. 3. The origin of the admissible task frames generated in a Monte Carlo simulation for LIMA harvesting task.

as they reach the LIMA for harvesting, while the arms are posed as suggested by the planner when the right, left and the endoscope ports are placed in the 3$^{rd}$, 6$^{th}$ and 5$^{th}$ intercostal spaces on the patient's rib cage.

The actual reliability achieved with the proposed plan $\Pi$ is evaluated by a Monte Carlo simulation. With the resulting task covariance matrices $\mathbf{Q}_{x,r}$ and $\mathbf{Q}_{x,l}$, a set of random task frames is generated and the satisfaction of the chance constraints is investigated. The origins of the admissible task frames are illustrated in Fig. 3, and the statistics of the chance constraints evaluated for the task frames are illustrated in Fig. 4. The bars show the proportion of the generated task frames that satisfy the chance constraints along the LIMA (with $N = 20$). The slight degradations from the expected 98% constraint satisfaction probability can be associated with nonlinearities, the fusion of the criteria in the objective function, and the assumption of identical covariance matrices for both instrument arms.

## V. Conclusion

In this paper, the planning of robotics-assisted interventions under task uncertainty was addressed. In order to accommodate more surgeons with different levels of skill and experience into the planning, it is essential that the plan accommodates a larger task uncertainty. The ultimate goal of

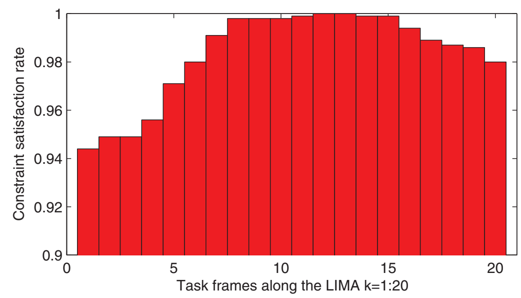


Fig. 4. Evaluation of the actual reliability with respect to the task frame uncertainty using a Monte Carlo simulation.

such a plan is to increase the chance of success by ensuring that the chances of collisions and joint limit violations remain sufficiently small. Therefore, the planning was formulated as a chance-constrained programming problem in terms of the instrument tip pose uncertainty in the vicinity of the desired task frame, minimizing the information regarding the task. To avoid using sampling-based techniques for solving the resulting stochastic optimization problem, the unscented transformation was utilized. This transformation yields more accurate estimation of the statistics of the constraints while the complexities pertaining to the linearization of the nonlinear constraints are avoided. The efficiency of the proposed formulation was demonstrated by a case study addressing optimal planning of the *da Vinci* robotic system for robotics-assisted LIMA harvesting in minimally invasive coronary artery bypass surgery.

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