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System identification to characterize human use of ethanol based on generative point-process models of video games with ethanol rewards

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Abstract— **The influence of family history and genetics on the risk for the development of abuse or dependence is a major theme in alcoholism research. Recent research have used endophenotypes and behavioral paradigms to help detect further genetic contributions to this disease. Electronic tasks, essentially video games, which provide alcohol as a reward in controlled environments and with specified exposures have been developed to explore some of the behavioral and subjective characteristics of individuals with or at risk for alcohol substance use disorders. A generative model (containing parameters with unknown values) of a simple game involving a progressive work paradigm is described along with the associated point process signal processing that allows system identification of the model. The system is demonstrated on human subject data. The same human subject completing the task under different circumstances, e.g., with larger and smaller alcohol reward values, is assigned different parameter values. Potential meanings of the different parameter values are described.**

I. INTRODUCTION

A major theme in alcoholism research is the influence of genetics on risk, e.g., [1]. Accurate endophenotyping is critical for genetic studies. However, characterization of human use of alcohol in the community, particularly in those with a substance use disorder, is difficult. One approach is to replace community use by use in controlled laboratory settings, e.g., [2].

O'Connor and colleagues have developed electronic cognitive tasks, essentially video games, where alcohol is the reward based upon completion of a progressive work paradigm. These paradigms provide a measure of how much effort a human subject is willing to invest in order to receive alcohol. In a progressive work game, the first dose of alcohol (given by intravenous infusion) requires relatively little work but the amount of work required to get successive doses progressively increases. Using the ideas of [3], the dose can be normalized to achieve brain alcohol concentration changes that are the same for all subjects independent of age, sex, weight, *etc*.

The most simple such game is considered. Specifically, the "work" consists of a simple button press with a schedule that prescribes a strictly increasing number of presses to receive the next successive dose of alcohol. The rate of button presses is limited; more than four presses per second were not counted. The subject knows whether a particular button push is counted. When a dose of alcohol is received, there is a pause while the subject receives the alcohol reward and any self-imposed delay at the beginning of the next work set. For that reason, our generative model is really a model for one epoch between doses of alcohol. The game is explained to the subject but no goal is imposed on the subject.

The manner in which the subject plays the game of the previous paragraph is exactly described by the sequence of times at which the subject presses the button. Therefore, a point process model, whose arrivals are the button-pressing times, provides a generative model of the subject's performance and a basis for analyzing the subject's performance. The point process model is described Section II, a method for computing the parameters of the model is described in Sections III-IV, and simulations and experimental results are given in Sections V–VI.

II. A POINT-PROCESS MODEL

Let $t \in [0,\infty)$ be time. A point process is a stochastic process $N(t)$ which takes values in $\{0,1,2,\dots\}$ where $N(t)$ evaluated at a time t_0 is the number of arrivals that have occurred between the starting time $t = 0$ and time t_0. The interarrival times of the point process are the time intervals between arrivals, i.e., the time intervals between the step increases in $N(t)$. The simplest point process is the Poisson process, which is completely described by a single positive real number, denoted by λ, which is the rate. In this process, the interarrival times are independent and identically distributed with an exponential probability density function (pdf) with parameter λ and this property (along with the choice that $N(0) = 0$) is one of the many equivalent definitions for the Poisson process.

The point process used to model the button presses is more general than the Poisson process, in particular, is a doubly-stochastic Poisson process [4, Chapter 7] in which λ is not a deterministic constant but instead is itself a stochastic process. This allows the model to describe the idea that the human subject's pattern of button pushes can change over the course of the video game experiment. In particular, $\lambda(t)$ is related to a first-order Gauss Markov process, which is denoted by $x(t)$, by the following equations:

$$\lambda(t) = \exp(\mu + x(t)) \quad (1)$$

$$\mathrm{d}x/\mathrm{d}t = \alpha x(t) + w(t) \quad (2)$$

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where $w(\cdot)$ is a zero-mean Gaussian white noise process with power spectral density $N_0/2$, the value of $x(0)$ is chosen so that $x(\cdot)$ is a wide-sense stationary stochastic process, and the three parameters in the model, all real valued, are μ, α, and $N_0/2$. The meanings of the parameters are described in the following paragraph.

Because $x(\cdot)$ is wide-sense stationary and Gaussian, $x(\cdot)$ is completely described by its mean function (denoted by $\bar{x}(t)$) and auto correlation function (denoted by $R_x(\tau)$) which have values

$$\bar{x}(t) = E[x(t)] = 0 \tag{3}$$

$$R_x(\tau) = E[x(t)x(t-\tau)] = \sigma^2 \exp(-|\tau|/\alpha) \tag{4}$$

where $E[\cdot]$ is expectation. (One implication of these results it that the statistics of $x(\cdot)$ and $-x(\cdot)$ are identical). Based on Eq. 4, the characteristic time over which changes in $x(\cdot)$ occur is $1/\alpha$ and the power in $x(\cdot)$ (i.e., $R_x(\tau = 0)$) is controlled jointly by α and $N_0/2$. Based on Eq. 3, the parameter μ controls the typical value of the time-varying rate $\lambda(t)$. Finally, the exponential function in Eq. 1 transforms a stochastic process that takes values in $(-\infty, +\infty)$ into a stochastic process that takes values in $(0, \infty)$ which is necessary if $\lambda(\cdot)$ is to be interpreted as a rate. While other transformations would also achieve this goal, choosing $\lambda(t)$ as an exponential furthermore allows exact computation of certain expectations in Section IV.

As will be described in Section IV, we compute certain expectations based on the idea that the rate $\lambda(\cdot)$ is constant over intervals of duration Δ. Therefore, in order to make this choice exact rather than an approximation, we replace Eqs. 1–2 by the discrete time (sampling interval Δ) system

$$\lambda_n = \exp(\mu + x_n) \tag{5}$$

$$x_{n+1} = \rho x_n + w_n \tag{6}$$

where w_n is a zero-mean Gaussian white noise process with variance σ^2. Therefore the three parameters in the model, all real valued, are μ, ρ, and σ^2 and

$$\bar{x}_n = E[x_n] = 0 \tag{7}$$

$$R_x(l) = E[x_n x_{n-l}] = \sigma^2 \rho^{-|l|}/(1-\rho^2). \tag{8}$$

III. SYSTEM IDENTIFICATION VIA MAXIMUM LIKELIHOOD PARAMETER ESTIMATION

Given the sequence of arrival times (i.e., button pressing times) on the interval $[0, T]$, the goal is to determine the values of the parameters μ, ρ, and $N_0/2$ in Equations 1-2. This estimation problem is solved by a maximum likelihood (ML) estimator, i.e., the estimated values of μ, α, and $N_0/2$, denoted by $\hat{\mu}$, $\hat{\alpha}$, and $\widehat{N_0/2}$ are defined by

$$\hat{\mu}, \hat{\alpha}, \widehat{N_0/2} = \arg\max_{\mu,\alpha,N_0/2} p(\{N(t) : 0 \le t \le T\} | \mu, \alpha, N_0/2) \tag{9}$$

where $p(\cdot|\cdot)$ is the conditional pdf on the arrival times given the parameter values. To solve this problem requires dealing with the real-valued time of each arrival. In order to formulate a simpler discrete-time problem [5], the arrivals are lumped into time bins of width Δ and the data is taken to be the number of arrivals in the bins, which is denoted by $\mathrm{d}N_k$ for the kth bin, i.e., $\mathrm{d}N_k = N(\Delta(k+1)) - N(\Delta k)$,where $k \in 0, \ldots \lfloor (T/\Delta) \rfloor - 1$. The corresponding equations for the dynamical system are Eqs. 5–6. Then Eq. 9 is replaced by

$$\hat{\mu}, \hat{\rho}, \widehat{\sigma^2} = \arg\max_{\mu,\rho,\sigma^2} p(\{\mathrm{d}N_k : k \in \{0, \ldots, K\}\} | \mu, \rho, \sigma^2) \tag{10}$$

where $T = K\Delta$.

Let $\mathrm{d}N$ without a subscript be the entire trajectory of $\mathrm{d}N_k$ for $k \in \{0, \ldots, K\}$ and likewise for x and x_k. Let $\theta = (\mu, \rho, \sigma^2)$. Then $p(\mathrm{d}N|\theta, x) = \prod_{k=1}^{K} f(\mathrm{d}N_k; \exp(\mu + x_{k-1})\Delta)$, where f is the Poisson probability mass function, and $p(\mathrm{d}N|\theta)$ can be computed from $p(\mathrm{d}N|\theta, x)$ by multiplying by the pdf $p(x|\theta)$ and integrating with respect to x.

IV. COMPUTATION OF THE MAXIMUM LIKELIHOOD ESTIMATOR

An expectation-maximization (EM) algorithm [6] is used to compute the ML estimate described in Eq. 10. The nuisance parameters in the algorithm are x. Let $\theta^{(l)}$ be the parameter values θ at the lth iteration of the EM algorithm. The expectation step is to compute $Q(\theta|\theta^{(l)}) = \int \ln[L(\theta|\mathrm{d}N, x)] p(x|\mathrm{d}N, \theta^{(l)}) \mathrm{d}x$ where $L(\theta|\mathrm{d}N, x) = p(\mathrm{d}N, x|\theta^{(l)})$ and the maximization step is to determine the θ which maximizes $Q(\theta|\theta^{(l)})$. The new estimate of the parameters, denoted by $\theta^{(l+1)}$, is this maximizing value of θ.

An iteration of the EM algorithm for this problem has two parts. The first part is to compute first and second order conditional moments of x given the data $\mathrm{d}N$. Q can be written in terms of these moments and the second part is to determine the value of θ that maximizes Q, which can also be written in terms of these moments. The necessary moments are $x_{k|k} \doteq E[x_k | \{\mathrm{d}N_l : l \in \{0, \ldots, k\}\}]$, $\sigma^2_{k|k} \doteq E[(x_k - x_{k|k})^2 | \{\mathrm{d}N_l : l \in \{0, \ldots, k\}\}]$, $W_k \doteq E[x_k^2 | \{\mathrm{d}N_l : l \in \{0, \ldots, K\}\}]$, and $W_{k,k-1} \doteq E[x_k x_{k-1} | \{\mathrm{d}N_l : l \in \{0, \ldots, K\}\}]$. In terms of these moments, the exact $\theta = (\mu, \rho, \sigma^2)$ that maximizes Q is

$$\rho^{(l+1)} = \frac{\sum_{k=1}^{K} W_{k,k-1}}{\sum_{k=1}^{K} W_{k-1}} \tag{11}$$

$$\left(\sigma^2\right)^{(l+1)} = \frac{1}{K}\left\{ W_k - 2\rho^{(l+1)} W_{k,k-1} + \left[\rho^{(l+1)}\right]^2 W_{k-1} + W_0 \left[1 - \left[\rho^{(l+1)}\right]^2\right] \right\} \tag{12}$$

$$\mu^{(l+1)} = \ln \sum_{k=1}^{K} dN(k\Delta) - \ln \sum_{k=1}^{K} \exp\left(x_{k|K} + (1/2)\sigma^2_{k|K}\right)\Delta. \tag{13}$$

Eq. 13 benefits from the choice of an exponential in Eq. 5 because that choice leads to computing the expectation of the exponential of a Gaussian random variable (essentially x_k), which can be done exactly via log-normal methods. Unlike Eqs. 11–13, which are exact, the computing of the moments cannot be done exactly with a feasible amount of

calculation. In the calculations reported in this paper, the method used is based on several nonlinear filtering ideas. The calculations divide into two phases which are a forward filtering phase and a backward smoothing phase. The forward phase equation for $x_{k|k}$ is computed by finding the value of $x_{k|k}$ that maximizes Q, and hence taking a maximum likelihood approach. The forward phase equation for $\sigma^2_{k|k}$ is calculated by setting the variance of the Gaussian random variable to the inverse of its Fisher information matrix [7]. The equations for $x_{k|k-1}$ and $\sigma^2_{k|k-1}$ follow standard Kalman filter equations [8]. The results are

$$x_{k|k} = \rho x_{k-1|k-1} + \sigma^2_{k|k-1}[y(k\Delta) - \exp(\mu + x_{k|k})\Delta \quad (14)$$

$$\sigma^2_{k|k} = -\left[-1/\sigma^2_{k|k-1} - \exp(\mu + x_{k|k})\Delta\right]^{-1} \quad (15)$$

$$x_{k|k-1} = \rho x_{k-1|k-1} \quad (16)$$

$$\sigma^2_{k|k-1} = \rho^2 \sigma^2_{k-1|k-1} + \sigma^2 \quad (17)$$

where Eq. 14 is solved for $x_{k|k}$ by a Newton method and the initial conditions are $x_{0|0} = x_0$ and $\sigma^2_{k|k} = \sigma^2(1-\rho^2)^{-1}$. The backward phase uses the backwards part of a forward-backward Kalman smoother [9]:

$$A_k = \rho \sigma^2_{k|k}(\sigma^2_{k+1|k})^{-1} \quad (18)$$

$$x_{k|K} = x_{k|k} + A_k(x_{k+1|K} - x_{k+1|k}) \quad (19)$$

$$\sigma^2_{k|K} = \sigma^2_{k|k} + A^2_k(\sigma^2_{k+1|K} - \sigma^2_{k+1|k}) \quad (20)$$

where $k = K-1,\ldots,1$ and the initial conditions are $x_{K|K}$ and $\sigma^2_{K|K}$ from the forward phase. Finally, using state-space covariance ideas [10],

$$W_{k,k+1} = A_k \sigma^2_{k+1|K} + x_{k|K} x_{k+1|K} \quad (21)$$

$$W_k = \sigma^2_{k|K} + x^2_{k|K}. \quad (22)$$

The EM algorithm requires an initial condition. The initial condition used in the calculations described in this paper is $\mu = [N(T) - N(0)]/K$, $\rho = 0$, $\sigma^2 = \mu/4$ and $x_0 = 0$.

V. SIMULATIONS

The algorithm of the previous section has two components: computing the moments and computing the ML estimator of μ, ρ, and σ^2. In simulation, it is possible to separately evaluate the performance of both components.

The first simulation covers the filter that computes $x_{k|k}$. The simulated data is from Equations 5-6 with $\mu = 6$, $\rho = 0.8$, $\sigma^2 = 1$, $K = 100$, $\Delta = 0.3$ and x_0 set to the corresponding steady state value. The initial conditions for the filter are the initial conditions for the overall algorithm described following Eq. 22. In summary of the results, Figure 1 shows the $x_{k|k}$ from Eqs. 14–17 for one trajectory demonstrating that many qualitative features of x_k are successfully preserved in $x_{k|k}$ in spite of the exponential nonlinearity.

The second simulation covers the entire maximum likelihood estimator for the parameters μ, ρ and σ^2. We have done Monte Carlo calculations ($J = 20$ trajectories, each $K = 1000$ samples long) of the bias and the variance of the estimates for μ, ρ and σ^2 when the true values are $\mu \in [0.2, 2]$, $\rho \in [0.002, 0.92]$, $\sigma^2 \in [0.01, 0.5]$ and $\Delta \in 1, 2$, which covers

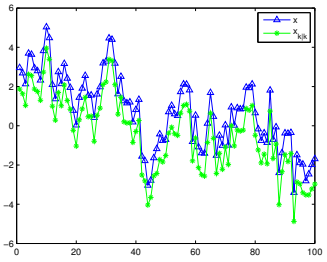


Fig. 1. An example of truth (x) and estimate ($x_{k|k}$) for the filtering problem.

the values we have observed in the experimental data. The EM algorithm is always started for the initial conditions described following Eq. 22.

Overall, the results show only small biases and small variances around the biased value. We summarize the results for μ because large values of μ versus small values of μ strongly influence whether the rate of arrivals is large or small. Among the results concerning μ, we focus on simulated data where the true values of all the parameters are near the values seen in the experimental data, specifically, $\mu \in [1, 1.25, 1.5, 1.75, 2.0]$, $\rho = 0.1$, $\sigma^2 = 0.1$ and $\Delta = 2$. The mean of the estimate of μ as a function of the true parameter values by Monte Carlo is $\bar{\mu} = (1/J)\sum_{j=1}^{J} \hat{\mu}^{(j)}$ where j indexes Monte Carlo trials. The bias is $b_\mu = \bar{\mu} - \mu^{\text{true}}$, and the standard deviation is the square root of $s^2_\mu = (1/(J-1))\sum_{j=1}^{J}(\mu^{(j)} - \bar{\mu})^2$. As shown in Table I, the

TABLE I

SUMMARY OF SIMULATION RESULTS CONCERNING μ. b_μ IS THE BIAS AND s_μ IS THE SQUARE ROOT OF THE ESTIMATION ERROR VARIANCE.

μ	1	1.25	1.5	1.75	2.0
b_μ	-0.0391	-0.0371	-0.0261	-0.0350	-0.0209
s_μ	0.0220	0.0182	0.0157	0.0160	0.0173

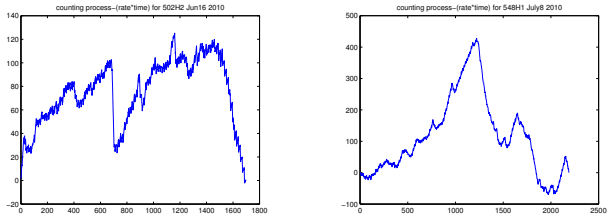
bias is less than 4% and the standard deviation is less than 2% of the true values and the dependence on the true value is weak.

VI. EXPERIMENTAL RESULTS

The studies were approved by the Indiana University Institutional Review Board. All subjects provided written informed consent. The experimental results for two subjects are shown in Figure 2. The two curves have differences at both large and small time scales. For instance, at the time scale of 10^3 samples the deviations from 0 are entirely different in pattern, even in direction. On a small time scale, Figure 2(a) has more variation than Figure 2(b) shown here as the appearance of a thicker curve for Figure 2(a) than Figure 2(b).

The histogram of interarrival times is shown in Figure 3. The data is only measured to within 1 second so the histogram is discretized. In spite of the discretization, the histogram does not appear to originate from an exponential interarrival time pdf (since the curve is not a straight line), motivating the doubly stochastic model of Eqs. 5–6.

The estimated dynamical system parameters μ, ρ, σ^2 are shown in Table II for the 8th epoch between ethanol rewards for two experiments on each of four subjects using



(a) (b)

Fig. 2. Experimental results on two subjects. The curve is the counting process for button presses as a function of the time minus the average rate multiplied by time so that the difference remains near zero. The average is exactly the μ value defined following Eq. 22 which is used in initialization of the estimator.

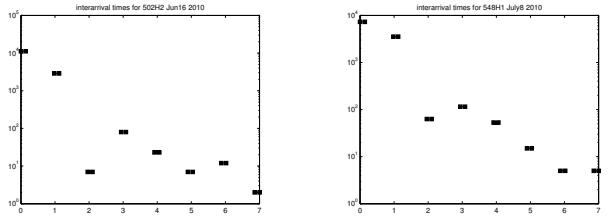


Fig. 3. Log-histogram of the interarrival times (seconds) for experimental results on two subjects.

an estimator with $\Delta = 2$. The differences between the two experiments are the amount of ethanol provided at the completion of the task. An estimate of the variance of $x(\cdot)$ from Eq. 8 is $\hat{\sigma}^2/(1-\hat{\rho}^2)$. For the values in Table II, this estimate takes the values .0755, .0927; .161, .114; .124, .212; 1.013, .634 where the results are in the same order as the rows in Table II. With the exception of Subject 515, larger rewards lead to a larger value of $\hat{\mu}$ which is the constant part of the rate function for button pushing. Larger rewards had a less consistent effect on the time-varying properties of the rate function since in half of the cases larger reward lead to a larger value of $\hat{\sigma}^2$ while in the other half it did not and, similarly, in half of the cases larger reward lead to a larger value for the estimated variance of $x(\cdot)$ while in the other half it did not. The $\hat{\mu}$ results are consistent with expectations for these subjects, all of whom are non-treatment-seeking alcoholics.

VII. DISCUSSION AND CONCLUSIONS

In light of the estimator performance described in Section V (e.g., Table I), the differences in dynamical system parameters μ, ρ, and σ^2 described in Section VI (Table II) may be sufficient to allow the dynamical system parameter estimates to act as features in pattern recognition and clustering algorithms. In particular, $\hat{\rho}$ ranges over an order of magnitude which implies large differences in the characteristic time over which the x_k process, and therefore the λ_k rate process, is strongly correlated.

TABLE II

PARAMETER ESTIMATES FROM EXPERIMENTAL DATA. "HIGH" VERSUS "LOW" IS THE AMOUNT OF ETHANOL PROVIDED AT THE COMPLETION OF THE TASK.

Subject	Reward	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\mu}$
527	High	0.289486	0.0691861	1.60211
527	Low	0.308521	0.0838568	1.16288
548	High	0.289161	0.147671	1.14713
548	Low	0.00054163	0.0113837	0.992117
502	High	0.0974935	0.101417	1.43138
502	Low	0.0856287	0.210045	1.25287
515	High	0.852337	0.277011	1.06055
515	Low	0.860755	0.164302	1.28342

An attractive characteristic of this approach is that it provides information on the temporal dynamics of the rate function, e.g., $\hat{\rho}$. In different types of experiments, alcoholics are known to have different temporal character to their responses and we hope that this will be apparent in the parameter estimates.

An attractive feature of an ML estimator is that standard theory provides an estimate of the covariance of the difference between the parameter estimates and the true parameter values [7]. This estimate requires computation of the Hessian of the log likelihood at the parameter values that maximize the log likelihood. While this computation is not currently implemented in the system described in this paper, it will be a key component of statistical tests for whether differences in parameter values are significant. In the future we will also investigate alternative ideas to compute the moments defined in Section IV, including particle filter ideas.

The values of μ, ρ, and σ^2 summarize the data: μ describes the average rate of button pushing, ρ describes how rapidly the instantaneous rate of button pushing changes with respect to time, and σ^2 describes the size of the changes. Potentially these features, or similar features from a more complex model, will be useful as the input to classifiers for distinguishing subjects who use alcohol in different ways and, through such classification, aid the selection of more appropriate therapy. Similarly, comparing these features when a subject is taking versus not taking a drug may be useful in the development of the drug and/or in the selection of an appropriate drug for therapy.

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