# An Optimization Algorithm For Joint Mechanics Estimate Using Inertial Measurement Unit Data During A Squat Task

Vincent Bonnet<sup>1</sup>, Claudia Mazzà<sup>1</sup>, Philippe Fraisse<sup>2</sup> and Aurelio Cappozzo<sup>1</sup>

Abstract— The use of dynamic optimization as a tool to estimate joint kinematics and kinetics, and ground reaction forces using data from a single inertial measurement unit (IMU) positioned on the lower trunk was investigated. The feasibility of this approach and its accuracy was explored for the analysis of a squat task, focusing on the ankle, knee and hip joints. An optimal motor control strategy aimed at minimizing the sum of the intersegmental couples and of their time derivatives was imposed to estimate the mechanics of a threesegment sagittal model. Moreover, in the optimization process constraints to the measured vertical acceleration, to the maximal vertical IMU excursion, and with regard to the maintenance of dynamic balance were imposed. Experiments were performed using 10 volunteers. Data were collected from the IMU, from a stereophotogrammetric system (SS) and from a force platform for validation purposes. Results showed a very good consistency of the model output with the lower limb joint trajectories, as obtained using the SS, and with the measured vertical component of the ground reaction (low root mean square differences (<10%) and high correlation coefficients (0.98)).

#### I. INTRODUCTION

**C**QUAT tasks are widely used in rehabilitation programs  $\checkmark$  for different pathologies [1]-[2] and in sports training to develop lower limb musculature [3]. Squatting is a paradigmatic task which allows the evaluation of basic skills of daily living, which include squatting to pick up an item, negotiating stairs, or sit-to-stand transfer [3]. Escamilla et al. [4] provided a description of the joint angles and forces involved in the execution of this exercise. This information is essential for clinicians and trainers as an aid to decisionmaking while prescribing a squat exercise. These variables are usually obtained from data measured using a SS and force plates. These systems allow accurate results but imply considerable economic investments and a complex experimental protocol. The challenge for clinical applications aimed at easy-to-use low-cost instruments and straightforward interpretation of the numerical outcome remains an open issue [5]. As an alternative, the estimation of lower limb joint mechanics during a squat exercise using the data provided by a single forceplate has been proposed [6]. Recently, IMUs have gained popularity, mainly for the purpose of ambulatory motion capture, thanks to their ease-

V. Bonnet, C. Mazzà and A. Cappozzo were with the LABLAB, Department of Human Movement and Sports Sciences, University of Rome "Foro Italico", Italy. (e-mail: Vincent.bonnet@gmail.com).

P. Fraisse was with the LIRMM UMR 5506 CNRS, University of Montpellier 2, France

of-use, their robust design and their low-cost [7]. Usually, a number of IMUs are attached to adjacent body segments to estimate joint kinematics [7]-[10], and the number of sensors increases with the number of joints involved.

In the above-described context, this paper proposes a method to estimate lower limb joint kinematics and ground reaction forces (GRFs) during the execution of a squat task, based only on the output of one single IMU located on the lower trunk in association with a biomechanical model and an optimization process. Joint kinetics may thereafter be estimated using these results.

#### II. METHODS

# A. Biomechanical model

The biomechanical model used to represent the human body was a planar (sagittal) chain composed of four rigid segments (feet, shanks, thighs, trunk) connected by hinge joints (Fig. 1). De Leva tables [11] were used to estimate segment lengths and inertial parameters of the model segments as a function of the height and body mass of each volunteer. Inverse dynamics was computed using recursive Newton-Euler equations, and the GRFs as described in [12].



Fig. 1: Biomechanical model of the human body during a squat task and position of IMU sensor.

The acceleration  $(C\ddot{o}M)$  of the CoM of the model was obtained through the symbolic double differentiation of the CoM coordinates:

$$C\ddot{o}M = \dot{J}_{CoM}\dot{\theta} + J_{CoM}\ddot{\theta} \tag{1}$$

where  $J_{COM}$  is the Jacobian matrix that expresses CoM velocity in a global reference frame (*X*, *Y*) with the origin on the malleolus (Fig. 1).

This work was supported in part by the Regione Lazio – Filas (agreement n. 11226 – July 9, 2009) and in part by the authors' University.

#### B. Optimization process

The redundant squat task mechanics formulated as a mathematical optimization problem with an objective function based on the minimization of a dynamic cost function. Kuzelicki *et al.* [13] proposed a cost function, specific to demanding mechanical postural tasks, that is the combination of the sum of the intersegmental couples and of their time derivatives:

$$C_n = \sum_{i=1}^n \Gamma_j^2(t_i) \Delta t + \sum_{i=1}^n \left(\frac{d\Gamma_j}{dt}\right)^2(t_i) \Delta t \qquad (2)$$

where *n* is the number of discrete time instants  $t_i$ ,  $\Delta t$  is the sampling interval and *j*=1,2,3 for ankle, knee, and hip joints, respectively.

As the investigated task was pseudo-periodic, the Fourier series decomposition:

 $\theta_j(t_i) = a_{0j} + \sum_{k=1}^N (a_{kj} \cos(k\omega t_i) + b_{kj} \sin(k\omega t_i))$  (3) was chosen to represent the joint positions  $\theta_{i..}$  where  $a_{kj}$  and  $b_{kj}$  are the amplitude coefficients for the  $k_{th}$  harmonic of the  $j_{th}$  joint,  $\omega = 2\pi/T_F$  is the pulsation of the motion, and  $T_F$  is the task execution time, detected using appropriate thresholds on the IMU vertical acceleration expressed in the (X, Y) frame.  $(A_{py})$ . A constraint was associated with dynamical balance maintenance:

$$CoP_{min} \le CoP(t) \le CoP_{max}$$
 (4)

where  $CoP_{min}$ , and  $CoP_{max}$  are defined by subject foot length. Equality constraints, based on measured values, were used to drive the biomechanical model while performing the squat task:

$$(CoM(1) - CoM(T_c)) = h_{IMU} \pm \varepsilon_{CoM}$$
<sup>(5)</sup>

where  $h_{IMU}$  is the maximal vertical displacement of the IMU, obtained by double integrating  $A_{py}$ ,  $T_c$  is the percentage of the cycle at which  $h_{IMU}$  occurs (described in Fig. 3), and  $\varepsilon_{CoM}$  is a tolerance parameter. Note that the subject specific values of  $h_{IMU}$  and  $T_c$  allow the optimization search to account for possible asymmetries in the duration of the descending and ascending phases that characterize the squat task. The system of equations reported in (6) ensures that, within a prescribed tolerance  $\varepsilon_{\theta}$ , the initial and final joint positions are equal to those measured ( $\theta_m$ ).

$$\theta_j(1) = \theta_{mj}(1) \pm \varepsilon_{\theta} \theta_j(n) = \theta_{mj}(n) \pm \varepsilon_{\theta}$$
 (6)

The measured angles  $\theta_{mj}$  coincide with upright posture angles at the beginning and end of the task.

Finally,  $A_{py}$  was used as a target constraint for the vertical acceleration of the *CoM* of the model:

$$C\ddot{o}M(t) = A_{py} \pm \varepsilon_{Acc} \tag{7}$$

The optimization problem consisted of minimizing  $C_n$  by finding the 2(2*N*+1) coefficients of the Fourier series that represent the joint angles and of making the model move while respecting the above constraints, the measured IMU acceleration, and its vertical peak-to-peak displacement.

## C. Experimentation

Ten young healthy volunteers (6 males and 4 females,  $age=30.2\pm5.0$  years, mass=69±10 kg, stature=1.70±0.07 m) were included in the study after signing an informed consent. Anthropometric parameters and upright posture

joint angles were initially measured. Volunteers were asked to assume a natural standing posture, keeping a laser pointer in one of their hands. The position of the laser was marked on a panel in front of the subject (Fig. 2). Starting from this position, they were then asked to perform a squatting task, continuously moving at self-selected speed, to reach a lower position in which the laser pointed a second mark, placed 0.2m below the initial one and then returning to their natural standing posture. This very simple way of providing the feedback to the subjects was chosen to create a low cost, and easy to reproduce, experimental setup for further applications. Participants were asked to keep their arms straight alongside their body and the soles of their feet flat on the ground. The task was repeated 10 times with rests between the trials. A force plate (Bertec Inc) was used to record the GRFs and a single IMU (MTx, Xsens Motion Technologies) attached to the lower trunk (Fig. 2) to record angular velocities and linear accelerations. Sensor measures were expressed in the (X, Y) frame by rotating the local frame of the IMU around its z axis (see Fig.1). The angle used for this rotation was computed by integrating the corresponding angular velocity.



Fig. 2: Experimental setup of the squat task.

Kinematic variables were simultaneously measured using a SS (9 Mx cameras, VICON). Eight reflective markers were located on: the second metatarsal head, the malleolus of the dominant leg, the anterior and posterior superior iliac spines, and the back and front of the head (Fig.2). From these points, the position of the following virtual markers was computed: H=midpoint between the head markers, and P=midpoint between the pelvis markers. These markers were used in combination with the mechanical model described in Figure 1.

Figure 3 shows the general features of the squat task obtained experimentally for all subjects and all trials. The ankle, knee and hip joint angles vary with an amplitude of  $a_1=40.0\pm6.5^\circ$ ,  $a_2=65.9\pm6.5^\circ$ ,  $a_3=45.2\pm12.3^\circ$ . These average values were used as common initial conditions for the optimization process of all trials. The maximum value of  $h_{IMU}=0.16\pm0.02m$ , corresponding to the crouch position occurred at  $T_c=52.1\pm5.3\%$  of the squat cycle time, showing a

relatively good consistency in the execution of the task across subjects.



Fig. 3: Experimental results for all subjects, all trials. Solid line and dashed lines indicate respectively mean and standard deviation values. (top) Joint angles. (bottom) Vertical displacement of IMU.

### D. Model input data

The input data to the model were chosen to perform reconstructions of squat task mechanics that could ensure a good compromise between accuracy, computational time, and robustness. A preliminary power spectral density analysis performed on the measured joint angles did not show any activity above the 4<sub>th</sub> harmonic. The number of harmonics in the Fourier series was then set to N=2 for ankle and hip, and N=4 for the knee in the optimization search, in order to reproduce the complexity of the measured  $A_{pv}$  and to minimize the number of parameters. The optimization algorithm was always run with the same initial conditions, setting a<sub>0j</sub> and a<sub>1j</sub> as described above, and all the other harmonics to 0. Based on the authors' experience, the tolerance parameter values were set to the following values:  $\varepsilon_{CoM}=0.025m$ ,  $\varepsilon_{Acc}=0.45m$ .s<sup>-2</sup>,  $\varepsilon_0=2^\circ$ .

#### E. Assessment of model accuracy

To assess the model accuracy, its outputs were compared to the joint angles estimated from the measured SS data and to the vertical component of the GRF. Normalized root mean square difference (NRMS, eq. 8), and correlation coefficient (r) were calculated for the comparison.

$$NRMS(\theta_j) = 100 \cdot \frac{\sqrt{\frac{1}{N} \sum_{i=1}^{N} (\theta_{mj} - \theta_j)^2}}{\sqrt{\frac{1}{N} \sum_{i=1}^{N} (\theta_{mj})^2}}$$
(8)

# III. RESULTS

Figures 4 and 5 show typical results obtained by our model using the parameters described in section II. In particular, two representative trials from two different subjects characterized by markedly different features of the GRFs have been reported. Despite these differences, the model was able to reproduce the vertical GRF with a NRMS difference that in both cases was on average less than 5%, and joint angles for the ankle, knee and hip with a NRMS difference less than 2%, 10% and 10%, respectively.



Fig. 4: Typical results obtained for one subject. Solid line and dashed lines indicate respectively output form model and measured data. (top) Joint angles. (bottom) Vertical GRF.

The low NRMS difference for the vertical GRF component is obtained mainly thanks to the constraint on vertical acceleration of the CoM (eq. 5). The tracking of CöM leads de facto to an accurate reproduction of GRF. For all joints and all trials, the mean NRMS difference in the joint angles was less than 10% (Table 1), with the highest difference exhibited at the hip. The r values show that the timing of the task execution is respected at all joints with a mean correlation of 0.98. Reported results, obtained over 100 trials show a good fit with the data points obtained through SS and force plate. The time pattern of the joint angles appear to be analytically reproducible using a pure sinusoidal function, however, the influence of harmonics of order greater than one could not be neglected in the reproduction of GRFs. Figure 6 shows, for all the subjects, the mean values of the coefficients of the Fourier series that were identified by the model. As it can be seen from the figure, relative small variations were found between the subjects. Further studies will investigate the influence and the link between these parameters in order to reduce their number and improve the computation time.

TABLE I
SUMMARY OF MODEL VALIDATION RESULTS

Variable	NRMS (%)	r
Ankle angle $(\theta_1)$	3.5±2.2	$0.98 \pm 0.02$
Knee angle $(\theta_2)$	9.3±4.5	$0.98 \pm 0.02$
Hip angle $(\theta_3)$	9.4±4.8	$0.98 \pm 0.02$
Vertical GRF	4.3±2.4	$0.98 \pm 0.02$



Fig. 5: Typical results obtained for one subject. Solid and dashed lines indicate output from model and measured data respectively. (top) Joint angles. (bottom) Vertical GRF.



Fig. 6: Evolution for all the subjects of the identified Fourier series parameters. (a) First harmonic. (b) Rest of the harmonics. (top) Ankle joint. (middle) Knee joint. (bottom) Hip joint.

#### IV. CONCLUSION

The proposed methodology for the estimation of kinematics and kinetics variables for lower limb joints proved to be accurate for 100 squats. Since it allows the management of joint redundancy it could, in principle, be extended to 3D motion and more complex tasks, which is essential for enlarging the spectrum of possible applications. Eventual limits imposed by the loss of cyclicity in the movements could be overcome by using a different representation of the joint angles, such as the B-spline. A sensitivity analysis examining the effects of the choice the algorithm parameters is also expected to improve the fitting and the robustness of our model. Similarly, better inertial parameters estimation by using the methodology described in [14] could improve the reported results. The proposed approach could also find applications in the motor control field, in the framework of the Bernstein problem [15] and computational model [16], since, conversely from other studies [6], a biomechanical cost function was used to drive the optimization process instead of a pure mathematical criterion.

#### REFERENCES

- M. Fredericson, C.M. Powers,"Practical management of patellofemoral pain", *Clin J Sport Med*, Vol 12, 2002, pp. 36-8.
- [2] F.C. Kuo, W.P. Kao b, H.L. Chen a, C.Z. Hong, "A Squat-to-reach task in older and young adults: Kinematic and electromyographic analyses", *Gait & Posture*, Vol. 33, 2011, pp. 124–129.
- [3] F.S.M. Alves, F.S. Oliveira, C.H.B.F. Junqueira, B.M.S. Azevedo, and V.C. Dionísio, "Analysis of electromyographic patterns during standard and declined squats", *Rev Bras Fisioter*, Vol. 13, 2009, p. 164-72.
- [4] R. F. Escamilla,"Knee biomechanics of the dynamic squat exercise.", Med. Sci. Sports Exerc., Vol. 33, 2001, pp. 127-141.
- [5] Cappozzo A., "Minimum measured-input models for the assessment of motor ability", J. of Biomechanics, Vol. 35, 2002, pp. 437-446.
- [6] Mazzà C. and A. Cappozzo, "An optimization algorithm for human joint angle time-istory generation using external force data", *Annals of Biomedical Engineering*, Vol. 32, 2004, pp. 764-722.
- [7] R. Williason and B.J. Andrews, "Detecting absolute human knee angle and angular velocity using accelerometers and rate gyroscopes", *Med. Bio. Eng. Comput*, Vol 39, 2001, pp.1-09.
- [8] Mayagoitia R.E., Nene A.V and Veltink P.H, "Accelerometer and rate gyroscope measurement of kinematics: an inexpensive alternative to optical motion analysis systems", *J.of Biomechanics*, Vol. 35, 2002, pp. 537-542.
- [9] Ferrari A., Cutti A.G., Garofalo P., Raggi M., Heijboer M., Cappello A. and Davalli A., "First in vivo assessment of "Outwalk": a novel protocol for clinical gait based on inertial and magnetic sensors", *Med. Bio. Eng. Comput*, Vol. 48, 2010, pp. 1-15.
- [10] Picerno P., Cereatti A., Cappozzo A., "Joint kinematics estimate using wearable inertial and magnetic sensing modules", *Gait & posture*, Vol. 28, 2008, pp. 588:595.
- [11] DeLeva P., "Adjustments to Zatsiorsky-Seluyanov's segmant inertia parameters", J. of Biomechanics, Vol. 29, 1996, pp. 1223-1230.
- [12] V. Cahouët, L. Martin, D. Amarantini,"Static optimal estimation of joint accelerations for inverse dynamic problem solution." J. of Biomechanics, Vol. 35, 2002, pp. 1507-13.
- [13] J. Kuželicki , M. Zefran, H. Burger and T. Bajd,"Synthesis of standing-up trajectories using dynamic optimization", *Gait & Posture*, Vol. 21, 2005, pp. 1-11.
- [14] S. Cotton, A. Murray and P. Fraisse, "Estimation of the Center of Mass: From humanoid robots to human beings", *IEEE/ASME Trans.on Mechatronics*, Vol. 14, 2009, pp. 707-712,
- [15] N. Bernstein,"The Co-ordination and Regulation of Movements". Oxford, UK:Pergamon Press, 1967.
- [16] E. Guigon, "Models and architectures for motor control: Simple or complex?", *Motor Control*, Vol. 20, pp 478-502.