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Compressive Sensing Imaging with Randomized Lattice Sampling: Applications to Fast 3D MRI

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Abstract—**Fast MRI makes it possible to visualize dynamic biological phenomena and can potentially reduce the cost of diagnostic imaging. Constrained imaging methods such as compressive sense (CS) and optimal lattice sampling (OLS) have proven to be effective for speeding up MRI. In doing so, CS takes advantage of the image sparsity or compressibility and OLS utilizes the known signal/spectrum support. Interestingly, while CS requires sampling to be "randomized" to obtain incoherent artifacts which is critical for reconstruction, OLS mandates sampling to be on a structured lattice. In this paper, we proposed a method to integrate CS with OLS so that both the sparsity and support constraints can be used simultaneously. The method randomizes the sampling on the lattice and minimizes a convex cost function with sparsity constraint and data fidelity terms. Computer simulations in 3D MRI show that the proposed method allows greater accelerations with minimal degradation of the image quality.**

Keywords: ***MRI; compressive sensing; lattice sampling; fast imaging; image reconstruction***

I. Introduction

Magnetic resonance imaging (MRI) is a relative slow imaging modality as compared with X-ray CT, ultrasound imaging, and optical imaging. A number of technologies have been proposed to improve the MR imaging speed. These include fast pulse sequences such as EPI or spirals, parallel imaging with multiple-channel receiver and array coils, and constrained imaging. Compressive sense (CS) and optimal lattice sampling (OLS) are two constrained imaging methods proven to be effective for fast MRI [1-7].

The two methods are fundamentally different in their assumptions, data acquisition schemes, and reconstruction algorithms. CS takes advantage of the image sparsity or compressibility. It was first introduced in the literature of Information Theory and Approximation Theory. The basic idea is that a sparse signal can be reconstructed from its data samples (projected) even if they are sampled at a rate below the Nyquist criterion (undersampling). This is true if the following conditions hold: First, the signal must have a sparse representation in a transformed domain (i.e., it must be compressible by some transform coding scheme). Second, the aliasing artifacts in a linear reconstruction caused by undersampling must be incoherent (noise-like) in the sparsifying transform domain. Under these conditions, the image can be reconstructed using a non-linear convex optimization method which enforces both sparsity representation of the signal and consistency of the reconstruction with the acquired samples. CS has been successfully implemented and applied in fast MRI in the recent few years [1-3,8-15].

On the other hand, fast imaging with OLS is based on multiple-dimensional signal sampling theory to achieve most efficient "spectrum" packing [4-6]. In MRI, this means a less dense k-space sampling is often sufficient if the support of the image ("spectrum" of the k-space data) can be packed. Many previous methods are based on this fundamental principle. For example, finite spatial support methods such as mrMRI (multi-region MRI) and reduced FOV (field of view) imaging assume that the objects (or its dynamic content) are confined to some sub-regions of the whole FOV [16,17]. Finite temporal spectrum support assumption is used in DIME (Dynamic Imaging by Motion Estimation) [7]. Finite spatiotemporal spectrum distributions of the (k, t)-space signals have also been exploited in methods such as UNFOLD (UNaliasing by FOurier-encoding the overLaps using the temporal Dimension) [18], PARADISE [5], k-t BLAST [19], k-t FOCUSS [15], and the related methods. The image reconstruction is these method normally only involves zero-padding, inverse Fourier transform and image cropping, which is much more straightforward and efficient than the CS reconstruction.

Interestingly, while CS requires sampling to be "randomized" to obtain incoherent artifacts which is critical for reconstruction, OLS mandates sampling to be on a structured lattice. In this paper, we propose a method to integrate CS with OLS so that both the sparsity and support constraints can be used simultaneously. The method randomizes the sampling on the lattice and minimizes a convex cost function with sparsity and data fidelity terms. Computer simulations in 3D MRI show that the proposed method allows greater accelerations with minimal degradation of the image quality.

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II. Materials And Methods

In this section, we introduce the basic idea of the method and algorithm. We will start with the k-space data acquisition scheme. Then the image reconstruction algorithm and the computer simulations will be described.

A. Data Acquisition

The proposed method is unique in its data acquisition scheme. The data coverage is designed to benefit the imaging from both the CS and the OLS principles. First, the sampling points in the k-space always fall on the lattice grid determined by the support of the object to be imaged. Second, the sampling points include only a subset of lattice grids, which is randomized according to the CS theory.

To design such sampling scheme, we first specify the OLS sampling lattice based on the support of the object. A number of literatures has covered this topic [4-6]. Therefore only a brief summary is provided here. For simplicity, we use a 2D example shown in Fig. 2 to illustrate the concept. Let $\vec{v}_1$ and $\vec{v}_2$ are two spatial packing vectors that optimally pack the support of the object in the 2D space. Fig. 2 (a) shows two such vectors for an object with a support as the shaded diamond. Repeating the diamond on all grid points generated by the linear combinations of the two vectors packs the supports tightly, but causes no real aliasing. According to the OLS, the optimal k-space signal sampling lattice can be generated by $m\vec{u} + n\vec{u}_2$ where m and n are integers, where $\vec{u}_1$ and $\vec{u}_2$ are two sampling vectors that satisfy

$$\begin{cases} \vec{v}_1 \bullet \vec{u}_1 = 1 & \vec{v}_1 \bullet \vec{u}_2 = 0 \\ \vec{v}_2 \bullet \vec{u}_1 = 0 & \vec{v}_2 \bullet \vec{u}_2 = 1 \end{cases} \tag{1}$$

where $\bullet$ represents the inner product. The two sampling vectors corresponding to the vectors in Fig. 2(a) is shown in Fig. 2(b). For the lattice sampling, the sampling density is defined as

$$D = \frac{1}{|\vec{u}_1 \times \vec{u}_2|} \tag{2}$$

Note that the conventional rectilinear sampling corresponds to the case where the support is rectangle-shaped and the two packing vectors are orthogonal. Due to the more compact packing of the object support in OLS, the sampling density becomes smaller. As the result, only a reduced number of k-space data is required for a particular k-space coverage and the imaging speed can be accelerated accordingly.

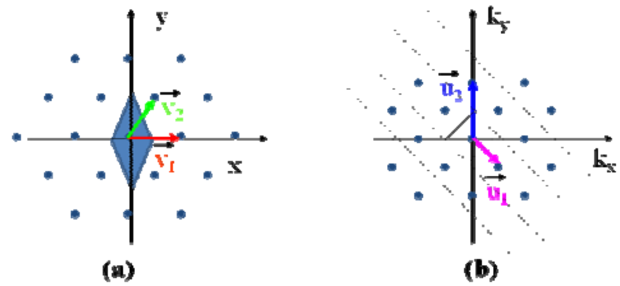

Fig. 2: Illustration of k-space sampling lattice for an object with a diamond-shaped support: (a) the object support and the compact packing vectors; and (b) the corresponding optimal sampling vectors and the lattice.

The second step in the data sampling design is to select a subset of randomized grid points from the lattice. A number of methods have been proposed in the literature. A simple approach is to generate the set based on a 2D uniform random distribution. Alternatively, 2D Gaussian distribution can be used to enforce weighting so that the central k-space will be sampled more densely than the outer k-space. This can be justified by the fact that k-space energy of most MR images is concentrated in the lower spatial frequencies. In this paper, the method that uses Poisson disc is adopted to generate the set from the lattice. This method can generate "randomized" points that are more favorable for CS reconstruction. A comparison of the regular CS sampling pattern (with central weighting) and the proposed CS sampling pattern is shown in Fig. 3. Note that the overall sampling is reduced by two complementary factors: (1) the random undersampling of the grids; and (2) the reduction of sampling density due to the optimal lattice structure. In the example shown in Fig.2, an additional factor-2 time saving can be achieved by the proposed method, as compared with the conventional CS imaging.

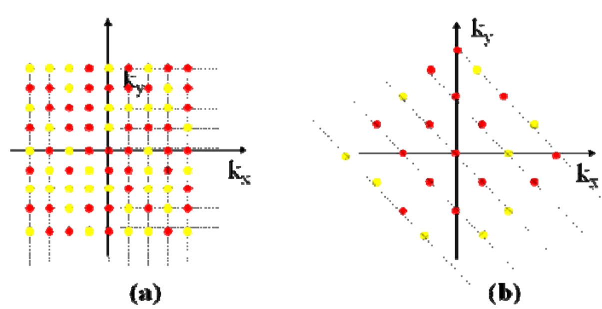

Fig. 3: Illustration of the randomized k-space sampling patterns for the object support in Fig. 2 in: (a) Cartesian coordinate as in the conventional CS imaging; and (b) the lattice grid in the proposed method. The red dots represented the sampled points (in 3D MRI, each dot represents one frequency encoding).

In practical 3D MRI, each dot in Fig.2 and Fig.3 can represent one frequency encoding. On this dimension, spatial localization can be simply resolved by an inverse Fourier transform because no undersampling exists along the frequency encoding direction.

B. Image Reconstruction

Given the acquired k-space data as specified, the image reconstruction in the proposed method involves two steps: the first step is to reconstruct an image using the CS. The image has a rectangular FOV which may contains more than one whole or partial aliases of the object support. In the second stop, a spatial mask is applied to crop the desirable image from the FOV according to the object's support.

The acquired k-space data on the lattice is related to the underlying image ***x*** by

$$y(m,n) = FT(\mathbf{x},\vec{k})\delta(\vec{k} - m\vec{u}_1 - n\vec{u}_2) \qquad (3)$$

where $FT(\mathbf{x},\vec{k})$ represents the continuous Fourier transform of the image at the spatial frequency $\vec{k}$, and $\delta(\bullet)$ is the spatial sampling function. Note that the set of (*m,n*) only covers the selected random points in the proposed method. For simplicity, we rewrite Eq. (3) in a vector form $\mathbf{y} = \mathbf{\Phi x}$ where $\mathbf{\Phi}$ represents the Discrete Fourier transform (DFT). Note that here ***y*** represents the zero-padded k-space data on a rectilinear coordinate of which the lattice grids is a subset; and ***x*** represent the image in the FOV with potential partial aliasing so that its DFT will be consistent with the data $\mathbf{y}$.

To reconstruct the image in the first step, we invoke the problem formulation on in the CS reconstruction

$$\min_{\mathbf{x}} \left|\Psi(\mathbf{x})\right| \text{ subject to } \mathbf{y} = \mathbf{\Phi x} \qquad (4)$$

where $\left|\Psi(\mathbf{x})\right|$ represents the image sparsity which can be based on the Total Variation (TV), or the L1 norm of the signal in an appropriate compression domain (such as Wavelet transform or DCT). The above problem can be turned into an unconstrained convex optimization problem:

$$\min_{\mathbf{x}} \left|\Psi(\mathbf{x})\right| + \lambda\left\|\mathbf{y} - \mathbf{\Phi x}\right\|_2 \qquad (5)$$

where λ is a regularization parameter. Note that the first term enforces the image sparsity and the second term imposes the data consistency on the sampled lattice grids (in the L_2 sense). In this paper, TV is used to measure the image sparsity, which is optimal for piece-wise constant or smooth images. The minimization is achieved by using a non-linear conjugate gradient method which is very efficient. Note that extra small regularization parameters lead to oversmoothing in the reconstructed image. Typically λ is set by experimental trials or the L-curve method to strike a balance between the data fitting term and the oversmoothness of the images.

The second step is simply to crop the image according to its object support to eliminate the aliasing. All algorithms are implemented in Matlab.

C. Validation Using Computer Simulations

To test the algorithm, computer simulations are performed which are based on a set of real brain MRI dataset. The data were acquired on a healthy volunteer on a 1.5 T scanner using a head coil and fast spin-echo sequence. The data size is 128 by 128 by 32. In simulations, the third dimension is assumed to be the frequency encoding. The two leading dimensions are assumed to be phased encodings, which are retrospectively decimated according to the sampling pattern derived (as illustrated in Fig. 3). The acceleration factor (R) is set to be 3 for the conventional CS and 6 for the proposed method. Reconstructions from the conventional CS and the proposed method will be compared.

III. Results

The reconstruction result is shown in Fig. 4. A representative slice reconstructed from 1/3 of the data using the conventional CS method (left) and from 1/6 of the data using the proposed method (right) are compared. As can be seen, comparable reconstruction quality was obtained using the proposed method, even it used only 1/6 of the full data. This indicates that the proposed method, by utilizing the randomized lattice sampling in CS, can further improve the acquisition efficiency without sacrificing the image quality.

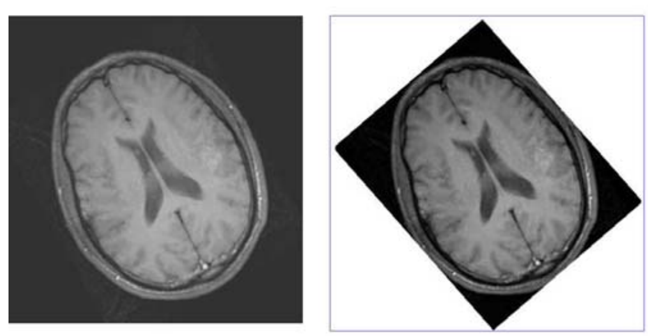

Fig. 4: Representative slice reconstructed from 1/3 of the total data using the conventional CS method (left) and from 1/6 of the data using the proposed method (right).

IV. Discussion

Compressive sensing and optimal lattice sampling use different constraints and have drastic different requirements on the k-space sampling scheme. The former requires "randomized" sampling while the latter needs structured sampling. In this work, we propose a new imaging method to integrate the two so their complementary advantages can further improve the MRI speed.

The brain anatomical imaging example was used to illustrate the utility of the method in this paper. The method can be potentially applied to other applications. For example, in dynamic cardiac imaging where the spatial-spectral structure of the (k,t) signal allows for efficient lattice sampling on non-rectilinear grid, as shown in [5]. In addition, although the object support may not be naturally limited, in real applications, intentional aliasing in certain areas can be tolerated without affect the diagnostic value. This is often the case in cardiac or abdominal MR where the region of interest is only part of the FOV (for example, the heart in the whole chest area). In such scenarios, the proposed method can still be utilized to achieve improved efficiency.

There are a number of limitations to the current implementation and studies. First, in applying the proposed method, the support of the image is *a priori* information. Therefore in practice, a low resolution scout image might need to be acquired to provide this information so that the data acquisition scheme can be designed. Second, the method applies only to scenarios where the FOV of the object or its spectrum allows a more efficient lattice sampling. Otherwise, no gains from OLS can be achieved. Finally, more systematic studies are required to fully validate and characterize the proposed method. In the present studies, no quantitative analysis of the reconstructed image quality is performed. Quantitative measures such as signal-to-noise ratio (SNR) and artifacts power will be included in the future studies. In addition, more systematic comparisons with the conventional CS reconstruction and/or the conventional OLS imaging methods may provide further insights to the proposed method.

The proposed method is currently limited to single-channel k-space data. Although straightforward extension to multi-channel data acquired using parallel receivers is possible, more synergetic integration of the method with the parallel imaging systems can potentially provide additional benefits and deserve further investigations.

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