

# A model of motoneuron behavior and muscle-force generation for sustained isometric contractions

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**Abstract**—A model for the simulation of motoneuron firing behavior and muscle force during sustained constant-force isometric contractions was developed. It provides a non-linear relationship between the excitation to the motoneuron pool of a muscle and the firing behavior of motor units; it implements muscle mechanical changes induced by fatigue and it comprises a feedback loop to maintain the muscle force at a given target level. We simulated a series of repeated force contractions sustained at 20% MVC with the first dorsal interosseous muscle of the hand and the vastus lateralis muscle of the thigh. The model generates force and firing behaviors which are consistent with experimental findings and underscores the influence of muscle mechanical changes on the control behavior of motor units during sustained contractions. The model predicts the increase of force fluctuation with fatigue in both muscles, likely due to recruitment of high-threshold high-amplitude twitch motor units. Force variability is greater in the first dorsal interosseous muscle than in the vastus lateralis muscle at any time during the contraction series, due to the different electrical and mechanical properties of the muscles.

## I. INTRODUCTION

The manner in which the nervous system controls muscles to maintain force during sustained contractions is still subject to debate [1-2]. We developed a model to simulate the control of motoneurons and the generation of muscle force during sustained isometric contractions. The model is a continuation of preliminary work done by Adam [3]. It provides a direct relationship between the excitation to the motoneuron pool of a muscle and the firing behavior of motor units. It generates the muscle mechanical response (force), which is altered with contraction times and the progression of muscle fatigue. It comprises a feedback loop to maintain the muscle force at a given target level. The model is based on recent physiological observations that have been obtained by our surface electromyographic (sEMG) decomposition technology [4]. It employs the concept of “common drive”, which states that all motor units in a pool are modulated by the same excitation [5]. Firing

characteristics follow the “onion skin” principle, which describes a hierarchical relationship between recruitment threshold and firing rates, with later-recruited motor units having progressively lower firing rates [6]. The model sheds light on the mechanisms implicated in the regulation of muscle force during sustained contractions; on the changes in motor unit firing rate and recruitment behavior; as well as on the cause of increased force fluctuation with fatigue.

## II. METHODS

The block diagram of the model is depicted in Fig. 1. Firing rate and force behaviors were simulated for the first dorsal interosseous (FDI) muscle of the hand and the vastus lateralis (VL) muscle of the quadriceps. The muscles were chosen because they have different recruitment and firing rate properties.

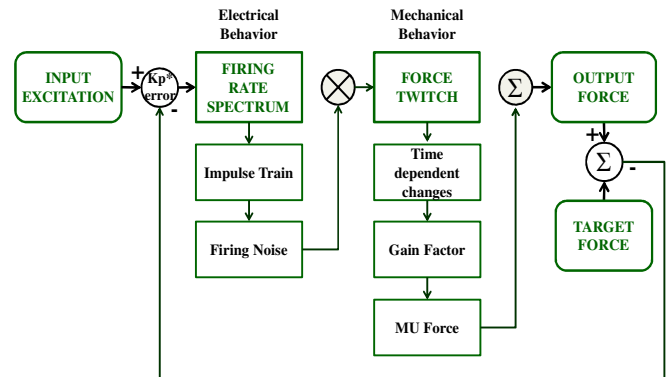


Fig. 1. The input excitation determines the electrical behavior of motor units. The mechanical behavior is then generated and summed to compute the total output muscle force, which is compared to the target force.

### A. Input

The input consists of an excitatory signal  $\phi$  common to all motor units in the pool (common drive), representing the excitation required to produce a given force level. In the absence of excitation ( $\phi=0$ ), no motor unit is active and no force is produced. As the excitation increases, additional motor units are recruited, the firing rates of the active motor units and the force output increase. The maximal level of excitation ( $\phi=1$ ) is the excitation required to exert the maximal voluntary contraction (MVC) force.

### B. Electrical characteristics of motor units

1) *Firing rate spectrum:* It provides a non-linear relationship between the excitation to the motoneuron pool

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and the firing rates of the active motor units. It was obtained by decomposing the sEMG signals detected during constant-force contractions performed at increasing target force level into the constituent motor unit action potentials [7]. A hierarchical structure of firing rate was observed at all force levels (onion skin). An equation was derived for describing the firing rate  $\lambda_i$  behavior of each  $i$ -th motor unit in the pool as a function of excitation  $\varphi$  and recruitment threshold  $\tau_i$ :

$$\lambda_i(\varphi, \tau_i) = D\varphi + (C - A \exp(-\varphi/B))\tau_i + E \quad (1)$$

with  $0 < \tau_i < 1$  and  $\tau_{i+j} > \tau_i$ . A, B, C, D, and E are muscle-dependent parameters with values 0.85, 0.32, -0.23, 6.93, 20.9 and 1.16, 0.15, -0.21, 8.03, 19.0 for the FDI and VL muscle respectively. The number of motor units in the pool ( $n$ ) is set to 120 for the FDI [8], and 600 for the VL muscle [9]. The range of recruitment (RR) varies from 0-67% and from 0-95% MVC in the FDI and VL muscles [7]. For the FDI muscle, the distribution of recruitment threshold in this range is modeled as an exponential of the form [10]:

$$\tau_i = \exp(ai), \text{ with } a = \ln(\text{RR})/n \quad (2)$$

A similar function with a shallower distribution is used for the larger VL muscle [11].

The firing rate spectrum is shown in the top panel of Fig. 2 for ten motor units progressively recruited as the excitation to the motoneuron pool increases between 10% and 90% of maximal excitation. The distribution of recruitment thresholds is reported in the bottom panel of Fig. 2.

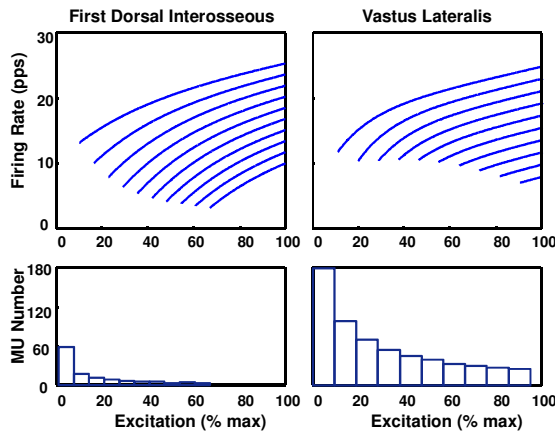


Fig. 2. Firing rate spectrum and distribution of recruitment thresholds for the FDI and VL muscles.

2) *Impulse train*: For each motor unit  $i$ , a pulse train is generated using the Integral Pulse Frequency Modulation method, which produces a spike train with frequency equal to the numerical value of the firing rate input.

3) *Noise*: Noise is added to each pulse in the train by modeling the inter-pulse intervals (IPIs) as Gaussian random variables with mean given by the predicted IPIs and

coefficient of variation  $CV=0.2$ . Each discharge time in the train is modified as [10]:

$$t_{i,j} = t_{i,j-1} + \mu + \sigma Z \quad (3)$$

where  $t_{i,j}$  is the time occurrence of the  $j$ -th discharge of motor unit  $i$ ;  $t_{i,j-1}$  indicates the preceding discharge;  $\mu$  is the mean firing rate of the impulse train;  $\sigma$  is the standard deviation (SD) of the IPIs; and  $Z$  is the Z-score, representing how far a generated value of IPI deviates from the mean of the distribution.  $Z$  is randomly picked between -3.9 and 3.9, allowing the IPIs to deviate less than four standard deviations from the mean of the distribution.

### C. Mechanical characteristics of motor units

1) *Force twitch*: For each motor unit, the model generates the force twitch, which is the force produced in response to a single pulse. It is described using three parameters: the peak amplitude  $P$ ; the rise time  $T_r$ , defined as the time from the start of the twitch to the peak value; and the half-relaxation time  $T_{hr}$ , which is the time from the peak value  $P$  to the point where the amplitude is reduced to  $P/2$ . Lower-threshold motor units produce lower-amplitude longer-duration twitches than higher-threshold motor units [12].

Values for the three parameters are obtained from the literature for the FDI muscle.  $P$  is modeled as a linear function of recruitment threshold over the recruitment range  $RR=130$  [10].  $T_r$  and  $T_{hr}$  are generated from a Weibull distribution with  $\text{mean} \pm \text{SD}$  of  $65 \pm 13$  ms and  $63 \pm 14$  ms respectively [13]. No data on individual motor unit twitches are available in the literature for the VL muscle. In one of our previous studies [2], the VL was electrically stimulated at supramaximal intensity to obtain the whole muscle twitch. The average whole muscle twitch  $P$  measured from three young healthy subjects was 46 N, the average  $T_r$  was 135 ms, and the average  $T_{hr}$  was 79 ms. In lack of more precise information, we assumed that similar exponential and Weibull distributions apply to both muscles. Values for  $T_r$  and  $T_{hr}$  were varied to generate different distributions of force twitches, which were summed to obtain the whole muscle twitch. We then chose the distributions which resulted in whole muscle twitch parameter values closest to the experimentally observed values.  $RP$  is set to 150; the  $\text{mean} \pm \text{SD}$  of the resulting distribution for  $T_r$  and  $T_{hr}$  are  $160 \pm 26$  ms and  $84 \pm 16$  ms. As more precise data for the force twitch distribution of the VL muscle become available, the model will be updated to reflect new evidences.

Force twitches are generated by using a mathematical equation which is a function of  $P$ ,  $T_r$ , and  $T_{hr}$  [14]:

$$\begin{aligned} f(t) &= pt^m e^{-kt} \\ p &= P \exp(-kT_r (\log T_r - 1)) \\ m &= kT_r \\ k &= \log 2 / [T_{hr} - T_r \log((T_r + T_{hr})/T_r)] \end{aligned} \quad (4)$$

The modeled motor unit twitches are reported in Fig. 3 for the motor units of the VL and the FDI muscles.

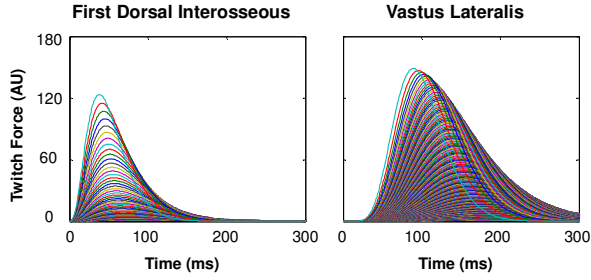


Fig. 3. Motor unit force twitches displayed in arbitrary units: the peak amplitude is set to 1 for the first recruited motor unit and to the peak range (130 and 150 for the FDI and VL) for the last recruited motor unit.

2) *Time dependent changes*: The force generating capacity of the muscle changes with contraction time: the amplitude of the force twitch increases at the beginning of a sustained contraction (potentiation), and later decreases as fatigue progresses. In our previous study [2], the peak tetanic force, measured during electrical stimulation of the VL muscle at 50 Hz, increased to approximately 1.08 of the initial value in the first 40 s of a 20% MVC isometric contraction, and decreased to 0.53 of the initial value as the contraction was repeated to exhaustion (reached after 8 min). The same protocol was applied to the FDI muscle (unpublished data): peak twitch amplitude increased to 1.20 of the initial value in the first 60 s, and then decreased to 0.40 of the initial value at endurance time (after 14 min). There was a tendency for  $T_r$  and  $T_{hr}$  to decrease with fatigue in both muscles, but the change was not significant or not consistent in all subjects.

Thus, in the model, values of  $T_r$  and  $T_{hr}$  are maintained constant. Only the peak twitch of the individual motor units is adjusted with contraction time: it increases to 1.08% and 1.2% of the initial value in the first 40 s and 60 s of activation, and subsequently decreases to 0.53% and 0.4% of the initial value after 8 min and 14 min for the VL and FDI muscle respectively.

3) *Firing rate dependent gain factor*: When the muscle is electrically stimulated at increasing frequencies, the generated force does not increase linearly with stimulus rate. The relation between the force generated by the individual motor units and the rate of firing presents a sigmoidal shape, which is similar for all motor units if the stimulus rate is normalized as a function of  $T_r$  [15]. In previous experiments [2], we electrically stimulated the FDI and the VL muscles at frequencies increasing from 1 to 100 Hz, and recorded the whole muscle force response. We obtained the force-frequency curves, and computed a gain function to scale the amplitude of the force twitch as a function of normalized rate  $f_n$ :

$$g_{ij}(f_n) = 0.4/f_n^{ij} (1-r)[1-r \exp([0.4-f_n]/c)], f_n > 4 \quad (5)$$

$$g_{ij}(f_n) = 1, 0 < f_n < 4$$

where  $g_{ij}$  is the gain assigned to the  $j$ -th firing of motor unit  $i$  and  $f_{n_{ij}}$  is the normalized instantaneous firing rate,  $f_{n_{ij}} = T_r / IPI_j$  ( $T_r$  is the rise time of motor unit  $i$  and  $IPI_j$  is the  $j$ -th interpulse interval). The values of the parameters used in (5) are  $r=0.87$  and  $c=2.82$ ,  $r=0.85$  and  $c=2.13$  for the FDI and VL muscle.

#### D. Force output

The time-varying force produced by the individual motor units is computed by convolving the impulse train with the motor unit force twitch. The total muscle force is then calculated by summing the individual responses of the active motor units  $k$ :

$$F_i(t) = \sum_j f_{ij}(t - t_{ij}) \quad (6)$$

$$F_{tot}(t) = \sum_k F_k(t)$$

where  $t_{ij}$  is the  $j$ -th firing time of motor unit  $i$ ,  $f_{ij}$  is the force twitch of motor unit  $i$  at the time of the  $j$ -th firing,  $F_i(t)$  is the resulting force output of motor unit  $i$ , and  $F_{tot}$  is the total output muscle force.

Lastly, the total output force is low-pass filtered at a cutoff frequency of 5 Hz to introduce the filtering effect of the muscle tissues.

#### E. Feedback loop

The feedback loop adjusts the value of the input excitation so that the output force is maintained at a value similar to that of the target force. This is achieved by computing the output force in interval  $dt=0.5$  s duration, and calculating the difference between the mean output force and the mean target force in the interval  $dt$ . If the error surpasses a predetermined threshold (set at 5% of the target force), the excitation is adjusted proportionally to the error: it is either increased or decreased until the output force matches the target force. This step is repeated until the error is within limits, at which point the simulation proceeds to the following time interval  $dt$ . The sample time is 1 ms.

### III. RESULTS

We modeled the force produced by the FDI and VL muscles during a series of repeated consecutive force contractions sustained at 20% MVC. The model behavior was validated by comparing the simulated force output with experimental evidences derived from a similar protocol of repeated contractions performed at the same target force level [2, 16]. We also modeled the force produced by the two muscles in the absence of feedback control.

The top and middle panels of Fig. 4 show the muscle force output (simulated with force feedback) during repetition #1 and #63 of the contraction series for the VL muscle and during repetition #1 and #40 for the FDI muscle. For the FDI, the duration of each repeated trajectory was 40 s. It was

decreased to 20 s for the VL muscle to reduce the simulation time and computation burden given by the greater number of motor units. The model was able to produce a force maintained at a constant value of 20% MVC. For both muscles, an increase in the fluctuations of the force can be observed as fatigue develops, likely due to the recruitment of higher-threshold higher-twitch amplitude motor units. The results of the simulations agree with previous observations [2, 16]. The force produced by the VL muscle is smoother than that produced by the FDI muscle at any time during the contraction series, probably due to the greater number of active motor units, greater range of recruitment, and longer duration of the force twitches in the quadriceps muscle.

The bottom panel of Fig. 4 shows the results for the FDI muscle in the simulation without feedback control. In this paradigm, the excitation is maintained at a constant value producing a 20% MVC force at the onset of the contraction. As the contraction progresses, the excitation remains constant, whereas the force cannot be sustained at the initial level: it increases as the force twitches potentiate and later decreases as the muscle fatigues. The force becomes smoother, since the number of active motor units does not change but their force twitches progressively decrease in amplitude. Similar results were obtained with the VL muscle.

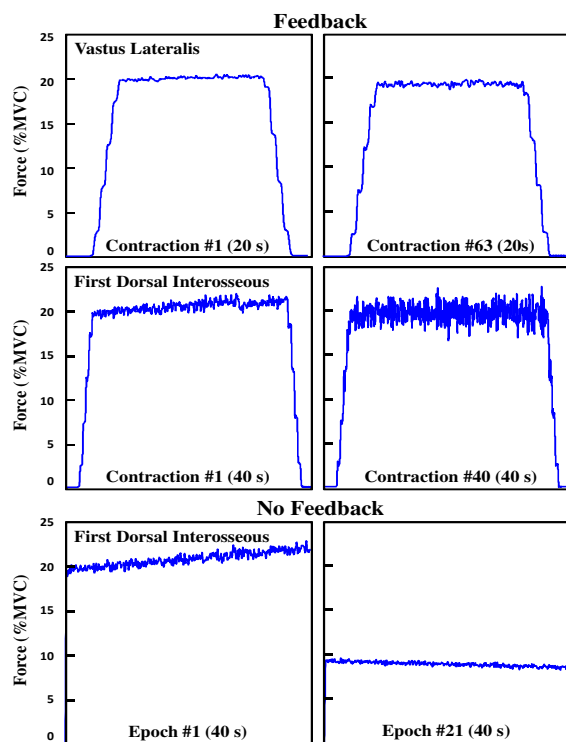


Fig. 4. The force output simulated at the beginning and further into the contraction series is shown with active feedback for the VL and FDI muscle (top and middle panel) and with no feedback for the FDI muscle only (bottom panel).

#### IV. DISCUSSION

When feedback is active, the model predicts the increased force fluctuations that occur during sustained contractions.

The peak twitch force of each motor unit was the only parameter allowed to change during the sustained contraction time. Different feedback controls can be tested. However, a simple feedback control, where the excitation is adjusted proportionally to the force error, is able to maintain the output force at the required target level despite changes in the force twitches. When no feedback is introduced, the force cannot be maintained at a constant level.

The results strongly suggest that, during voluntary isometric fatiguing contractions, the excitation to the motoneuron pool is adjusted to compensate for the varying muscle-force generating capacity.

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