

Robust Identification of Multi-Joint Human Arm Impedance Based on Dynamics Decomposition: A Modeling Study

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Abstract—Multi-joint/multi-degree of freedom (DOF) human arm impedance estimation is important in many disciplines. However, as the number of joints/DOFs increases, it may become intractable to identify the system reliably. A robust, unbiased and tractable estimation method based on a systematic dynamics decomposition, which decomposes a multi-input multi-output (MIMO) system into multiple single-input multi-output (SIMO) subsystems, is developed. Accuracy and robustness of the new method were validated through a human arm and a 2-DOF exoskeleton robot simulation with various magnitudes of sensor resolution and nonlinear friction. The approach can be similarly applied to identify more sophisticated systems with more joints/DOFs involved.

I. INTRODUCTION

THIS paper proposes a robust, unbiased, and yet tractable linear stochastic estimation of human arm multi-joint/multi-DOF impedance transfer function matrix (TFM) based on a systematic dynamics decomposition method (Fig. 1).

Extensive research has been carried out with the aim of identifying multi-joints/DOFs impedance TFM [1-7]. Most of the studies utilized linear stochastic estimation method with force perturbations [1-3, 5-7] owing to a number of advantages of stochastic estimation compared with previous methods [8-11]. These advantages include that a model structure is not assumed, although the system is assumed to behave *linearly* for small perturbations; that the unpredictable stochastic perturbations minimize the likelihood of voluntary reactions; and that the perturbations obviate the need for separate measurements in different directions and provide a frequency-rich input to the subject in a relatively short time [1, 2, 5, 7].

Although these researches succeeded in estimating multi-joints/DOFs impedance TFM, practically, it may become inapplicable with the increase of number of joints/DOFs to be identified. Even in the case of 2 or 3 DOF, human arm dynamics becomes *intractably complex* [4, 11]. Moreover, to estimate an n DOF impedance TFM, it will be shown in section II that, even with the most recent multiple-input multiple-output (MIMO) estimation method

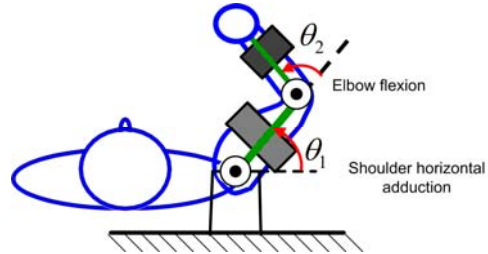


Fig. 1. Schematic diagram of simulation conditions. The human shoulder and elbow joints are assumed to be perfectly aligned and firmly connected with those of the exoskeleton with torque transducers.

[7], $2n^2$ transfer functions (TF) are needed to be computed *simultaneously* without numerical errors. Furthermore, in order to use a MIMO estimation method [1, 2, 5, 7], coupling(s) between elements of perturbation force vector (input coupling) should be minimized to reduce the estimation error [1, 5, 7, 12].

Besides, under force perturbations, it is important to use a closed-loop dynamics (human arm and robot) identification [2, 5-7, 13] for the unbiased estimation [12-14]; and to compensate for nonlinear friction in robot joints [15-17], which degrades estimation accuracy and reliability [2, 5-7].

Therefore, it is our goal to propose a robust, unbiased, and most importantly tractable human arm impedance TFM estimation method. Specifically, we propose an estimation method that enables us to estimate an unbiased and accurate human arm multi-joints/DOFs impedance TFM regardless of nonlinear friction in robot joints and sensor resolution while keeping the complexity of the method low enough to put into practice even with the increase of number of joints/DOFs.

The *key idea* is to *systematically* decompose the complex dynamics into manageable multiple single-input multiple-output (SIMO) subsystems – essentially a set of single-input single-output (SISO) systems – using internal model based impedance control (IMBIC) [5, 6, 18] and to apply then a SIMO identification, which can be easily found in literatures [12], on each decomposed subsystem.

Accuracy and robustness against nonlinear friction and sensor noise of the proposed identification method were verified through simulations with a 2 DOF human arm model [10] and a 2 DOF exoskeleton robot model together with the widely used LuGre friction model [5, 16, 17] (Fig. 1).

This paper is structured as follows. In section II, we propose the dynamics decomposition method. Section III provides combined use of stochastic estimation and IMBIC for unbiased and robust human arm impedance TFM

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estimation. Section IV presents accuracy and robustness of proposed estimation method through simulations. Finally, in section V, we summarize the results and draw conclusions.

II. SYSTEMATIC DYNAMICS DECOMPOSITION WITH IMBIC

A. Human Arm Dynamics

To investigate the robustness of the proposed estimation method against nonlinear friction and sensor noise only, a linear human arm model, which was verified in previous studies [1-3, 5, 9, 10], was considered.

$$\boldsymbol{\tau}_h = \mathbf{Z}_h(s) (\boldsymbol{\theta}_h - \boldsymbol{\theta}_0), \text{ where} \quad (1)$$

$$\mathbf{Z}_h(s) = \mathbf{M}_h s^2 + \mathbf{B}_h s + \mathbf{K}_h, \text{ and} \quad (2)$$

$\boldsymbol{\tau}_h = [\tau_{h1} \tau_{h2} \dots \tau_{hn}]^T \in \mathfrak{R}^n$ denotes the torque vector applied to the human arm joints; $\mathbf{Z}_h \in \mathfrak{R}^{n \times n}$ the human arm joint impedance TFM; s the Laplace variable; $\boldsymbol{\theta}_h = [\theta_1 \theta_2 \dots \theta_n]^T \in \mathfrak{R}^n$ the human arm joint angle vector; $\mathbf{M}_h, \mathbf{B}_h, \mathbf{K}_h \in \mathfrak{R}^{n \times n}$ denote the human arm joint space inertia, damping, and stiffness matrices, respectively, defined at the constant equilibrium position vector $\boldsymbol{\theta}_0 \in \mathfrak{R}^n$. Without loss of generality, $\boldsymbol{\theta}_0$ can be set to be a zero vector and (1) can then be written as follows

$$\boldsymbol{\tau}_h = \mathbf{Z}_h(s) \boldsymbol{\theta}_h. \quad (3)$$

B. Robot Under IMBIC

The goal of IMBIC [6, 18] is, regardless of nonlinear friction in robot joints, to replace the nonlinear robot dynamics with the following linear desired dynamics:

$$\mathbf{M}_{rd} \ddot{\boldsymbol{\theta}}_r + \mathbf{B}_{rd} \dot{\boldsymbol{\theta}}_r + \mathbf{K}_{rd} (\boldsymbol{\theta}_r - \boldsymbol{\theta}_{rd}) = -\boldsymbol{\tau}_h + \boldsymbol{\tau}_\Delta, \quad (4)$$

where $\mathbf{M}_{rd}, \mathbf{B}_{rd}, \mathbf{K}_{rd} \in \mathfrak{R}^{n \times n}$ denote the desired inertia, damping, and stiffness matrices of the desired linear model; $\boldsymbol{\theta}_r, \dot{\boldsymbol{\theta}}_r, \ddot{\boldsymbol{\theta}}_r \in \mathfrak{R}^n$ the robot joint angle and its first and second time derivatives, respectively; $\boldsymbol{\theta}_{rd} \in \mathfrak{R}^n$ the constant desired test location vector and may be equal to $\boldsymbol{\theta}_0$ in the case of unimpaired subjects within the ROM [2, 5]; $\boldsymbol{\tau}_\Delta \in \mathfrak{R}^n$ the vector of torque perturbations. In the Laplace domain, after simple manipulation of (4), we obtain

$$\mathbf{Z}_{rd}(s) (\boldsymbol{\theta}_r - \boldsymbol{\theta}_{rd}) = -\boldsymbol{\tau}_h + \boldsymbol{\tau}_\Delta, \quad (5)$$

where $\mathbf{Z}_{rd} \in \mathfrak{R}^{n \times n}$ denotes the desired impedance TFM of the robot and is defined as

$$\mathbf{Z}_{rd}(s) = \mathbf{M}_{rd} s^2 + \mathbf{B}_{rd} s + \mathbf{K}_{rd}. \quad (6)$$

Similar to (3), without loss of generality, using desired admittance TFM \mathbf{Y}_{rd} – the inverse of \mathbf{Z}_{rd} and whose i, j -th element is Y_{rij} – (5) can be written as follows

$$\boldsymbol{\theta}_r = \mathbf{Y}_{rd}(s) (-\boldsymbol{\tau}_h + \boldsymbol{\tau}_\Delta). \quad (7)$$

By applying IMBIC to the nonlinear robot dynamics, the robot can closely follow the dynamic behavior of desired linear dynamics (5) (or (7)). Details of IMBIC including superior desired impedance realization accuracy, experimentally confirmed friction compensation performance, tuning procedure can be found in [6, 18].

Since $\boldsymbol{\tau}_h$ is measured torque, by replacing the measured

torques of some joints in (4) with zero values, almost infinite impedance can be realized for those joints.

C. Systematic Dynamics Decomposition

Since no relative motion exists between the human arm and the robot (i.e., $\boldsymbol{\theta}_r = \boldsymbol{\theta}_h$), (3) and (7) represent the closed-loop dynamics (i.e., human arm and the apparent robot dynamics).

To perturb k^{th} DOF only, all elements of \mathbf{Y}_{rd} , except Y_{rkk} , and all elements of $\boldsymbol{\tau}_\Delta$, except $\tau_{\Delta k}$, must be zero

$$Y_{rij} = 0, \text{ if } i \neq k \text{ or } j \neq k, \text{ and} \quad (8)$$

$$\boldsymbol{\tau}_\Delta = [\mathbf{0}_{1 \times (k-1)} \tau_{\Delta k} \mathbf{0}_{1 \times (n-k)}]^T. \quad (9)$$

Substitution of (8) into (7) and simple manipulation yield

$$\theta_i = \begin{cases} 0 & \text{if } i \neq k \\ Y_{rkk} (-\tau_{hk} + \tau_{\Delta k}) & \text{if } i = k \end{cases}. \quad (10)$$

With the substitution of apparent robot dynamics (7) into (3) with the values of \mathbf{Y}_{rd} in (8) and $\boldsymbol{\tau}_\Delta$ in (9), and some simple manipulations, one can get

$$\tau_{hi} = T_{ik} \tau_{\Delta k}, \text{ where} \quad (11)$$

$$T_{ik} = Z_{hik} Y_{rkk} (1 + Z_{hkk} Y_{rkk})^{-1}. \quad (12)$$

Substituting (11) and (12) into (10) yields

$$\theta_k = R_{kk} \tau_{\Delta k}, \text{ where} \quad (13)$$

$$R_{kk} = Y_{rkk} (1 + Z_{hkk} Y_{rkk})^{-1}. \quad (14)$$

This procedure can be repeatedly applied for all joints/DOFs needed to be identified in this *systematic* manner.

From (12) and (14), one can see that the whole n DOF closed-loop dynamics after the decomposition is described with a matrix $\mathbf{T} \in \mathfrak{R}^{n \times n}$ whose i, k -th element is T_{ik} , and a vector $\mathbf{R} \in \mathfrak{R}^n$ whose k^{th} element is R_{kk} , whereas original dynamics given in (3) and (7) needs two matrices $\mathbf{Z}_h \in \mathfrak{R}^{n \times n}$ and $\mathbf{Y}_{rd} \in \mathfrak{R}^{n \times n}$. Thus, after the decomposition, the same dynamics can be described with fewer elements. Further, after the decomposition, still all the elements of human arm impedance TFM \mathbf{Z}_h can be found in \mathbf{T} . In other words, the decomposition preserves all the information of the human arm impedance.

Since the complex n DOF closed-loop MIMO dynamics is *systematically* decomposed into n SIMO systems, essentially any SISO identification method can be applied.

III. AN UNBIASED ROBUST HUMAN ARM IMPEDANCE ESTIMATION METHOD

In this section, an unbiased robust estimation method, specifically designed for the decomposed system, is proposed.

A. Estimation of Human Arm Impedance

With close examination of T_{ik} in (12) and R_{kk} in (14), one can find that

$$T_{ik} = Z_{hik} R_{kk}. \quad (15)$$

Thus, if one can compute \hat{R}_{kk} , estimate of R_{kk} , and \hat{T}_{ik} , estimate of T_{ik} , one can then compute \hat{Z}_{hik} , estimate of human arm impedance TF Z_{hik} , as follows

$$\hat{Z}_{hik} = \hat{T}_{ik} \hat{R}_{kk}^{-1}. \quad (16)$$

B. Computation Method

In the case of the proposed method, because the input $\tau_{\Delta k}$ is known exactly without any input noise, *without any bias*, \hat{R}_{kk} – estimate of TF from $\tau_{\Delta k}$ to θ_k – and \hat{T}_{ik} – estimate of TF from $\tau_{\Delta k}$ to τ_i – can be computed as follows [12]

$$\hat{R}_{kk} = G_{\tau_{\Delta k} \hat{\theta}_{k-k}} G_{\tau_{\Delta k} \tau_{\Delta k}}^{-1}, \text{ and} \quad (17)$$

$$\hat{T}_{ik} = G_{\tau_{\Delta k} \hat{\tau}_{i-k}} G_{\tau_{\Delta k} \tau_{\Delta k}}^{-1}. \quad (18)$$

Here $\hat{\theta}_{k-k}$ and $\hat{\tau}_{i-k}$ denote measured k^{th} joint angle and i^{th} joint torque with uncorrelated noise, respectively, when k^{th} joint is perturbed. Substituting (17) and (18) into (16) gives us human arm impedance (\hat{Z}_{hi}). Note that, thanks to the decomposition, it is not necessary to check the input coupling.

Similar to [7], a derived ordinary coherence function for each sub-system was derived as a reliability measure with the assumption that noise output spectrum for the proposed estimation is equal to that of the direct estimation. The higher the coherence, the more reliable the estimate.

IV. SIMULATION STUDY

A. Simulation Condition

For simulations, 2 DOF linear human arm model was used with the dynamic and kinematic variables in [10] (Table I).

Dynamic parameters of MIT-MANUS were used for exoskeleton robot [5, 19, 20], and link lengths of exoskeleton are set to be the same as those of subject's arm [10].

To verify the robustness against nonlinear friction, LuGre friction model, which is widely used in robotics area, has been employed [5, 16, 17]. Four different levels, given in [5], were tested. Further, four different levels of resolution (Table II) have also been employed to evaluate the robustness against noise. In each case, the same parameters (both friction and resolution) were applied to the two joints.

Random torque perturbations (τ_{Δ}) were generated for each joint by filtering a uniformly distributed random signal with an 8th order Butterworth low pass filter with a 15Hz cut-off frequency by following [5, 7]. Magnitude of perturbation may affect estimation performance [3, 12]. Thus, three levels (Table III) of perturbations were applied.

Fourth order Runge-Kutta method with 0.01ms time step was used for a reliable simulation by following [21]. Sampling times for control and data acquisition were set to be 1 ms.

Spectral analysis parameters are given in Table IV. Trials lasted for 50 s (50,000 data points), allowing a number of sequential epochs of data to be averaged to reduce random error while allowing an acceptable spectral resolution.

B. Simulation Results

Estimated human arm impedance TFMs are always accurate regardless of resolution and friction (Fig.2).

From Fig. 4, one can see that regardless of friction level, resolution level, and perturbation magnitude, estimation results were always reliable because ordinary coherence

TABLE I
PARAMETERS OF HUMAN ARM DYNAMICS ADOPTED FROM [10] (THEIR TABLE 3 LOCATION 1 SUBJECT A, RESPECTIVELY)

Joint angle (deg)	M_h (Kg·m ²)		B_h (Nm·s/rad)		K_h (Nm/rad)	
$\theta_1=62.708$	0.187	0.084	0.651	0.239	7.967	3.187
$\theta_2=77.260$	0.079	0.060	0.236	0.407	1.663	6.901

TABLE II
FOUR DIFFERENT LEVELS OF SENSOR RESOLUTION

Resolution Level	Encoder (deg)	Torque sensor (Nm)
0 (No resolution)	0	0
1	1.758e-3	0.005
2	3.516e-3	0.010
3	7.031e-3	0.020

TABLE III
PEAK MAGNITUDE OF THREE LEVELS OF PERTURBATIONS

Perturbation Level	$\tau_{\Delta 1}$ (Nm)	$\tau_{\Delta 2}$ (Nm)
1	3	1.8
2	6	3.6
3	9	5.4

$\tau_{\Delta 1}$: shoulder perturbation; $\tau_{\Delta 2}$ elbow perturbation, respectively.

TABLE IV
SPECTRAL ANALYSIS PARAMETERS

N_{FFT}	N_{WND}	N_{OVL}	f_r (Hz)
8192	8192	6144	0.122

N_{FFT} number of data points included in the FFT calculation; N_{WND} the length of the hanning window function; N_{OVL} the number of overlapping samples; f_r minimum resolvable frequency.

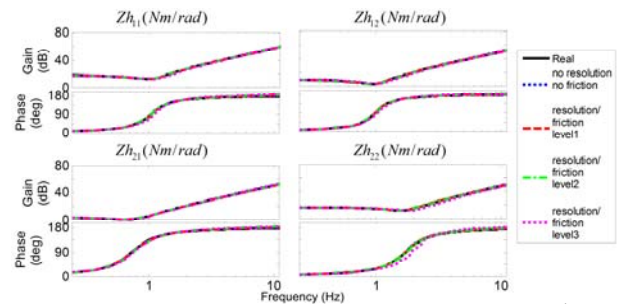


Fig. 2. Real and estimated human arm impedance TFMs with 3rd level torque perturbations under 4 different levels of friction and resolution. Regardless of nonlinear friction in robot joints and sensor resolutions, human arm TFM is accurately estimated with the proposed estimation method.

functions of all four elements are always close to unity. Further, in Fig. 3, VAFs are always close to 100% and R^2 is close to unity. In short, from Figs. 2-4, it can be concluded that the proposed estimation method combined with IMBIC is robust against nonlinear friction in robot joints and sensor resolution, which are some of the most practical sources of error.

V. CONCLUSIONS

A robust, unbiased, and tractable human arm impedance estimation method based on systematic dynamics decomposition is proposed. Through realistic simulations, it is validated that the proposed method enables us to estimate human arm impedance TFM reliably and accurately regardless of magnitude of nonlinear friction and sensor resolution.

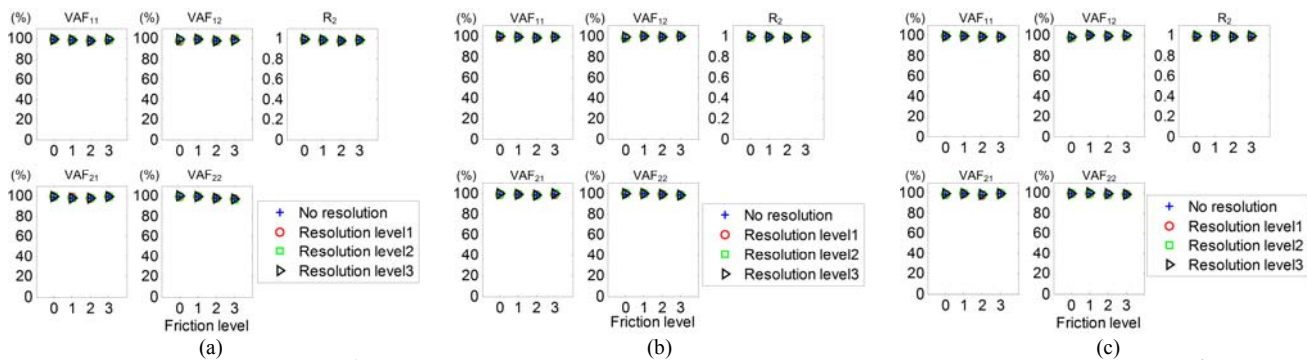


Fig. 3. Accuracy measures, VAFs and R^2 , under 4 different friction levels and resolution levels; (a) with 1st level perturbations; (b) with 2nd level torque perturbations; (c) with 3rd level torque perturbations.

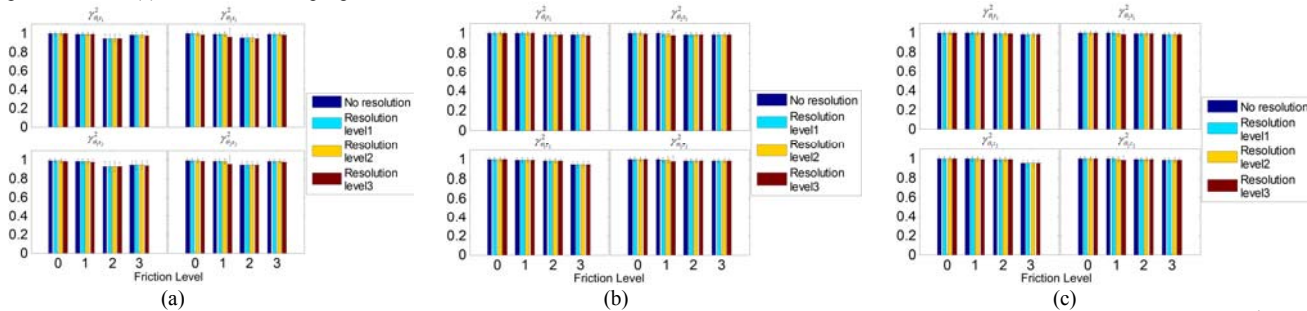


Fig. 4. Reliability measure, mean of ordinary coherence functions, that correspond to Fig.3. (a) with 1st level perturbations; (b) with 2nd level perturbations; (c) with 3rd level perturbations.

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