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T Waves Segmentation and Analysis Using Inverse Normalized Integrals

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Abstract

Stress tests are typical protocols that exhibit T waves changes for healthy and ischemic subjects. ST-T elevation is one example among others that characterizes ischemic patient. A more complex description of T wave changes needs extended models with a cost of higher identification pitfalls. We propose here to work in the inverse normalized integrals domain. This domain permits us an estimation of all pertinent parameters for the characterization of T waves shape. These parameters stand for the delay, the width (defined in the paper as the scaling factor) and the offset. In contrast, direct application of PCA on the data (defined in the paper as time-domain approach) assumes perfectly aligned waves with identical widths. Time-varying parameters values corresponding to exercise tests are estimated by using both approaches. It is shown that results are consistent with clinical knowledge.

1. Introduction

It as been shown in [1] that T waves from the ECG can be characterized by a set of parameters that can be estimated in the normalized integral domain. Each observation, indexed by i, of the T waves set has been modeled as:

$$x_i(t) = k_i s(\alpha_i t - d_i) + n_i(t) \;\; with \;\; \alpha_i > 0; k_i > 0 \quad (1)$$

, with k_i, α_i, d_i the amplitude coefficient, the scaling factor and the delay or shift, respectively. $s(t)$ is assumed to be a deterministic unknown signal and the noise $n(t)$ will be omitted in the following for the sake of clarity. In practice, the estimation is unbiased under the assumption that the waves are correctly segmented. Unfortunately, a procedure that guarantees this perfect segmentation doesn't exist because the wave of interest is not fully described and may vary from beat to beat. Additionally, this model can be extended by introducing an offset in (1). We propose here to address the problem of segmentation and offset estimation in the same domain than in [1]. The relevance of this extended model and the proposed solution is demonstrated by characterizing the T waves of one patient with ischemia and one healthy volunteer, during exercise test carried out on ergometer.

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2. Mathematical developments

When using model (1) and omitting the noise, the normalized integral of $s(t)$ and $x_i(t)$ are computed by :

$$S(t) = \left(\int_0^t s(u)du\right) / \left(\int_0^T s(u)du\right) \quad (2)$$

$$X_i(t) = \left(\int_0^t x_i(u)du\right) / \left(\int_0^T x_i(u)du\right) \quad (3)$$

These functions are strictly increasing assuming the positivity of the observations. This assumption is easily verified when recording T waves with the proper leads. From (1), $x_i(t)$ is related to $s(t)$ by the application of an increasing affine function called φ_i, that implies:

$$X_i = S \circ \varphi_i \Leftrightarrow X_i(t) = S(\varphi_i(t)) \;\; \text{with} \;\; 0 \le t \le T \quad (4)$$

The functions S and X_i being increasing, for any value of t we get:

$$y = S(t) = X_i(t_i) \Leftrightarrow t = S^{-1}(y) \;\; \text{with} \;\; t_i = \psi_i(t) \quad (5)$$

According to (1), we have the relation:

$$t_i = \frac{S^{-1}(y)}{\alpha_i} + \frac{d_i}{\alpha_i} \quad (6)$$

When the y axis is sampled with a sampling period δ_y, the values of t_i that correspond in the continuous case to $t_i = X_i^{-1}(y)$ are gathered in a vector

$\mathbf{t}_i = [X_i^{-1}(0)\ X_i^{-1}(\delta_y) \cdots X_i^{-1}(1)]$. Using the vector formulation relation (6) is replaced by:

$$\mathbf{t}_i = a_i\mathbf{v} + b_i\mathbb{1} = \frac{1}{\alpha_i}\mathbf{t} + \frac{d_i}{\alpha_i}\mathbb{1} \tag{7}$$

where $\mathbf{t}$ and $\mathbb{1}$ stand for the sampled $S^{-1}(y)$ that is unknown but common to all observations and the unit vector, respectively. Considering all the observations, not only the set of parameters (a_i,b_i) has to be estimated but also vector $\mathbf{t}$. In order to solve this problem we propose to decorrelate the estimation of the a_i's and b_i's by imposing orthogonality of $\mathbf{t}$ and $\mathbb{1}$. This is simply achieved by zeroing the mean of each $\mathbf{t}_i$. This leads to a two stage estimation: estimation of the a_i's and $\mathbf{t}$ followed by the estimation of the b_i's.

Using the entire set of observations, the first estimation solves the minimization:

$$\check{\mathbf{t}} = arg\ \min_{\mathbf{t}}(\sum_i \|\mathbf{t}_i - a_i\mathbf{t}\|^2) \tag{8}$$

with theoretically unique a_i's. Imposing the constraint $\mathbf{t}^T\mathbf{t} = 1$ leads to the equivalent problem:

$$\check{\mathbf{t}} = arg\ \max_{\mathbf{t}} \mathbf{t}^T\mathbf{R}\mathbf{t} \tag{9}$$

where $\mathbf{R}$ stand for the correlation matrix of the observations $\mathbf{t}_i$'s. The solution is given by the eigenvector decomposition of the matrix $\mathbf{R}$ where the estimation $\check{\mathbf{t}}$ corresponds to the first eigenvector. In order to derive this decomposition as an equivalent Principal Component Analysis, a matrix $\mathbf{T}$ is defined as $\mathbf{T} = [\mathbf{t}_1 \cdots \mathbf{t}_N]$ and the PCA is computed such that $\mathbf{T} = \mathbf{V}\Sigma\mathbf{U}'$. The first column of $\mathbf{V}$ is thus the normalized $\check{\mathbf{t}}$.

The solution in (8) is equivalent to a weighted averaging when the model (1) is relevant. When it is no longer appropriate this analysis provides us with a global description of the mutual information shared by the $\mathbf{t}_i$'s with the property that the first column of $\mathbf{V}$, namely $\mathbf{v}_1$, plays the role of the mean shape. Let us recall that the transformation that is under the scope of this paper is an affine function applied to the time axis of a reference shape. In other words, we model each realization of the random process (1) as a shifted and scaled in time version of a reference shape. The model corresponding to (8) doesn't exhibit the time shift but the scaling factor. It means that the information related to this parameter will be shared by all the eigenvectors from the PCA. The vector θ_i corresponding to the time scale and shift coefficients for the realization i is estimated by using a least square minimization whose solution is given by:

$$\hat{\theta}_i = [\mathbf{v}_1\mathbb{1}]^{\#}\mathbf{t}_i \tag{10}$$

The first and second components of $\hat{\theta}_i$ will correspond to the scaling and shift parameters, respectively. The scale and delays parameters refer to a virtual vector and not to a given observation from the dataset, the first for instance. How to relate any $\mathbf{t}_i$ to $\mathbf{t}_1$ assuming the affine (6)? From (10), we have the relation $\mathbf{t}_1 = a_1\mathbf{v}_1 + b_1\mathbb{1}$ and thanks to the property of the model formulated in the inverse normalized integral we have $\mathbf{t}_i = \frac{1}{\alpha_i}\mathbf{t}_1 + \frac{d_i}{\alpha_i}\mathbb{1}$ but also $\mathbf{t}_i = a_i\mathbf{v}_1 + b_i\mathbb{1}$. By using appropriate substitution in these expression, we get:

$$\alpha_i = \frac{a_1}{a_i} \tag{11}$$

$$d_i = \alpha_i b_i - b_1 \tag{12}$$

Then, scale and delay parameters estimated by using (10) can be transformed in order to refer to the first observation. This transformation allows to track the continuous effect, increasing exercise for instance, of the experiment onto the signals under the scope with respect to causality. This estimation relies on a correct initial wave segmentation. Unfortunately it is unlikely to record ECG containing waves that fulfill conditions for a good estimation. The use of the normalized integral assumes that waves vanish on the bounds of the segmentation window. We suggest to adapt the segmentation location to the criteria that is being minimized in (8). The objective is to find the best location of the window to reduce the number of significant vectors in $\mathbf{V}$ that explain the entire set of data. In other words, each observation $x_i(t)$ is transformed such that $\tilde{x}_i(t) = x_i(t + dec_i + I)$, where dec_i corresponds to the adjustment delay. This adjustment is estimated by maximizing the criteria $\lambda_1^2/\sum_i \lambda_i^2$. The λ_i's correspond to the singular values sorted in descending order computed with the matrix $\mathbf{T}$ obtained with the transformed observations. When the model (1) is verified for all segments, and omitting the noise, the criteria should be equal to one since only the first singular value is different than zero.

Using the proposed estimation of the a_i's and b_i's and the relations (11), (12), the observations $\tilde{x}_i(t)$ can be referred to $\tilde{x}_1(t)$ using the relation $\tilde{x}_i(t) = \tilde{x}_1(\alpha_i t - d_i)$. The difficulty that has to be solved is to return to the original observations x_i using the previous development. In summary, we have at disposal the two relations:

$$\begin{cases} \tilde{x}_i(t) = x_i(t + dec_i + I) \\ \tilde{x}_i(t) = \tilde{x}_1(\alpha_i t - d_i) \end{cases}$$

Using these relations we get $x_i(t + dec_1 + I) = x_1(\alpha_i t - d_i + dec_1 + I)$. Comparing this relation with $x_i(t) = x_1(\gamma_i t - \beta_i)$, we finally get:

$$\begin{cases} \gamma_i = \alpha_i \\ \beta_i = \alpha_i dec_i + d_i - dec_1 - (1 - \alpha_i)I \end{cases}$$

An additional difficulty is related to the baseline wander exhibited in any stress test records. Albeit a high-pass filtering is applied to the ECG records, a residual component still reduces the estimation performance by increasing the bias.The proposed solution consists in adding a varying offset such that the minimum value in the segmentation window is maintained greater than zero during the minimization of $J = \lambda_1^2 / \sum_i \lambda_i^2$. In summary, the global minimization problem is solved by using a combination of an alternated least square with an iterative approach such in [5]. The first step consists in iteratively minimizing J with a grid search for each observation, with respect to the offset. The second step deals with the adjustment delays dec_i by using an equivalent scheme. This process is repeated until convergence.

An alternative to the inverse normalized integral is to process the data in the time domain by using similar approach. In contrast to the use of the normalized integral, conventional PCA applied in the time domain relies on the perfect alignment of signals in addition to a constant scaling factor. Furthermore it is not affected by the addition of a variable offset by using a correct modeling. Since a time delay is expected during the stress test protocol, a grid search will be introduced during the minimization of the criteria similar to J. This corresponds to an extension of an usual approach when faced with such observations. The presence of a probable scaling factor is not addressed in the time domain because it needs interpolation unlike the inverse normalized integrals where this parameter appears linearly in the model.

3. Application

The ECG signals were measured at rest and during an exercise test carried out on ergometer. The 67-channel high-resolution ECG measurement system was used. Leads were located according to the University of Amsterdam lead system [2],[3]. ECG signals were acquired with 4096 Hz sampling frequency, and digitized at 24-bit resolution [4]. ECG signal from precordial lead V5 was used in this study. The multi-stage protocol was used. After obtaining 10 minutes of ECG recorded at rest, patients started to pedal at constant speed with load of 50 W which was increased by 25 W every 2 minutes. Tests were terminated in case of chest pain, fatigue, arrhythmias, or marked ST-T segment change. A test was considered negative only when the 85% of predicted maximum heart rate was achieved by patient and there was no distinct positive ST-T changes. One patient with ischemia and one healthy volunteer with no history of cardiovascular disease were analyzed. In order to reduce the influence of the baseline wander, each R-R interval has been corrected such that the silent interval (P-Q) is zeroed. For this analysis, an ensemble of 500 consecutive beats during the exercise are recorded and one T wave out of every ten is considered in order to reduce the amount of data to process. Thus, 50 T waves coarsely pre-segmented with respect to the R wave location are processed for each subject. The R-R intervals are given in fig. 1. Comparing fig. 2 and fig. 5, we can notice that the estimations of the delays are similar for both approaches but different for the two subjects. It is expected that during exercise, QT intervals of healthy subject decrease while ischemic patient exhibits the inverse trend. The trends of the offsets plotted in fig. 6 correspond to the ST-T elevation expected for ischemic subject while the healthy one doesn't exhibit any particular trends (see fig. 3). In contrast to the healthy case, the ischemic subject presents a scaling factor in fig. 7 corresponding to a widening of the T waves as long as the exercise increases. Because ground truth is lacking it is difficult to consider the accuracy of such results. However, the proposed method being time-scale invariant unlike the time-domain approach, we assume that the offset in fig. 6 (thick line) corresponds to the typical ST-T elevation for ischemic subjects.

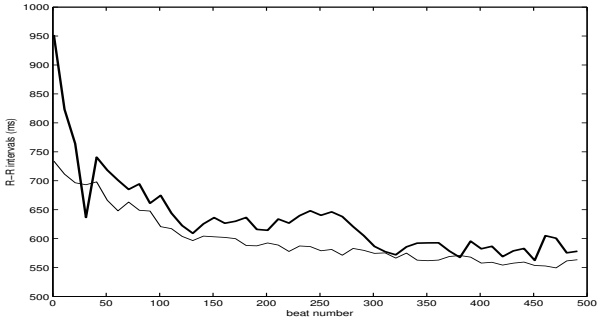

Figure 1. R-R intervals of the healthy (thick line) and ischemic (thin) subjects

4. CONCLUSIONS

The model of T wave observations has been extended in order to account for physiological elevation. While this model is more complete, the segmentation issue still remains because of the variability of the T wave shape. We have proposed a global procedure that estimates the parameters of interest with respect to clinical expectations. Examples on healthy and ischemic subject show that conventional approaches that works in the time domain can be replaced by the proposed one, at least because it provides an additional parameter that is the scaling factor.

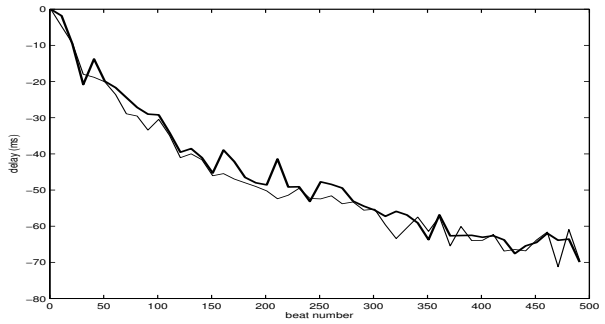

Figure 2. Estimated delays of the healthy subject using time domain (thin line) and inverse normalized integrals (thick line)

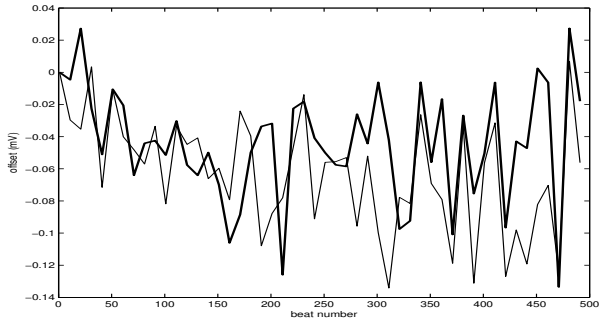

Figure 3. Estimated offset of the healthy subject using time domain (thin line) and inverse normalized integrals (thick line)

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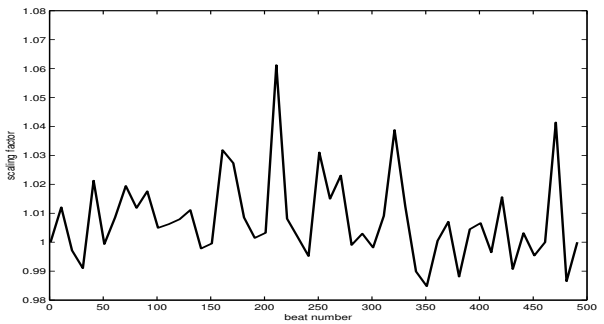

Figure 4. Estimated scaling factor of the healthy subject using inverse normalized integrals

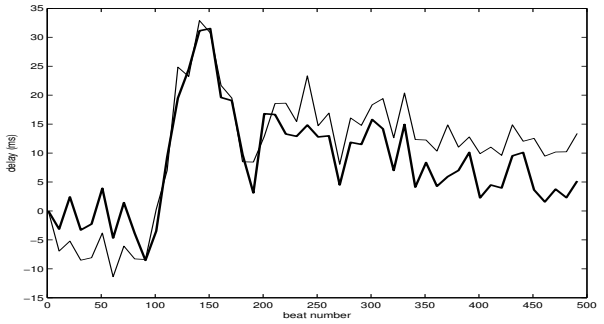

Figure 5. Estimated delays of the ischemic subject using time domain (thin line) and inverse normalized integrals (thick line)

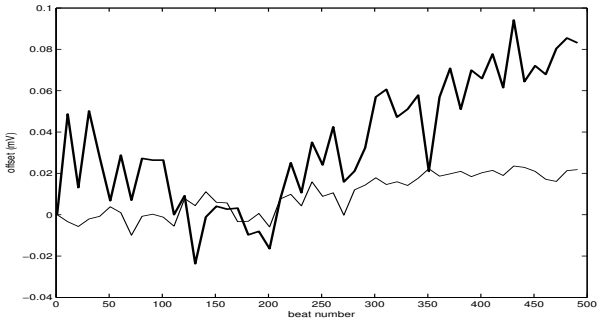

Figure 6. Estimated offset of the ischemic subject using time domain (thin line) and inverse normalized integrals (thick line)

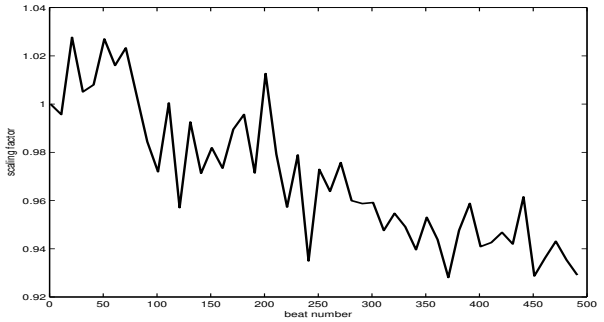

Figure 7. Estimated scaling factor of the ischemic subject using inverse normalized integrals