33rd Annual International Conference of the IEEE EMBS
Boston, Massachusetts USA, August 30 - September 3, 2011

Symmetrical Modified Dual Tree Complex Wavelet Transform for Processing Quadrature Doppler Ultrasound Signals

G. Serbes, and N. Aydin, *Member, IEEE*

Abstract—**Dual-tree complex wavelet transform (DTCWT), which is a shift invariant transform with limited redundancy, is an improved version of discrete wavelet transform. Complex quadrature signals are dual channel signals obtained from the systems employing quadrature demodulation. An example of such signals is quadrature Doppler signal obtained from blood flow analysis systems. Prior to processing Doppler signals using the DTCWT, directional flow signals must be obtained and then two separate DTCWT applied, increasing the computational complexity. In this study, in order to decrease computational complexity, a symmetrical modified DTCWT algorithm is proposed (SMDTCWT). A comparison between the new transform and the symmetrical phasing-filter technique is presented. Additionally denoising performance of SMDTCWT is compared with the DWT and the DTCWT using simulated signals. The results show that the proposed method gives the same output as the symmetrical phasing-filter method, the computational complexity for processing quadrature signals using DTCWT is greatly reduced and finally the SMDTCWT based denoising outperforms conventional DWT with same computational complexity.**

I. Introduction

Many measurement systems such as magnetic resonance and Doppler ultrasound systems employ quadrature demodulation techniques at the detection stage. In Doppler ultrasound systems used in blood flow analysis, the incoming signal from an ultrasonic transducer is multiplied by the transmitted radio frequency signal and 90 degree phase-shifted version of the transmitted signal [1]. After low pass filtering, in-phase and quadrature phase components of the audio Doppler signal are obtained. Flow direction is encoded in the phase relationship between in-phase and quadrature phase channels. A number of methods exist for extracting directional information from the quadrature Doppler signals [2, 3]. Fast Fourier transform (FFT) mapping the directional information in the frequency domain is widely used for the analysis of Doppler signals [2]. Similarly, a complex continuous wavelet transform algorithm mapping the directional information in the scale domain was introduced in [4].

Manuscript received June 20, 2011.
G. Serbes is with the Mechatronics Engineering Department, Bahcesehir University, Istanbul, Turkey (e-mail: gorkem.serbes@bahcesehir.edu.tr).
N. Aydin is with the Computer Engineering Department, Yildiz Technical University, Istanbul, Turkey (mail: naydin@ieee.org).

In the case of the discrete wavelet transform (DWT), which is becoming a popular tool for analysis of nonstationary biological signals, an algorithm mapping directional signals in the scale domain during analysis does not exist. Moreover, an important drawback of the DWT is that the distribution of energy between coefficients at different scales is very sensitive to shifts in the input data. As a solution to this problem, a complex DWT algorithm called dual tree complex discrete wavelet transform (DTCWT) was proposed in [5]. However, it does not provide directional signal decoding during analysis. In [6], a modified dual tree complex discrete wavelet transform (MDTCWT) capable of mapping directional signals at the transform output was presented. In this study a symmetrical version of MDTCWT (SMDTCWT) will be presented and denoising performance of SMDTCWT will be compared with the DWT and the DTCWT using simulated signals.

II. Complex Quadrature Doppler Signals

Complex quadrature Doppler signals are obtained at the detection stage of the Doppler ultrasound systems employing quadrature demodulation technique. Output of most commercial Doppler ultrasound systems is in quadrature format. Quadrature Doppler signals are dual channel signals. A quadrature Doppler signal can be assumed as a complex signal, in which the real and imaginary parts can be represented as the Hilbert transform of each other. Mathematically, a discrete quadrature Doppler signal can be modeled as

$$y(n) = D(n) + jQ(n) \tag{1}$$

where $D(n)$ is in-phase and $Q(n)$ is quadrature-phase components of the signal. $D(n)$ and $Q(n)$ can also be represented in terms of the directional signals as

$$D(n) = \pm s_f(n) \pm \mathrm{H}[s_r(n)] \tag{2}$$

$$Q(n) = \pm \mathrm{H}[s_f(n)] \pm s_r(n) \tag{3}$$

where $s_f(n)$ and $s_r(n)$ represent forward and reverse signals respectively and H[ ] stands for the Hilbert transform. The information concerning flow direction is encoded in the phase relationship between $D(n)$ and $Q(n)$. Symmetrical phasing filter technique (SPFT) is a widely used method for

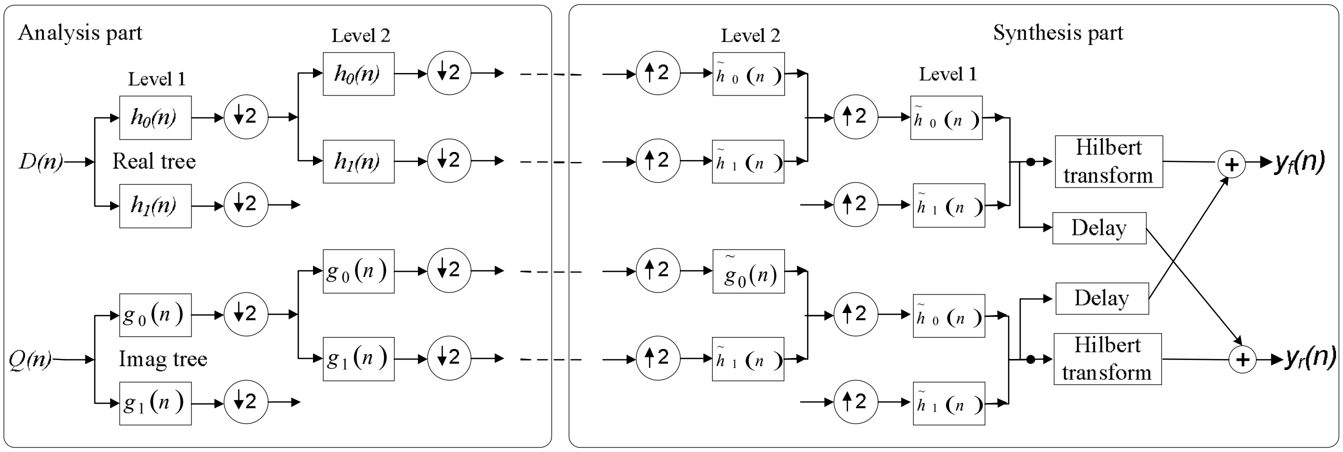

Fig. 1. Analysis and reconstruction stages of the SMDTCWT algorithm for two levels.

extracting directional signals from the quadrature signals. In the SPFT, two Hilbert transforms are applied to both $D(n)$ and $Q(n)$ resulting the directional signal outputs symmetrically [7]. Therefore in this work, the reconstructed directional outputs of the SMDTCWT are compared with the outputs of the SPFT.

III. METHOD

The DTCWT was developed to overcome the lack of shift invariance property of ordinary DWT. Also it has limited redundancy (2^m:1 for m dimensional signals, which is a very good ratio as compared with undecimated DWT). In the analysis of non-stationary Doppler signals (particularly embolic Doppler ultrasound signals which are similar to transients), any distortion in the phase of the signal cannot be tolerated as the direction of the flow information is encoded in the phase relationship of the in-phase and quadrature-phase components of the quadrature signal.

Conventionally, prior to applying the DTCWT to the quadrature Doppler signals, first it must be decoded into the directional signals and then two DTCWT algorithms should be applied to each signals. However, by combining SPFT and DTCWT, a modified transform with reduced computational complexity compared to conventional algorithm can be achieved. This is attained by combining the part of the SPFT with a modified DTCWT algorithm as illustrated in the Figure 1.

Conventional DTCWT consists of a pair of DWT trees, each representing real and imaginary parts of the transform. In both DWTs all the filters are real and these two real trees use two different sets of filters. These sets of filters are jointly designed so that the overall transform is approximately analytic. The details of the DTCWT implementation can be found in [5], [8], [9], and [10]. In the conventional DTCWT transform, a real signal is applied to the both trees for decomposition and the outputs of the both reconstructed trees are added at the end of the reconstruction stage.

In the SMDTCWT, two modifications are made to the conventional DTCWT as illustrated in Figure 1. At the analysis stage, instead of applying the complex quadrature signal to the both trees, the in-phase and quadrature-phase parts are applied to the real and imaginary trees seperately. The real and imaginary trees in this transform are the same as the conventional DTCWT. At the reconstruction stage, the outputs of reconstructed real and imaginary trees are applied through Hilbert transformers introducing a 90 degree phase shift into the real and imaginary parts of the signal. Also the reconstructed outputs are applied to delay filters which compansate the time delay introduced by Hilbert transform filters. Finally, in order to obtain blood flow signals, 90 degree phase shifted output of the real tree and time delayed output of imaginary tree is added resulting the blood flow in one direction. 90 degree phase shifted output of the imaginary tree and time delayed output of real tree is added resulting the blood flow in other direction. The result is the same as the conventional SPFT as described in [7], and the mathematical proof of the SMDTCWT would be the same as the SPFT. The described algorithm is the equivalent to first applying the SPFT to the quadrature signal and then taking two conventional DTCWTs, but with reduced computational complexity.

In order to show that the proposed algorithm works as intended, an embolic quadrature Doppler signal recorded from a patient was used [3]. The sampling frequency was 7150 Hz and only 512 points were used. This quadrature signal is illustrated in Figure 2(a). The signal was normalized to 1 and the in-phase and the quadrature-phase components of the signal were offset by 1 and -1 respectively for clarity. First, the forward and reverse signals were obtained by using the SPFT to compare with. Then the same quadrature signal was decomposed to five levels and then reconstructed by using the SMDTCWT resulting in the forward and the reverse signals.

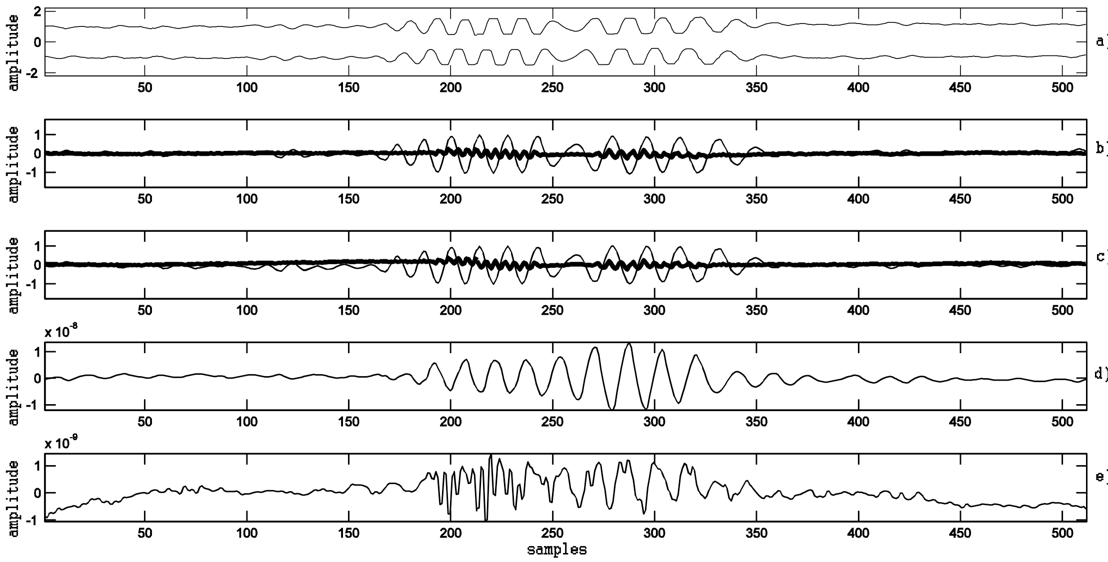

Fig. 2. (a) A quadrature embolic Doppler signal, (b) the forward (thin line) and the reverse (thick line) outputs using the SMDTCWT, (c) the forward (thin line) and the reverse (thick line) outputs using the SPFT, and corresponding differences of (d) the forward and (e) the reverse signals obtained by the SMDTCWT and the SPFT

In order to compare both results statistically, the percent root mean square difference (PRD) formula for both forward and reverse signals is used.

$$PRD = \frac{\sqrt{\sum (x_i - x_j)^2}}{\sqrt{\sum x_j^2}} \times 100 \quad (4)$$

where x_j is the resulting directional signal obtained by the SPFT and x_i is the resulting directional signal obtained by the SMDTCWT.

The computational complexity of the algorithm was also compared with the SPFT followed by two real DWTs, and the SPFT followed by two DTCWTs on a PC with Intel Dore Duo 2.26 GHz processor and 4 GB RAM. The algorithms were implemented in Matlab and tested using a quadrature Doppler signal having 512 samples. In order to minimize effect of any computational time used by any program, which might be running at the background, each algorithm was run 10000 times and average execution time of the algorithms were calculated.

Denoising performance of SMDTCWT was also evaluated and compared with DWT and DTCWT by using simulated signals in complex quadrature format which was constructed by using sinusoidal signals contaminated by a random synthetic Gaussian noise. For DWT and DTCWT denoising case, firstly directional signals are obtained using SPFT and 5 levels wavelet decomposition is applied to these signals. Afterwards, soft-thresholding method was used for removing noise and processed subbands were reconstructed. The outputs of the three methods were subtracted from the noise-free output of the SPFT algorithm, and the differences were computed as RMS error (RMSE). RMSE is given as

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(X_i - \hat{X}_i\right)^2} \quad (5)$$

where $\hat{X}_i$ is the resulting denoised directional signal of the related method and X_i is the original directional signal without noise.

Complex simulated quadrature signal containing the forward and reverse signal components was created by using the following equations, where A and B are the signal amplitudes, f_A and f_B are the frequency of the directional signals, f_S is the sampling frequency and $g(n)$ is the random noise signal.

$$D(n) = \text{A}\cos(2*\text{pi}*n*f_A/f_S) + \text{B}\sin(2*\text{pi}*n*f_B/f_S) + g(n) \quad (6)$$

$$Q(n) = \text{A}\sin(2*\text{pi}*n*f_A/f_S) + \text{B}\cos(2*\text{pi}*n*f_B/f_S) + g(n) \quad (7)$$

For the simulated signals used in this study, f_B/f_S ratio is chosen as 0.005 and noise amplitude is chosen as 0.2

IV. Results and Conclusion

The signals representing forward (thin line) and reverse (thick line) flow components of the embolic Doppler signal, which are obtained by using the SMDTCWT and the SPFT are shown in Figures 2(b) and 2(c) respectively. The error signals obtained by subtracting the signals in Figure 2(c) from the signals in Figure 2(b) are illustrated in Figures 2(d) and 2(e) respectively. It is remarkable that the difference signals for both forward and reverse flow signals are around -80 dB, indicating that the algorithm works as exactly intended. The PRDs for the reverse flow signals

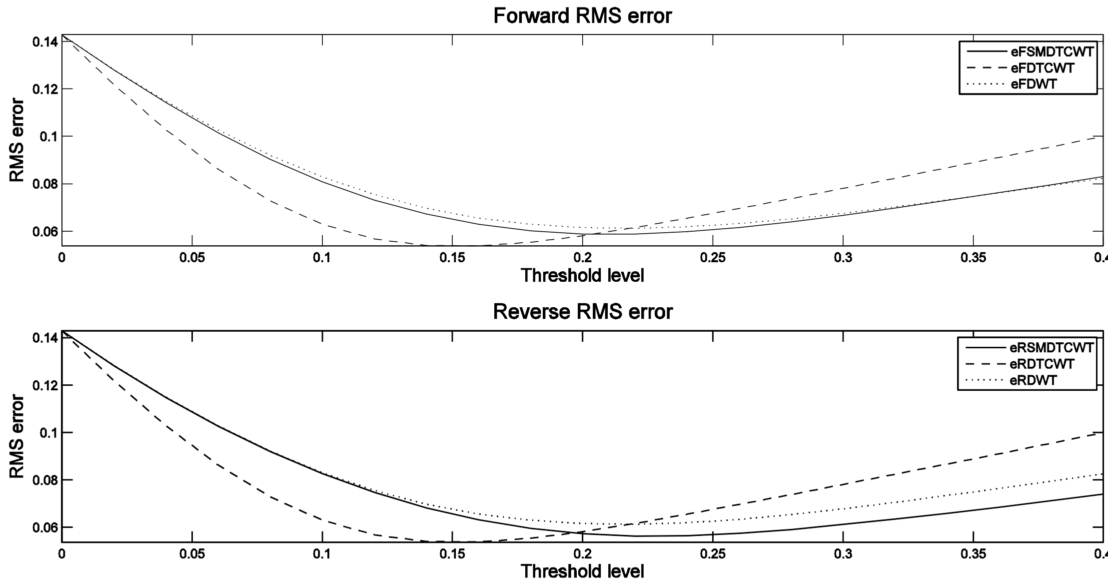

Fig. 3 RMSE values of SMDTCWT, DWT and DTCWT denoising. (eRSMDTCWT means "denoised with SMDTCWT", eRDTCWT means denoised with DTCWT, eRDWT means denoised with DWT)

(9.1487×10^{-7}) and for the forward flow signals (8.1651×10^{-7}) are extremely small and negligible. Therefore the outputs of the both algorithms can be assumed the same. It is obvious that these results are in good correlation with the qualitative results shown in the Figure 2.

When the processing times indicating the computational complexities of the three methods (the SPFT followed with two DWT, the SPFT followed with two DTCWT, and the SMDTCWT) are examined computational cost of the proposed algorithm (16.4 ms) is almost same as the SPFT algorithm followed by two DWTs (16.1 ms) and half of the SPFT algorithm followed by two DTCWTs (32.1 ms).

Finally, the Figure 3 shows the RMSE values for the SMDTCWT, DWT and DTCWT denoising algorithms for different threshold levels. Considering the denoising performance of SMDTCWT, as can be seen from Figure 3, the SMDTCWT has better overall denoising performance in both directions than the conventional DWT.

In conclusion, the SMDTCWT algorithm has less computational cost than conventional DWT, inherently offers advantages provided by the conventional DTCWT, and additionally maps directional signals at the end of the reconstruction stage. It also has better denoising performance than conventional DWT for various threshold levels. In the future, it may be possible to design new complex wavelet filters that will have properties similar to that of a Hilbert transformer for further reducing the computational complexity.

REFERENCES

[1] D. H. Evans, W. N. McDicken, R. Skidmore and J.P. Woodcock, *Doppler ultrasound: Physics, Instrumentation and Clinical Applications*. John Wiley, Chichester, 1989.

[2] N. Aydin, L. Fan and D. H. Evans, "Quadrature-to-directional format conversion of Doppler signals using digital methods", *Physiol. Meas*., vol. 15., pp. 181-199, 1994.

[3] N. Aydin, S. Padayachee and H. S. Markus, "The use of wavelet transform to describe embolic signals", *Ultrasound Med Biol*., vol 25, 6, pp. 953-958, 1999.

[4] N. Aydin and H. S. Markus, "Directional wavelet transform in the context of complex quadrature Doppler signals", *IEEE Signal Processing Letters*, vol. 10, no. 7, pp. 278-280, 2000.

[5] N. G. Kingsbury, "Complex wavelets for shift invariant analysis and filtering of signals", *Journal of Applied and Computational Harmonic Analysis*, vol 10, no 3, pp. 234-253, 2001.

[6] G. Serbes and N. Aydin, "A complex discrete wavelet transform for processing quadrature Doppler ultrasound signals", *9th International Conference on Information Technology and Applications in Biomedicine*, Larnaka, 2009

[7] Aydin N, " Computerised graft monitoring". Ph.D.dissertation, Dept. Med. Phys., *University of Leicester*, Leicester, UK, 1994

[8] I. W. Selesnick, R G Baraniuk, and N. G. Kingsbury: "The dual-tree complex wavelet transform", *IEEE Signal Processing Magazine*, vol. 22, no. 6, pp. 123-151, 2005.

[9] N. G. Kingsbury, "The dual-tree complex wavelet transform: a new technique for shift invariance and directional filters", *IEEE Digital Signal Processing Workshop, SP 98*, Bryce Canyon, paper no 86, 1998.

[10] http://taco.poly.edu/WaveletSoftware/